n次行列式の定義と性質

A-1 [多重線形性]

$$\left.\begin{array}{l}\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{a}_j\ \mathbf{a}_{j+1} \cdots \mathbf{a}_n], \quad \mathbf{B} = [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{b}_j\ \mathbf{a}_{j+1} \cdots \mathbf{a}_n] \\ \mathbf{C} = [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \alpha\mathbf{a}_j + \beta\mathbf{b}_j\ \mathbf{a}_{j+1} \cdots \mathbf{a}_n]\end{array}\right\}$$

$$\Longrightarrow\ D_n(\mathbf{C}) = \alpha D_n(\mathbf{A}) + \beta D_n(\mathbf{B})$$

A-2 [退化条件]

$$\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\mathbf{a}_j\ \mathbf{a}_{j+1} \cdots \mathbf{a}_{k-1}\ \mathbf{a}_j\ \mathbf{a}_{k+1} \cdots \mathbf{a}_n]$$

$$\Longrightarrow\ D_n(\mathbf{A}) = 0$$

A-3 [正規化条件]

$$D_n(\mathbf{E}) = 1$$

n次行列式の定義と性質

P-1 $\mathbf{A} = [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{O}\ \mathbf{a}_{j+1} \cdots \mathbf{a}_n]$

$$\Longrightarrow\ D_n(\mathbf{A}) = 0$$

P-2 [交代性]

$$\left.\begin{aligned} \mathbf{A} &= [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{a}_j\ \mathbf{a}_{j+1} \cdots \mathbf{a}_{k-1}\ \mathbf{a}_k\ \mathbf{a}_{k+1} \cdots \mathbf{a}_n] \\ \mathbf{B} &= [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{a}_k\ \mathbf{a}_{j+1} \cdots \mathbf{a}_{k-1}\ \mathbf{a}_j\ \mathbf{a}_{k+1} \cdots \mathbf{a}_n] \end{aligned}\right\}$$

$$\Longrightarrow\ D_n(\mathbf{A}) = -D_n(\mathbf{B})$$

P-3

$$\left.\begin{aligned} \mathbf{A} &= [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{a}_j\ \mathbf{a}_{j+1} \cdots \mathbf{a}_{k-1}\ \mathbf{a}_k\ \mathbf{a}_{k+1} \cdots \mathbf{a}_n] \\ \mathbf{B} &= [\mathbf{a}_1 \cdots \mathbf{a}_{j-1}\ \mathbf{a}_j + \alpha\mathbf{a}_k\ \mathbf{a}_{j+1} \cdots \mathbf{a}_{k-1}\ \mathbf{a}_k\ \mathbf{a}_{k+1} \cdots \mathbf{a}_n] \end{aligned}\right\}$$

$$\Longrightarrow\ D_n(\mathbf{A}) = D_n(\mathbf{B})$$