PAPER

Integer Discrete Cosine Transform via Lossless Walsh-Hadamard Transform with Structural Regularity for Low-Bit-Word-Length

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SUMMARY This paper presents an integer discrete cosine transform (IntDCT) with only dyadic values such as $k/2^n$ (k, $n \in \mathbb{N}$). Although some conventional IntDCTs have been proposed, they are not suitable for lossless-to-lossy image coding in low-bit-word-length (coefficients) due to the degradation of the frequency decomposition performance in the system. First, the proposed M-channel lossless Walsh-Hadamard transform (LWHT) can be constructed by only ($\log_2 M$)-bit-word-length and has structural regularity. Then, our 8-channel IntDCT via LWHT keeps good coding performance even if low-bit-word-length is used because LWHT, which is main part of IntDCT, can be implemented by only 3-bit-word-length. Finally, the validity of our method is proved by showing the results of lossless-to-lossy image coding in low-bit-word-length.

key words: *integer discrete cosine transform (IntDCT), dyadic values, low-bit-word-length, lossless Walsh-Hadamard transform (LWHT), lossless-to-lossy image coding*

1. Introduction

Multimedia coding (compression) standards such as JPEG [1] and MPEG [2] use the discrete cosine transform (DCT) [3] which has excellent properties, several types and many fast algorithms [4]–[7]. In JPEG, DCT type-II (DCT-II) and -III (DCT-III), which is the inverse transform of DCT-II, are applied to lossy image coding, and the differential pulse code modulation (DPCM) is applied to lossless image coding. We have to prepare lossy and lossless compressed data separately according to application.

Recently, high speed transmission of high quality data has become very desirable due to the advances in Internet and multimedia contents. Several integer DCTs (IntDCTs), which are constructed by lifting structures [8] and rounding operations based on DCT-II, have been proposed for lossless-to-lossy image coding [9]–[14]. For example, [9] and [10] have presented the 8-channel IntDCTs composed of the integer rotation transforms (IRTs), the 4-channel integer DCT type-IV (IntDCT-IV) and the 4-channel integer Hadamard transforms (IHTs) etc. But, they require both floating-point coefficients and rounding operations. Then, [11], [12] and [13] have presented the low cost 8-channel IntDCTs which are easy to implement because they do not require floating-point coefficients. However, [11] and [12] can not be used for lossless image coding due to non-consideration of dynamic range and [13] is not suitable for lossless image coding in low-bit-word-length (coefficients) due to its construction. Although [14] is not only inexpensive in terms of hardware complexity but also operates well in lossless-to-lossy image coding due to the optimum-word-length-assignment (OWLA) of coefficients, it requires a higher-bit-word-length in some lifting structures and yields DC leakage.

In image coding, IntDCT should not have huge dynamic range of the output signals and DC leakage. Huge dynamic range requires many bits and DC leakage generates a checker-board artifact in low bit rate in lossy image coding. On the other hand, for high-speed implementation, it is required to approximate floating-point lifting coefficients to hardware-friendly dyadic values such as $k/2^n$ (k, $n \in \mathbb{N}$) which can be implemented by only bit-shift and addition operations. If lower-bit-word-length is allocated to every lifting coefficients, it is ideal for real hardware. However, the conventional IntDCTs generate a checker-board artifact in low-bit-word-length.

In this paper, we propose an IntDCT via LWHT with structural regularity even if low-bit-word-length is used. First, IntDCT is separated to Walsh-Hadamard transform (WHT) as the pre-processing part and the others as the post-processing part [12]. Second, we present an M-channel lossless WHT (LWHT) which can be constructed by only ($\log_2 M$)-bit-word-length. It is achieved by using the scaling shift and considering the two-dimensional (2-D) separable transform of one-dimensional (1-D) WHT. The proposed IntDCT has regularity which is imposed structurally by LWHT even if low-bit-word-length is used. As result, we can achieve an IntDCT with low cost and fast implementation. Finally, the validity is shown by the comparison of our methods and the conventional methods in coding gain, frequency response and the results of lossless-to-lossy image coding.

Notations: $\mathbf{I}$ is an identity matrix, $\mathbf{M}^T$ is a transpose of matrix $\mathbf{M}$, and $\mathbf{M}^{[N]}$ is an $N \times N$ square matrix, respectively.

2. Review

2.1 Multiplierless Lifting Structure

The lifting structure [8], also known as the ladder structure, is a special type of lattice structure, a cascading construction using only elementary matrices - identity matrices with single nonzero off-diagonal element.

Figure 1 shows a basic lifting structure. In this case,

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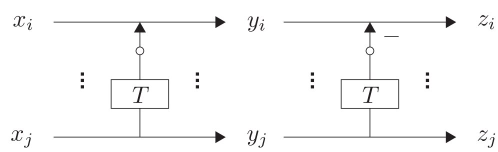

Fig. 1 A lifting structure (white circles: rounding operations).

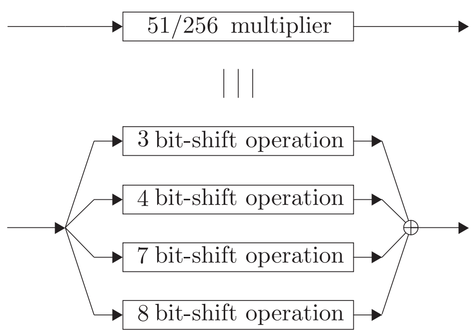

Fig. 2 An approximation from floating multiplication to bit-shift and addition operations.

the lifting matrix and its inverse matrix are as follows:

$$\begin{bmatrix}1 & T\\0 & 1\end{bmatrix}, \quad \begin{bmatrix}1 & T\\0 & 1\end{bmatrix}^{-1} = \begin{bmatrix}1 & -T\\0 & 1\end{bmatrix}.$$

Then, they are represented by

$$\begin{array}{l} y_i = x_i + round[Tx_j] \\ y_j = x_j \end{array} \rightarrow \begin{array}{l} z_i = y_i - round[Ty_j] = x_i \\ z_j = y_j = x_j \end{array}$$

where T is lifting coefficient and *round*[.] is rounding operation. Thus the lifting structure with rounding operation can achieve lossless-to-lossy coding.

For high-speed implementation, lifting coefficients are required to approximate floating-point to hardware-friendly dyadic values such as $k/2^n$ (k, $n \in \mathbb{N}$) which can be implemented by only bit-shift and addition operations. It performs fast implementation in a real time software encoder and reduce the circuit size. The multiplications are expressed by $k/2^n$ ($k, n \in \mathbb{N}$) which is n-bit-word-length. For example, a coefficient $51/2^8 = 51/256$ can be operated as

$$\begin{aligned}\frac{51}{256} &= \frac{32}{256} + \frac{16}{256} + \frac{2}{256} + \frac{1}{256}\\ &= \frac{1}{8} + \frac{1}{16} + \frac{1}{128} + \frac{1}{256}\\ &= \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^8}.\end{aligned} \tag{1}$$

From (1), we can replace the multiplier by 51/256 to sum results from 3 bit-shift, 4 bit-shift, 7 bit-shift, and 8 bit-shift operations as illustrated in Fig. 2. We can find that the perfect reconstruction in lifting structure is always kept even if lifting coefficients are approximated.

2.2 DCT and IntDCT

In this paper, only type-II and -III in DCT are described. DCT-II and -III are often used for image and video coding. The m-column and n-row element of M-channel DCT-II and -III matrix $\mathbf{C}_{II}^{[M]}$ and $\mathbf{C}_{III}^{[M]}$ are defined as

$$\left[\mathbf{C}_{II}^{[M]}\right]_{m,n} = \sqrt{\frac{2}{M}} c_m \cos\left(\frac{m\left(n+\frac{1}{2}\right)\pi}{M}\right)$$

$$\left[\mathbf{C}_{III}^{[M]}\right]_{m,n} = \sqrt{\frac{2}{M}} c_n \cos\left(\frac{\left(m+\frac{1}{2}\right)n\pi}{M}\right)$$

where $0 \le m, n \le M-1$ and $c_i = 1/\sqrt{2}$ ($i = 0$) or 1 ($i \ne 0$). For simplicity, let us define that $M = 2^n$ ($n \in \mathbb{N}$) and DCT means DCT-II.

Also, DCT can be constructed by cascade connection of Givens rotation matrices [4]–[7]. Givens rotation matrix $\mathbf{R}_\theta$ is expressed by

$$\mathbf{R}_\theta = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}.$$

In the past, many IntDCTs have been proposed for lossless-to-lossy image coding [9]–[14]. They are based on lifting structures and rounding operations. The lifting structure of Givens rotation matrix $\mathbf{R}_\theta$ is represented by

$$\mathbf{R}_\theta = \begin{bmatrix}1 & \frac{\cos\theta-1}{\sin\theta}\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \sin\theta & 1\end{bmatrix}\begin{bmatrix}1 & \frac{\cos\theta-1}{\sin\theta}\\ 0 & 1\end{bmatrix}. \tag{2}$$

2.3 WHT

As well known, WHT has interesting relationships with other discrete transforms such as DCT [3] etc. It is expressed by

$$\mathbf{W}^{[1]} = \begin{bmatrix}1\end{bmatrix}, \quad \mathbf{W}^{[2]} = \begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$\mathbf{W}^{[M]} = \mathbf{W}^{[2]} \otimes \mathbf{W}^{[\frac{M}{2}]} = \begin{bmatrix}\mathbf{W}^{[\frac{M}{2}]} & \mathbf{W}^{[\frac{M}{2}]}\\ \mathbf{W}^{[\frac{M}{2}]} & -\mathbf{W}^{[\frac{M}{2}]}\end{bmatrix}$$

where $\otimes$ is Kronecker product. The 8-channel WHT kernel arranged in Walsh order is

$$\mathbf{W}^{[8]} = \begin{bmatrix}1&1&1&1&1&1&1&1\\ 1&1&1&1&-1&-1&-1&-1\\ 1&1&-1&-1&-1&-1&1&1\\ 1&1&-1&-1&1&1&-1&-1\\ 1&-1&-1&1&1&-1&-1&1\\ 1&-1&-1&1&-1&1&1&-1\\ 1&-1&1&-1&-1&1&-1&1\\ 1&-1&1&-1&1&-1&1&-1\end{bmatrix}. \tag{3}$$

Note that (3) is still not normalized by the scaling factor $1/\sqrt{M}$.

2.4 IntDCT via WHT

Let us normalize $\mathbf{W}$ such that $\hat{\mathbf{W}} = (1/\sqrt{M})\mathbf{W}$ is an orthonormal matrix: $\hat{\mathbf{W}}^{-1} = \hat{\mathbf{W}}^T$. Then one can prove that

[3]

$$\mathbf{C}_{II}^{[M]} = \hat{\mathbf{Q}}^{[M]}\hat{\mathbf{W}}^{[M]} = \mathbf{P}^{[M]^T}\mathbf{Q}^{[M]}\mathbf{P}^{[M]}\hat{\mathbf{W}}^{[M]} \quad (4)$$

where $\hat{\mathbf{W}}^{[M]}$ and $\hat{\mathbf{Q}}^{[M]}$ are the pre- and post-processing part, respectively. In case of $M = 8$,

$$\mathbf{P}^{[8]} = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

and

$$\mathbf{Q}^{[8]} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}^{[2]} & \mathbf{0}^{[2]} \\ \mathbf{0}^{[2]} & \mathbf{R}_{(-\frac{\pi}{8})} \end{bmatrix} & \mathbf{0}^{[4]} \\ \mathbf{0}^{[4]} & \mathbf{S}^{[4]} \end{bmatrix}$$

where $\mathbf{R}_{(-\pi/8)}$ is a Givens rotation matrix and $\mathbf{S}^{[4]}$ is 4×4 orthogonal matrix which can be further factorized using only 4 angles as

$$\mathbf{S}^{[4]} = \begin{bmatrix} \cos\left(\frac{7\pi}{16}\right) & 0 & 0 & -\sin\left(\frac{7\pi}{16}\right) \\ 0 & \cos\left(\frac{3\pi}{16}\right) & -\sin\left(\frac{3\pi}{16}\right) & 0 \\ 0 & \sin\left(\frac{3\pi}{16}\right) & \cos\left(\frac{3\pi}{16}\right) & 0 \\ \sin\left(\frac{7\pi}{16}\right) & 0 & 0 & \cos\left(\frac{7\pi}{16}\right) \end{bmatrix}$$

$$\times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} \cos\left(\frac{3\pi}{8}\right) & 0 & -\sin\left(\frac{3\pi}{8}\right) & 0 \\ 0 & \cos\left(\frac{3\pi}{8}\right) & 0 & -\sin\left(\frac{3\pi}{8}\right) \\ \sin\left(\frac{3\pi}{8}\right) & 0 & \cos\left(\frac{3\pi}{8}\right) & 0 \\ 0 & \sin\left(\frac{3\pi}{8}\right) & 0 & \cos\left(\frac{3\pi}{8}\right) \end{bmatrix}$$

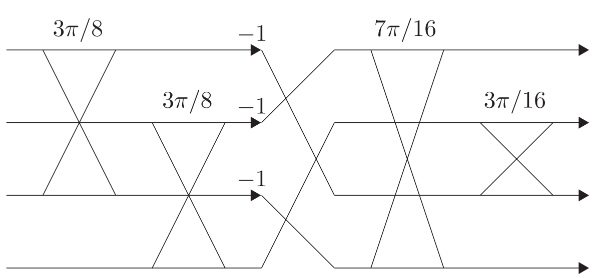

Fig. 3 Efficient decomposition of $\mathbf{S}^{[4]}$ consisting of 4 rotation angles.

and shown in Fig. 3. To obtain IntDCT via WHT, $\mathbf{W}^{[8]}$ in (3) instead of $\hat{\mathbf{W}}^{[8]}$ and the lifting factorization of Givens rotation matrix in (2) are used [12]. In this paper, all transform coefficients α_0-α_9 and β_0-β_4 shown in Fig. 4 and Table 1 are rounded into the n-bit-word-length, $k/2^n$ ($k, n \in \mathbb{N}$ and $3 \le n \le 6$).

3. IntDCT via LWHT with Structural Regularity for Low-Bit-Word-Length

The desired conditions ♯1 and ♯2 for IntDCT in lossless-to-lossy image coding are as follows:

Table 1 Lifting coefficients of Chen's IntDCT [12].

	floating-point [12]	rounded dyadic values			
		$k/2^3$	$k/2^4$	$k/2^5$	$k/2^6$
α_0	0.1989123674	1/4	3/16	3/16	13/64
β_0	−0.3826834324	−3/8	−3/8	−3/8	−3/8
α_1	0.1989123674	1/4	3/16	3/16	13/64
α_2	−0.6681786379	−5/8	−11/16	−21/32	−43/64
β_1	0.9238795326	7/8	15/16	15/16	59/64
α_3	−0.6681786379	−5/8	−11/16	−21/32	−43/64
α_4	−0.6681786379	−5/8	−11/16	−21/32	−43/64
β_2	0.9238795326	7/8	15/16	15/16	59/64
α_5	−0.6681786379	−5/8	−11/16	−21/32	−43/64
α_6	−0.8206787908	−7/8	−13/16	−13/16	−53/64
β_3	0.9807852804	1	1	31/32	63/64
α_7	−0.8206787908	−7/8	−13/16	−13/16	−53/64
α_8	−0.3033466836	−1/4	−5/16	−5/16	−19/64
β_4	0.5555702330	1/2	9/16	9/16	9/16
α_9	−0.3033466836	−1/4	−5/16	−5/16	−19/64

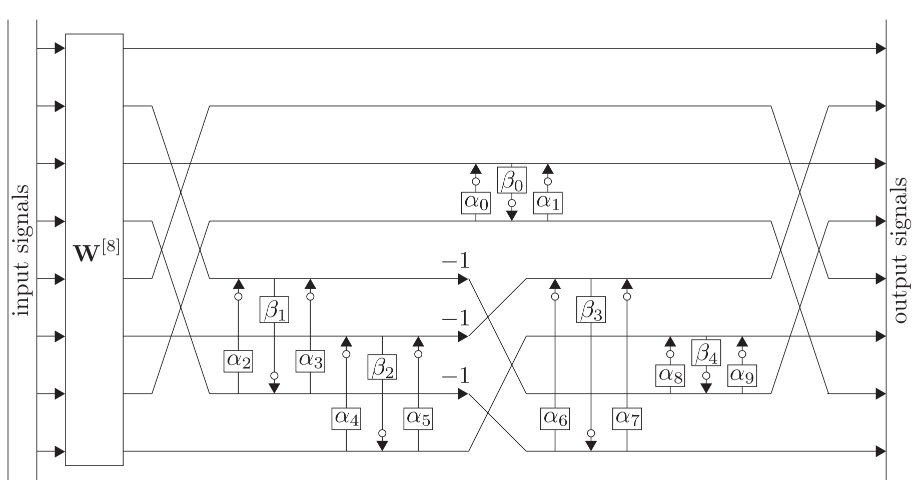

Fig. 4 Chen's IntDCT [12] (white circles: rounding operations).

♯1: The transform should not have DC leakage.
→ If DC leakage is generated, the signal energy decomposed to frequency domain is not sufficiently concentrated. And if so, high compression ratio can not be achieved.
⇒ Regularity should be imposed.

♯2: Dynamic range of the output signals should be as small as possible.
→ If dynamic range is huge, to avoid bit redundancy, quantization part would be required after transform part. And if so, realization of lossless image coding is impossible due to its error.
⇒ The transform should be normalized without using quantization part.

In low-bit-word-length, although most of the conventional IntDCTs do not have regularity, one in [12] satisfies the condition ♯1. However, it does not satisfies the condition ♯2 because the coefficients of the pre-processing part $\mathbf{W}^{[8]}$ are not normalized. Hence this paper solves the normalization problem of WHT: $\hat{\mathbf{W}}^{[8]} = (1/\sqrt{8})\mathbf{W}^{[8]}$, in IntDCT in [12] while keeping the other part as it is.

3.1 Two-Dimensional (2-D) Block Transform

When we apply a block transform matrix $\mathbf{F}$ into an 2-D input signal $\mathbf{x}$ in column- and row-wise separately, the 2-D output signal $\mathbf{y}$ is expressed by

$$\mathbf{y} = (\mathbf{F}(\mathbf{F}\mathbf{x})^T)^T = \mathbf{F}\mathbf{x}\mathbf{F}^T. \tag{5}$$

We call it 2-D separable block transform. If the transform $\mathbf{F}$ is factorized as $\mathbf{F} = \mathbf{F}_1\mathbf{F}_0$ where $\mathbf{F}_0$ and $\mathbf{F}_1$ are the pre- and post-processing part, (5) is represented by

$$\mathbf{y} = \mathbf{F}_1\mathbf{F}_0\mathbf{x}(\mathbf{F}_1\mathbf{F}_0)^T = \mathbf{F}_1\mathbf{F}_0\mathbf{x}\mathbf{F}_0^T\mathbf{F}_1^T.$$

This equation means that 2-D block transform by $\mathbf{F}_1$ is applied after 2-D block transform by $\mathbf{F}_0$. Therefore this process is shown in Fig. 5(b). Then we regard $\mathbf{F}_0$ and $\mathbf{F}_1$ as $\hat{\mathbf{W}}$ and $\hat{\mathbf{Q}}$ in (4), respectively.

3.2 LWHT with Structural Regularity for Low-Bit-Word-Length

In this subsection, we consider the normalization of $\mathbf{W}^{[M]}$ in (4) and present M-channel LWHT which has only ±1 and $\pm1/M$, $(\pm\log_2 M)$-bit-word-length, and structural regularity.

According to the condition ♯2, if $\mathbf{W}$ normalized by the scaling factor $1/\sqrt{M}$ is used in transform part, (5) can be represented by

$$\mathbf{y} = \hat{\mathbf{W}}\mathbf{x}\hat{\mathbf{W}}^T = \left(\frac{1}{\sqrt{M}}\mathbf{W}\left(\frac{1}{\sqrt{M}}\mathbf{W}\mathbf{x}\right)^T\right)^T. \tag{6}$$

In (6), one scaling factor $1/\sqrt{M}$ can be moved to another one as

$$\mathbf{y} = \left(\frac{1}{M}\mathbf{W}(\mathbf{W}\mathbf{x})^T\right)^T \quad \text{or} \quad \mathbf{y} = \left(\mathbf{W}\left(\frac{1}{M}\mathbf{W}\mathbf{x}\right)^T\right)^T. \tag{7}$$

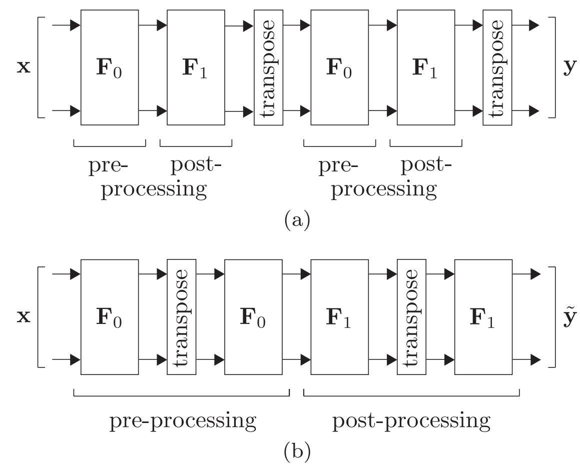

Fig. 5 2-D block transform: (a) standard transform, (b) separated transform to the pre- and post-processing part.

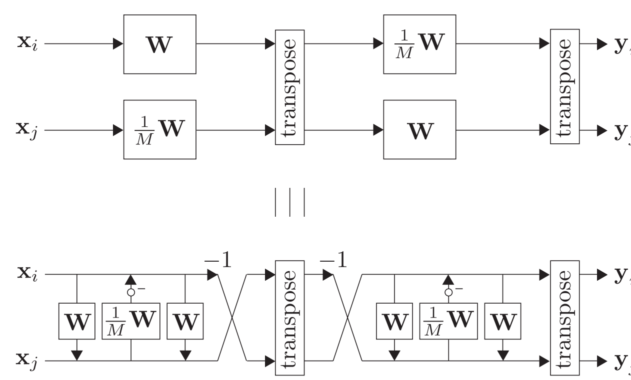

Fig. 6 LWHT with structural regularity for lower-bit-word-length (white circles: rounding operations).

Be careful that (7) can not be applied to lossless image coding as it is because real values generated by the merged scaling factor $1/M$ can not be eliminated. Now, we consider to transform two adjacent blocks simultaneously as

$$\begin{bmatrix}\mathbf{y}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{y}_j\end{bmatrix} = \left(\begin{bmatrix}\frac{1}{M}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}\end{bmatrix}\left(\begin{bmatrix}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \frac{1}{M}\mathbf{W}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{x}_j\end{bmatrix}\right)^T\right)^T \tag{8}$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ are adjacent $M \times M$ blocks in the 2-D image and $\mathbf{y}_i$ and $\mathbf{y}_j$ are the output signals of them. (8) is illustrated as the upper half in Fig. 6. Next, we can obtain a lifting factorization as follows [15]:

$$\begin{bmatrix}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \frac{1}{M}\mathbf{W}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\frac{1}{M}\mathbf{W}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}$$

and

$$\begin{bmatrix}\frac{1}{M}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\frac{1}{M}\mathbf{W}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}$$

where $(1/M)\mathbf{W} = \mathbf{W}^{-1}$. Consequently, the normalized LWHT can be constructed by only ±1 and $\pm1/M$ without losing structural regularity. The proposed LWHT is illustrated as the lower half in Fig. 6. Thus the 2-D separable transform by $\hat{\mathbf{W}}$ can be implemented by efficient lifting structure without any scaling.

3.3 Expansion from LWHT to IntDCT

In this paper, we set $M = 8$ as the most popular block size. First, an original image is separated to upper half and lower half. Second, the 2-D separable block transform of the images is implemented by our 8-channel LWHT such as Fig. 6. This implementation requires only 3-bit-word-length. Next, $\hat{\mathbf{Q}}^{[8]}$ in (4) is implemented. Thus, by using our normalized LWHT for the pre-processing part in [12] which satisfies the condition ♯1 but ♯2, our IntDCT achieves structural regularity and the normalized filter coefficients at the same time in more than 3-bit-word-length. If the floating-points in Table 1 are used and rounding operations are not used, our IntDCT shows the same performance as the original DCT. But, if dyadic values $k/2^n$ ($k, n \in \mathbb{N}$) in Table 1 and rounding operations are used, our IntDCT can achieve lossless-to-lossy image coding by the lower cost implementation.

4. Results

In this paper, the validity of the proposed IntDCTs is shown in lossless-to-lossy image coding by evaluating coding gain [dB], frequency response, lossless bit rate [bpp] in lossless image coding, and PSNR [dB] in lossy image coding. We chose [9]–[11] and [14] as the conventional IntDCTs.

4.1 Coding Gain and Frequency Response

Coding gain is one of the most important factors to be considered in compression applications. A transform with higher coding gain compacts more energy into a fewer number of coefficients. As a result, higher objective performances such as PSNR would be achieved after quantization. Since coding gain of the DCT approximates the optimal KLT closely, it is desired that our IntDCT has similar coding gain to that of the original DCT. The biorthogonal coding gain is defined as [16]

$$\text{Coding gain [dB]} = 10\log_{10}\frac{\sigma_x^2}{\prod_{k=0}^{M-1}\sigma_{x_k}^2 \parallel f_k \parallel^2}$$

where σ_x^2 is the variance of the input signal, $\sigma_{x_k}^2$ is the variance of the k-th subbands and $\parallel f_k \parallel^2$ is the norm of the k-th synthesis filter. Table 2 shows the comparisons of coding gain of the proposed IntDCT and the conventional ones.

Table 2 Comparisons of coding gain of the proposed IntDCT and the conventional IntDCTs.

Lifting coefficients	Coding gain [dB]				
	K's [9]	F's [10]	T's [11]	C's [14]	Prop.
float	8.8259	8.8259	6.2256	8.6960	8.8259
$k/2^3$	7.4420	8.4829	6.1446	8.1177	8.7344
$k/2^4$	8.6498	8.7724	6.2077	8.5560	8.8206
$k/2^5$	8.7685	8.8044	6.2258	8.6830	8.8155
$k/2^6$	8.8086	8.8257	6.2243	8.6774	8.8244

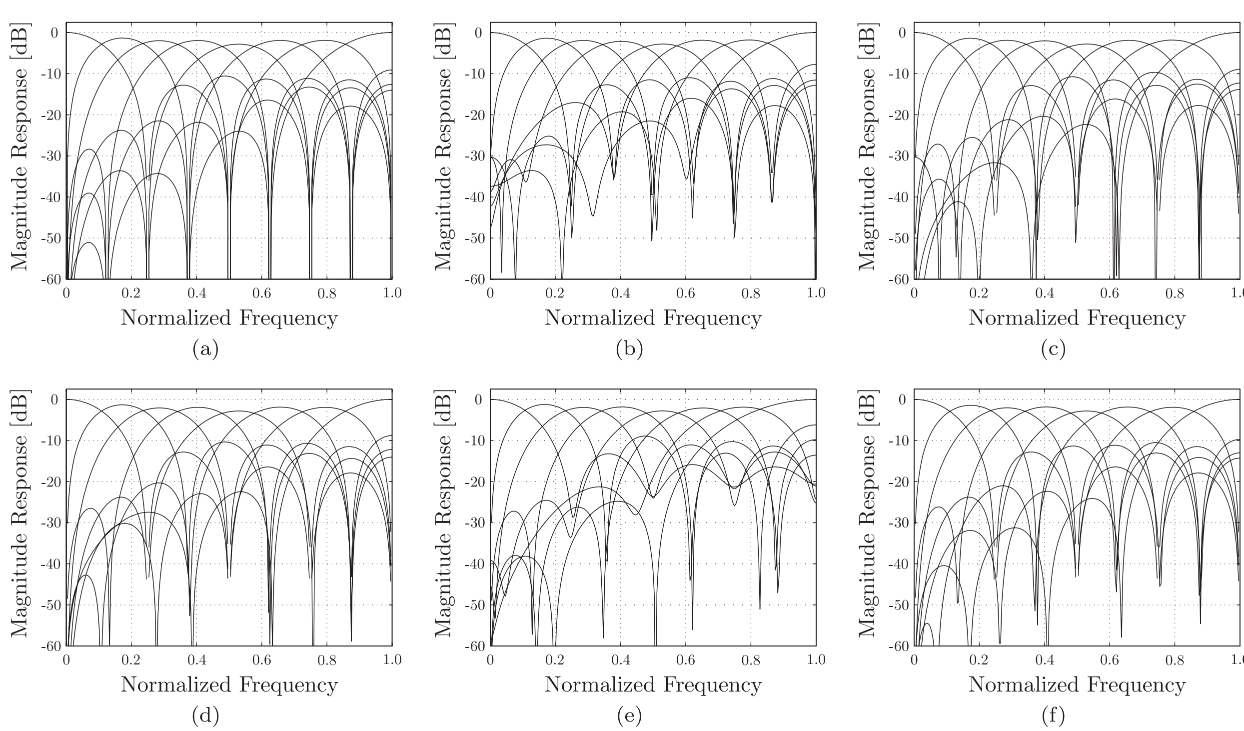

Fig. 7 Frequency responses of the proposed IntDCT and the conventional IntDCTs in 4-bit-word-length: (a) original DCT (infinite-word-length), (b) Komatsu's RDCT [9], (c) Fukuma's LDCT [10], (d) Tran's BinDCT [11], (e) Chokchaitam's IntDCT (OWLA) [14], (f) proposed IntDCT.

Table 3 Comparisons of lossless image coding by the proposed IntDCTs (Lossless bit rate [bpp]).

Image 512×512	Prop. IntDCT $k/2^3$	$k/2^4$	$k/2^5$	$k/2^6$
Baboon	6.29	**6.28**	**6.28**	**6.28**
Barbara	5.07	5.02	5.02	**5.01**
Boat	5.26	**5.22**	**5.22**	**5.22**
Elaine	5.27	**5.25**	5.26	**5.25**
Finger1	6.14	**6.08**	**6.08**	**6.08**
Finger2	5.98	**5.87**	5.89	**5.87**
Goldhill	5.19	**5.18**	**5.18**	**5.18**
Grass	6.19	**6.18**	**6.18**	**6.18**
Lena	4.72	**4.68**	4.69	**4.68**
Pepper	5.02	**4.98**	**4.98**	**4.98**
Avg.	5.51	**5.47**	5.48	**5.47**

Table 4 Comparisons of lossless image coding by the proposed IntDCT and the conventional IntDCTs in 4-bit-word-length (Lossless bit rate [bpp]).

Image 512×512	K's [9]	F's [10]	T's [11]	C's [14]	Prop. IntDCT
Baboon	6.55	6.48	6.86	6.36	**6.28**
Barbara	5.55	5.39	5.45	5.15	**5.02**
Boat	5.79	5.65	5.69	5.34	**5.22**
Elaine	5.87	5.75	5.86	5.43	**5.25**
Finger1	6.24	6.14	6.36	6.10	**6.08**
Finger2	6.15	6.00	6.14	5.93	**5.87**
Goldhill	5.57	5.46	5.66	5.29	**5.18**
Grass	6.30	6.26	6.65	6.23	**6.18**
Lena	5.45	5.26	5.16	4.88	**4.68**
Pepper	5.64	5.48	5.47	5.14	**4.98**
Avg.	5.91	5.79	5.93	5.59	**5.47**

It is clear that coding gains of the conventional IntDCTs are not kept in low-bit-word-length, but that of the proposed IntDCT is almost kept even in 3-bit-word-length.

Next, frequency responses of the proposed IntDCTs and the conventional ones are shown in Fig. 7. In Fig. 7, [9], [10] and [14] have DC leakage which may generate a checker-board artifact in low bit rate. The proposed IntDCT does not have DC leakage, because 8-channel LWHT as the pre-processing part can be constructed with only 3-bit-word-length and regularity can be always kept in more than 3-bit-word-length. Also, although [11] does not have DC leakage, it shows worse coding gain than ours.

4.2 Application to Lossless Image Coding

The proposed IntDCTs are applied to lossless-to-lossy image coding. The set partitioning in hierarchical trees (SPIHT) progressive image transmission algorithm [17] was used to encode the transformed images. The comparisons of

$$\text{Lossless bit rate [bpp]} = \frac{\text{Total number of bits [bit]}}{\text{Total number of pixels [pixel]}}$$

in lossless image coding are shown in Table 3. Table 3 shows the performance at the almost same level all without affecting different bit-word-length in more than 4-bit-word-length. We show also the comparisons of the proposed IntDCT and the conventional ones in 4-bit-word-length in Table 4. The proposed IntDCT showed better coding performance than all of the conventional ones.

Table 5 Comparisons of lossy image coding by the proposed IntDCTs (PSNR [dB]).

Image 512×512	Prop. IntDCT $k/2^3$	$k/2^4$	$k/2^5$	$k/2^6$
bit rate: 0.25 [bpp]				
Baboon	22.54	22.56	**22.57**	22.56
Barbara	26.83	26.92	**26.93**	**26.93**
Goldhill	29.26	29.34	29.32	**29.35**
Lena	31.62	31.80	31.76	**31.81**
Pepper	31.22	31.35	31.35	**31.36**
bit rate: 0.50 [bpp]				
Baboon	24.75	**24.80**	**24.80**	**24.80**
Barbara	30.48	30.62	30.62	**30.65**
Goldhill	31.78	31.87	31.86	**31.90**
Lena	34.98	35.35	35.31	**35.39**
Pepper	34.03	34.28	34.27	**34.31**
bit rate: 1.00 [bpp]				
Baboon	28.16	28.22	28.20	**28.25**
Barbara	35.50	35.78	35.78	**35.81**
Goldhill	34.97	35.12	**35.14**	35.13
Lena	38.10	**38.54**	38.49	**38.54**
Pepper	36.42	36.61	**36.62**	36.61

Table 6 Comparisons of lossy image coding by the proposed IntDCT and the conventional IntDCTs in 4-bit-word-length (PSNR [dB]).

Image 512×512	K's [9]	F's [10]	T's [11]	C's [14]	Prop. IntDCT
bit rate: 0.25 [bpp]					
Baboon	21.60	22.16	20.84	22.14	**22.56**
Barbara	25.42	26.30	23.86	26.43	**26.92**
Goldhill	27.11	28.19	27.41	28.85	**29.34**
Lena	27.28	29.16	29.03	31.32	**31.80**
Pepper	27.33	29.01	28.57	31.02	**31.35**
bit rate: 0.50 [bpp]					
Baboon	23.60	24.19	21.89	24.40	**24.80**
Barbara	27.95	28.95	27.27	29.91	**30.62**
Goldhill	29.57	30.55	29.54	31.38	**31.87**
Lena	29.50	31.57	32.34	34.12	**35.35**
Pepper	29.76	31.59	31.68	33.14	**34.28**
bit rate: 1.00 [bpp]					
Baboon	26.62	27.18	24.63	27.71	**28.22**
Barbara	31.84	32.97	31.25	34.94	**35.78**
Goldhill	32.72	33.77	31.33	34.51	**35.12**
Lena	33.03	35.47	35.70	36.95	**38.54**
Pepper	32.81	34.71	34.43	35.62	**36.61**

4.3 Application to Lossy Image Coding

If lossy compressed data is required, it can be achieved by interrupting the obtained lossless bit stream. The comparisons of

$$\text{PSNR [dB]} = 10\log_{10}\left(\frac{255^2}{\text{MSE}}\right)$$

where MSE is the mean squared error in lossy image coding are shown in Table 5. Similar to Sect. 4.2, we show also the comparisons of the proposed IntDCT and the conventional ones in 4-bit-word-length in Table 6. Furthermore, Fig. 8 shows the enlarged images of 'Lena' which are lossy compressed images by the proposed IntDCT and the conventional ones in 4-bit-word-length when bit rate is 0.25 [bpp].

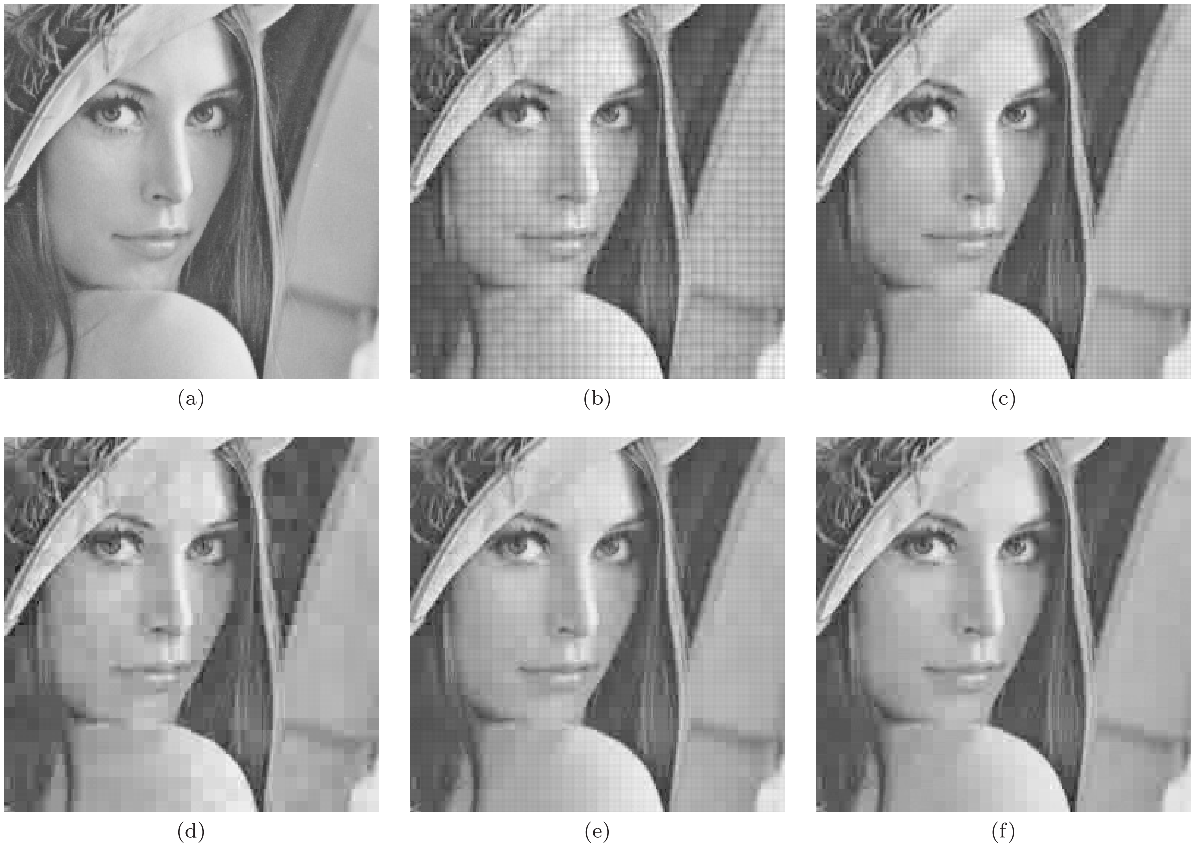

Fig. 8 Enlarged images of 'Lena' in 4-bit-word-length (bit rate: 0.25 [bpp]): (a) original image, (b) Komatsu's RDCT [9], (c) Fukuma's LDCT [10], (d) Tran's BinDCT [11], (e) Chokchaitam's IntDCT (OWLA) [14], (f) proposed IntDCT.

In Table 6 and Fig. 8, the proposed IntDCT presents better coding performance than the conventional ones in lossy image coding. Especially, in Fig. 8, we can find that a checkerboard artifact is not generated in the proposed IntDCT. Although [11] also does not have the artifact, it shows bad coding performance due to non-consideration of dynamic range and no quantization part.

5. Conclusion

This paper presents an integer discrete cosine transform (IntDCT) via lossless Walsh-Hadamard transform (LWHT) with structural regularity even in low-bit-word-length (coefficients). It can be achieved by using a novel LWHT considering normalization as the pre-processing part in IntDCT. Our M-channel LWHT can be constructed with only $(\pm \log_2 M)$-bit-word-length and has structural regularity. Therefore, our IntDCTs are more suitable for lossless-to-lossy image coding and show better coding performance than the conventional IntDCTs when lower-bit-word-length is allocated to every lifting coefficients.

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