PAPER

Design of M-Channel Perfect Reconstruction Filter Banks with IIR-FIR Hybrid Building Blocks

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SUMMARY This paper discusses a new structure of M-channel IIR perfect reconstruction filter banks. A novel building block defined as a cascade connection of some IIR building blocks and FIR building blocks is presented. An IIR building block is written by state space representation, where we easily obtain a stable filter bank by setting eigenvalues of the state transition matrix into the unit circle. Due to cascade connection of building blocks, we are able to design a system with a larger number of free parameters while keeping the stability. We introduce the condition which obtains the new building block without increasing of the filter order in spite of cascade connection. Additionally, by showing the simulation results, we show that this implementation has a better stopband attenuation than conventional methods.

key words: *M-channel infinite impulse response perfect reconstruction filter banks, state space representation, FIR-IIR hybrid building blocks*

1. Introduction

Recently, many researchers have been studying multirate signal processing. One of the most efficient techniques for processing wideband digital signals in communication systems and compressing audio, image and video signals is called filter bank (FB). A perfect reconstruction (PR) FB design involves its analysis and synthesis polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ [1]. These matrices consist of polynomials of z which is a delay. There are two types of transfer function of FBs, called finite impulse response (FIR) and infinite impulse response (IIR). In an FIR case, the denominator of transfer function is 1, so we do not need to consider its stability. On the contrary, in an IIR case, the denominator involves delays. Thus, it is necessary to consider its stability. Meanwhile, IIR FBs have superiority in terms of the order of the transfer function. In other words, they can be realized in lower order (much fewer multipliers and adders) than FIR FBs to obtain the desired response specifications [2]. While there are many well-developed FIR FB design theories, there are few satisfactory implementations for IIR FB design because of the difficultly to ensure its stability. Several design methods for the case of IIR PRFBs whose analysis FB is causal stable (poles of $\mathbf{E}(z)$ are inside the unit circle) and the synthesis FB is anti-causal stable (poles of $\mathbf{R}(z)$ are outside the unit circle) have already been introduced. For example, [3] proposed using allpass filters instead of delays in lossless matrices to design IIR PRFB, approximately linear-phase filters using complex allpass sections is proposed in [4], and orthogonal IIR 2-channel PRFBs are designed using all pass filters in [5]. For such FBs, synthesis filtering needs to be performed in anti-causal fashion by introducing time reversal [6], which increases the storage cost of the system and may only be acceptable for finite length signals such as images. Although some design methods have been proposed for the causal and stable case [7], they mostly treated the 2-channel FB cases. Recently, an M-channel real IIR FB [8] was presented, but the design method includes a complicated stabilization procedure of the synthesis filters.

Another problem of IIR FB design is optimization problem. High order IIR FB design needs a lot of computation. It is also difficult to optimize higher order FBs as a building block. To solve this problem, these FBs can be obtained by cascading lower order building blocks.

Organization: In this paper, we introduce a class of IIR causal stable PRFBs obtained by cascade connection of FIR and IIR building blocks. Firstly, we present the conventional method for IIR PRFB by using the state space representation [9] and the structure of FIR degree-1 building blocks in Sect. 2. In Sect. 3, we show the condition which obtains the new building block without increasing the order for the case of order-1 and 2. If we have the building block of order-1 and 2, we can easily obtain higher order IIR FBs by cascade connection of these building blocks. Frequency responses of the proposed FBs are presented and compared with the conventional method in Sect. 4.

Notation: Boldface small and capital letters represent vectors and matrices, respectively. $det()$, $adj()$ and $trace()$ denote determinant, adjoint and trace of a matrix. Superscripts † indicates the conjugate transpose of matrices and vectors. A McMillan degree of a building block is the minimum number of delay units required to realize the building block.

2. Review

2.1 Polyphase Structure for Filter Banks

Figure 1 shows a typical structure of an M-channel maxi-

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mally decimated FB, where $H_k(z)$ and $F_k(z)$ are the k-th (for $k = 0 \cdots M-1$) analysis and synthesis filter, respectively. In Fig. 2, the analysis and synthesis filters are shown by using the polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ represented as follows:

$$\begin{aligned} &[H_0(z)\ H_1(z)\ \cdots\ H_{M-1}(z)]^T = \mathbf{E}(z^M)\mathbf{e}(z)^T \\ &[F_0(z)\ F_1(z)\ \cdots\ F_{M-1}(z)] = \mathbf{e}(z)\mathbf{R}(z^M) \\ &\mathbf{e}(z) = [1\ z^{-1}\ \cdots\ z^{-(M-1)}]. \end{aligned} \tag{1}$$

It is clear that the condition for PR is $\mathbf{R}(z)\mathbf{E}(z) = c\mathbf{I}$ $(c \in \mathbb{R})$ (PR condition [1]).

2.2 IIR Filter Banks Using State Space Structure

The PR condition for general $M \times M$ polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are

$$\mathbf{R}(z) = \mathbf{E}^{-1}(z) = \frac{adj(\mathbf{E}(z))}{det(\mathbf{E}(z))} \tag{2}$$

where obviously $\mathbf{E}(z)$ is nonsingular. For IIR FBs, causal stable synthesis filters are obtained if $\det(\mathbf{E}(z))$ is minimum phase with $\mathbf{E}(z)$ being causal. The problem of constraining $\mathbf{E}(z)$ with minimum phase determinant is resolved by considering the minimal factorization of $\mathbf{E}(z)$ in the state space structure.

As it is known, any rational function $\mathbf{E}(z)$ can be expressed in the state space structure [9],

$$\mathbf{E}(z) = \mathbf{D} + \mathbf{C}'(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

where $\mathbf{D}$, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}'$ are $M \times M$, $m \times m$, $m \times M$ and $M \times m$ matrices respectively $(m \leq M)$. $\mathbf{A}$ is called the state transition matrix. Then to simplify, we can rewrite the above equation

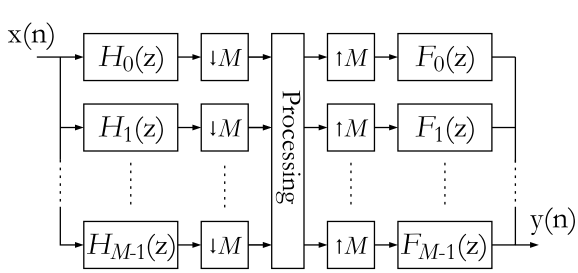


Fig. 1 An M-channel filterbank.

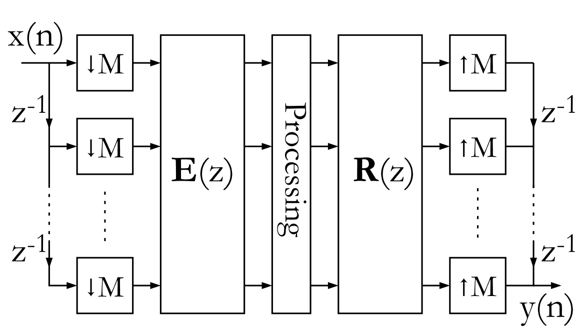


Fig. 2 The polyphase representation of FBs.

$$\mathbf{E}(z) = \mathbf{D}(\mathbf{I} + \mathbf{C}(z\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}) = \mathbf{D}\mathbf{E}'(z) \tag{3}$$

And the synthesis polyphase matrix $\mathbf{R}$(z) is given by

$$\mathbf{R}(z) = (\mathbf{I} - \mathbf{C}(z\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{B})\mathbf{D}^{-1} = \mathbf{R}'(z)\mathbf{D}^{-1} \tag{4}$$

where

$$\mathbf{A}^* = \mathbf{A} - \mathbf{B}\mathbf{C}. \tag{5}$$

Since $\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}$, this implies $\mathbf{R}'(z)\mathbf{E}'(z) = \mathbf{I}$. It is sufficient to focus on factorization of $\mathbf{E}(z)$ and $\mathbf{R}(z)$. $\mathbf{E}'(z)$ and $\mathbf{R}'(z)$ are called building blocks, and $\mathbf{D}$ is called a last block. The structure of the building block is shown in Fig. 3(a) and (b). Because $\mathbf{R}(z)$ can be calculated by matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ of $\mathbf{E}(z)$ uniquely, in this paper, we focus on only the structure of the building block in $\mathbf{E}(z)$

- *Free parameters*
  There is no restriction of every matrix $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$. These are all nonsingular matrices. Thus, the number of parameters is $M^2 + 2mM + m^2$.

2.2.1 Stability

The design of IIR FBs using the state space representation, all transfer functions have the same denominator. So the analysis filter bank is rewritten as

$$\mathbf{E}(z) = \frac{\mathbf{B}(z)}{A(z)},$$

where $\mathbf{B}(z)$ is same form as FIR polyphase analysis matrices and $A(z)$ is m-th order polynomial of z. As stated earlier, in designing IIR FBs, we have to take the stability into account.

From system theory, it is well known that poles of $\mathbf{E}(z)$ are same as the eigenvalues of $\mathbf{A}$ in the analysis bank. Similarly, the poles of $\mathbf{R}(z)$ are same as the eigenvalues of $\mathbf{A}^*$. For causal stable analysis and synthesis filters, poles of $\mathbf{E}(z)$ and $\mathbf{R}(z)$ have to be inside unit circle, which implies eigenvalues of $\mathbf{A}$ and $\mathbf{A}^*$ have to be inside unit circle. So the design problem boils down to characterizing the set of matrices $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\}$ such that the eigenvalues of $\mathbf{A}$ and $\mathbf{A}^*$ are inside the unit circle. However it is very hard to keep the stability on both analysis and synthesis bank when m is large.

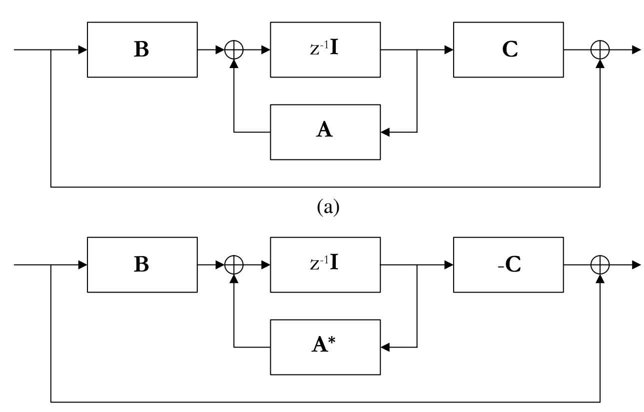


Fig. 3 A realization of the conventional (a) $\mathbf{E}'(z)$, (b) $\mathbf{R}'(z)$.

2.2.2 Controllability and Observability

In designing FBs using state space representation, we have to impose the both conditions observability and controllability to obtain minimal system $\mathbf{E}(z)$ [13].

- *Controllability condition*
  The $m \times Mk$ matrix

$$C(\mathbf{A}, \mathbf{B}) = [\mathbf{B}\ \mathbf{AB}\ \mathbf{A}^2\mathbf{B}\ \cdots\ \mathbf{A}^k\mathbf{B}]$$

  must be of rank m; this condition is called *controllability condition*.

- *Observability condition*
  The $m \times Mk$ matrix

$$O(\mathbf{C}, \mathbf{A}) = [\mathbf{C}^T\ (\mathbf{CA})^T\ (\mathbf{CA}^2)^T \cdots\ (\mathbf{CA}^k)^T]$$

  must be of rank m; this condition is called *observability condition*.

To keep $\mathbf{E}(z)$ minimal, additional constraints have to be imposed in the present design. It can be seen that if $m \le M$, full rank matrices $\mathbf{B}$ and $\mathbf{C}$ are enough to satisfy both minimality conditions discussed above, irrespective of the rank of $\mathbf{A}$. So, in the present design methods we assume $m \le M$ (the dimension of the matrix $\mathbf{A}$ never exceeds the number of channels M) and $rank(\mathbf{B}) = rank(\mathbf{C}) = m$.

2.3 FIR Degree-1 Building Block

A class of causal M-channel FIR biorthogonal (BO) FBs of order-L are factorized into [10]

$$\mathbf{E}(z) = \mathbf{W}_L(z) \cdots \mathbf{W}_1(z)\mathbf{E}_0 \quad (j = 1, 2, \cdots m) \tag{6}$$

where $\mathbf{E}_0$ is an $M \times M$ nonsingular matrix and is called the first block. $\mathbf{W}_m(z)(m = 1 \ldots L)$ is an $M \times M$ first-order BO building block given by

$$\mathbf{W}_m(z) = \mathbf{I} - \mathbf{U}_m\mathbf{V}_m^\dagger + z^{-1}\mathbf{U}_m\mathbf{V}_m^\dagger \tag{7}$$

where $\cdot^\dagger$ denotes conjugate transpose and the $M \times \gamma_m$ parameter matrices $\mathbf{U}_m$ and $\mathbf{V}_m$ satisfy

$$\mathbf{V}_m^\dagger\mathbf{U}_m = \begin{bmatrix} 1 & \times & \cdots & \times \\ 0 & 1 & \cdots & \times \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}_{\gamma_m \times \gamma_m}$$

for some integer $1 \le \gamma_m \le M$, where $\times$ indicates possibly nonzero elements. This is a generalization of the paraunitary factorization given in [11] where $\mathbf{U}_m = \mathbf{V}_m$, and has been used for a factorization of the biorthogonal lapped transform (BOLT) [12]. The properties of the matrix are described as follows:

- Since the rank of $\mathbf{V}_m^\dagger\mathbf{U}_m$ is γ_m, the McMillan degree of $\mathbf{W}_m(z)$ as in (7) is γ_m. When $\gamma_m = 1$ and $M/2$, the structure is called *degree-1* and *order-1* building block, respectively. In this paper, we focus on the order-1 structure, since more design parameters can be used than the degree-1 structure.
- The structure in (6) completely spans all causal FIR PRFBs having anticausal FIR inverses. The spanned analysis filters have filter lengths no greater than $M(L+1)$, and the McMillan degree of $\mathbf{E}(z)$ ranges from L to ML, where L is the order of the FB.
- The Type-II synthesis polyphase matrix $\mathbf{R}(z)$ is given by

$$\mathbf{R}(z) = \mathbf{E}_0^{-1}\mathbf{W}_1^{-1}(z) \cdots \mathbf{W}_L^{-1}(z) \tag{8}$$

  Due to possibly nonzero off-diagonal elements of $\mathbf{V}_m^\dagger\mathbf{U}_m$, the order of $\mathbf{W}_m^{-1}(z^{-1})$ can be greater than one. Thus the synthesis bank could have different filter lengths from $M(L+1)$. In this paper, we set $\mathbf{V}_m^\dagger\mathbf{U}_m = \mathbf{I}_\gamma$. Hence, $\mathbf{W}_m^{-1}(z^{-1}) = \mathbf{I} - \mathbf{U}_m\mathbf{V}_m^\dagger + z\mathbf{U}_m\mathbf{V}_m^\dagger$, which is anticausal and satisfies $\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}$ for PR.

3. PRFBs with IIR-FIR Hybrid Building Blocks

In this section, we introduce a novel building block obtained by cascade connection of the conventional IIR building blocks and the FIR BO building blocks.

As mentioned above, it is very hard to design IIR FB with higher order directly using (3) and (4) while keeping the stability. When we design the general IIR filters with higher order, the IIR transfer functions are implemented by cascade connection of first and second order transfer functions, which is easy to impose the stability. Similarly, we consider the IIR FB with higher order is implemented by cascade connection of lower order IIR FBs as

$$\mathbf{E}(z) = \prod_{k=1}^{K} \mathbf{E}_1^{(k)}(z) \prod_{n=1}^{N} \mathbf{E}_2^{(n)}(z) \tag{9}$$

where $\mathbf{E}_1(z)$ and $\mathbf{E}_2(z)$ are the polyphase matrices with first and second order denominators, respectively. Therefore the total order of the denominator is $K + 2M$.

This paper considers only cases where the order of the new building block is one or two.

3.1 Design of Order-1 Hybrid Building Block

We consider the condition for the case of order-1 ($m = 1$) in this part. The analysis matrix $\mathbf{E}'(z)$ can be rewritten by

$$\begin{aligned}\mathbf{E}'(z) &= \mathbf{I} + \mathbf{c}(z - \lambda)^{-1}\mathbf{b}^\dagger \\ &= \frac{\mathbf{I} - z^{-1}(\mathbf{cb}^\dagger - \lambda\mathbf{I})}{1 - \lambda z^{-1}}\end{aligned} \tag{10}$$

where $\mathbf{b}$ and $\mathbf{c}$ are $M \times 1$ vectors. Since λ is pole, $|\lambda|$ has to be less than 1. In order to increase the number of free parameters while keeping the order of the numerator, we connect

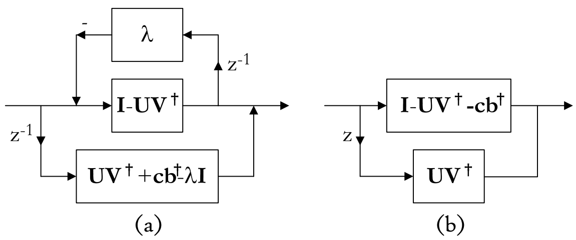


Fig. 4 The structure of order-1 IIR-FIR hybrid building blocks:(a) analysis filter (b) synthesis filter.

the first degree FIR building block to an IIR building block as

$$\mathbf{E}_1'(z) = \frac{\mathbf{I} - z^{-1}(\mathbf{cb}^\dagger - \lambda\mathbf{I})}{1-\lambda z^{-1}} \cdot (\mathbf{I} - \mathbf{UV}^\dagger + z^{-1}\mathbf{UV}^\dagger)$$
$$= \frac{\mathbf{I} - \mathbf{UV}^\dagger + z^{-1}\mathbf{G}_1 - z^{-2}\mathbf{G}_2}{1-\lambda z^{-1}} \quad (11)$$

where the size of $\mathbf{U}$ and $\mathbf{V}$ is $M \times 1$ and

$$\mathbf{G}_1 = \mathbf{UV}^\dagger - \mathbf{cb}^\dagger + \lambda\mathbf{I} + (\mathbf{cb}^\dagger - \lambda\mathbf{I})\mathbf{UV}^\dagger,$$
$$\mathbf{G}_2 = (\mathbf{cb}^\dagger - \lambda\mathbf{I})\mathbf{UV}^\dagger.$$

So as not to increase the order of the building block, the condition is

$$(\mathbf{cb}^\dagger - \lambda\mathbf{I})\mathbf{U} = 0.$$

This condition is satisfied by

$$\begin{cases} \lambda = \mathbf{b}^\dagger\mathbf{c} \\ \mathbf{U} = \mathbf{c} \end{cases}. \quad (12)$$

Therefore, the proposed order-1 building blocks $\mathbf{E}_1(z)$ and $\mathbf{R}_1(z)$ are expressed as

$$\mathbf{E}_1(z) = \frac{\mathbf{I} - \mathbf{UV}^\dagger + z^{-1}(\mathbf{UV}^\dagger + \mathbf{cb}^\dagger - \lambda\mathbf{I})}{1-\lambda z^{-1}} \quad (13)$$
$$\mathbf{R}_1(z) = [\mathbf{I} - \mathbf{UV}^\dagger - \mathbf{cb}^\dagger + z\mathbf{UV}^\dagger]. \quad (14)$$

As shown in (13) and (14), the building blocks are IIR system for $\mathbf{E}_1(z)$ and FIR system for $\mathbf{R}_1(z)$. Figure 4(a) and (b) show the structures of order-1 IIR-FIR hybrid building blocks.

- *Free parameters*
  In the conventional method, there are $(M+1)^2$ parameters for the case of order-1. In our method, the restriction imposed for $\lambda = \mathbf{b}^\dagger\mathbf{c} \neq 0$ and $\mathbf{V}^\dagger\mathbf{c} = 1$. There are no restrictions of matrices $\mathbf{D}$, $\mathbf{b}$ and $\mathbf{c}$. Thus, the total number of free parameters is $M^2 + 3m - 1$.

3.2 Design of Order-2 Hybrid Building Block

In this subsection, we introduce the new design for the case of order-2 ($m = 2$) similar to order-1. In this case, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are a 2×2, $2 \times M$ and $M \times 2$ matrix, respectively, and $(z\mathbf{I} - \mathbf{A})^{-1}$ can be rewritten as follows

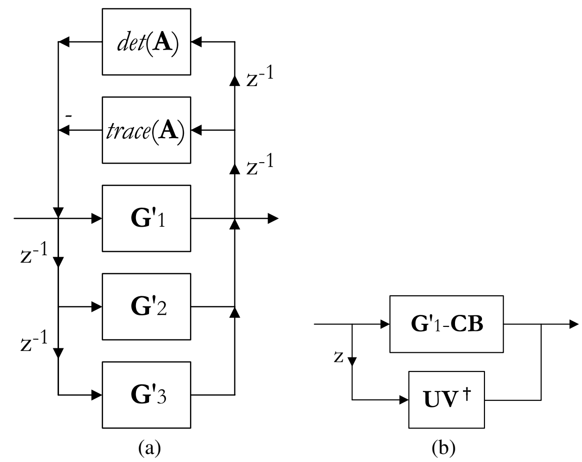


Fig. 5 The structure of IIR-FIR hybrid building blocks: (a) analysis filter (b) synthesis filter.

$$\det(z\mathbf{I} - \mathbf{A}) = z^2 - z\mathrm{trace}(\mathbf{A}) + \det(\mathbf{A}) \quad (15)$$
$$(z\mathbf{I} - \mathbf{A})^{-1} = \frac{z\mathbf{I} - \det(\mathbf{A})\mathbf{A}^{-1}}{\det(z\mathbf{I} - \mathbf{A})} \quad (16)$$

From (15) and (16), the $\mathbf{E}'(z)$ in (3) is formulated as

$$\mathbf{E}'(z) = \mathbf{I} - \frac{z\mathbf{CB} - \det(\mathbf{A})\mathbf{CA}^{-1}\mathbf{B}}{\det(z\mathbf{I} - \mathbf{A})}$$
$$= \frac{\mathbf{I} + z^{-1}\mathbf{G}_1 + z^{-2}\mathbf{G}_2}{1 - z^{-1}\mathrm{trace}(\mathbf{A}) + z^{-2}\det(\mathbf{A})}$$

where

$$\mathbf{G}_1 = \mathbf{CB} - \mathit{trace}(\mathbf{A})\mathbf{I},$$
$$\mathbf{G}_2 = \mathit{det}(\mathbf{A})(\mathbf{I} - \mathbf{CA}^{-1}\mathbf{B}).$$

Then, we denote the proposed building block of the analysis polyphase matrix as follows:

$$\mathbf{E}_2'(z) = \frac{\mathbf{I} + z^{-1}\mathbf{G}_1 + z^{-2}\mathbf{G}_2}{1 - \mathit{trace}(\mathbf{A})z^{-1} + \mathit{det}(\mathbf{A})z^{-2}} \cdot \left[\mathbf{I} - \mathbf{UV}^\dagger + z^{-1}\mathbf{UV}^\dagger\right]$$

where the size of $\mathbf{U}$ and $\mathbf{V}$ is $M \times 2$. To eliminate the term of z^{-3}, the constraints not to increase the order of numerator can be expressed as

$$\mathit{det}(\mathbf{A})(\mathbf{I} - \mathbf{CA}^{-1}\mathbf{B})\mathbf{UV}^\dagger = \mathbf{0}.$$

then it is rewritten by

$$\begin{cases} \mathbf{A} = \mathbf{BC} \\ \mathbf{U} = \mathbf{C} \end{cases}. \quad (17)$$

Hence, our proposed order-2 building blocks $\mathbf{E}_2(z)$ and $\mathbf{R}_2(z)$ are represented as

$$\mathbf{E}_2(z) = \frac{\mathbf{G}_1' + z^{-1}\mathbf{G}_2' + z^{-2}\mathbf{G}_3'}{1 - \mathit{trace}(\mathbf{A})z^{-1} + \mathit{det}(\mathbf{A})z^{-2}} \quad (18)$$
$$\mathbf{R}_2(z) = [\mathbf{G}_1' - \mathbf{CB} + z\mathbf{UV}^\dagger] \quad (19)$$

where

$$\mathbf{G}'_1 = \mathbf{I} - \mathbf{U}\mathbf{V}^\dagger$$
$$\mathbf{G}'_2 = \{\mathbf{CB} - trace(\mathbf{A})\mathbf{I}\}\mathbf{E}_0 + \mathbf{U}\mathbf{V}^\dagger$$
$$\mathbf{G}'_3 = det(\mathbf{A})(\mathbf{I} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}) - \{\mathbf{CB} - trace(\mathbf{A})\mathbf{I}\}.$$

Figure 5 shows the structure of analysis and synthesis IIR-FIR hybrid building blocks.

- *Free parameters*
  In the conventional method, the number of free parameters is $M^2 + 4M + 4$ for the case of order-2. In proposed method, there are the restrictions for $\mathbf{A} = \mathbf{BC}$ and $\mathbf{V}^\dagger\mathbf{C} = \mathbf{I}$. Thus, we have $M^2 + 6M - 3$ parameters totally.

3.3 Higher Order Hybrid Building Block

In previous section, we show the novel structure of order-1 and 2 hybrid building blocks. In Table 1, the form of TFs are shown. By cascading these building blocks, building blocks of all the arbitrary forms can be designed. Moreover, filter length differs with the analysis and synthesis filter of the proposed order-2 hybrid building block. Therefore, we can easily obtain unequal length filter banks.

4. Result

In this section, we present the design examples of proposed IIR PRFBs. In this paper, we optimize the cost function Φ to design FB which is calculated as weighted linear combination of following errors:

$$\Phi = \sum_{i=0}^{M-1} \left(E_{pass}^{(i)} + E_{stop}^{(i)} + R_{pass}^{(i)} + R_{stop}^{(i)} \right), \tag{20}$$

$$E_{pass}^{(i)} = \int_{\omega \in pass_i} W_p(\omega)(1 - |H_i(\omega)|)^2 d\omega$$

$$E_{stop}^{(i)} = \int_{\omega \in stop_i} W_s(\omega)|H_i(\omega)|^2 d\omega$$

$$R_{pass}^{(i)} = \int_{\omega \in pass_i} W_p(\omega)(1 - |F_i(\omega)|)^2 d\omega$$

$$R_{stop}^{(i)} = \int_{\omega \in stop_i} W_s(\omega)|F_i(\omega)|^2 d\omega$$

where $H_i(e^{j\omega})$ and $F_i(e^{j\omega})$ (for$i = 0, 1, \cdots M - 1$) are the frequency responses of the ith filter, which can be obtained by substituting designed FBs using the proposed building blocks into (1). $E_{pass}^{(i)}$ and $E_{stop}^{(i)}$ are error energies in the passband and stopband of the analysis filterbank, respectively. Similarly, $R_{pass}^{(i)}$ and $R_{stop}^{(i)}$ are error energies of the synthesis filterbank. W_p and W_s are positive weighting functions. We optimize the parameters of $\mathbf{B}, \mathbf{C}, \mathbf{V}$ and $\mathbf{D}$ using `fminunc.m` in Matlab 2006a such that the cost function in (20) is minimized.

Remark: Order-1 building blocks should be optimized by taking the λ inside the unit circle in order to achieve stability. Due to the condition $\mathbf{A} = \mathbf{BC}$, the stability might be lost. In order to impose the stability, the optimization should be done while checking the nonsingularity of $\mathbf{A}$ which is 2×2 matrix.

4.1 Design Example

Figure 6 show the magnitude responses in the analysis and synthesis banks of proposed 4-channel IIR PRFBs. The order-1 FB in Fig. 6(a) and (b) is designed by the proposed order-1 building block in Sect. 3.1. Figure 6(c) and (d) illustrate the frequency responses of FBs designed by the proposed building block in Sect. 3.2. Since the inverse of the proposed order-2 building block has order-1 structure as shown in Table 1, the order-2 IIR PRFB is unequal length FB. As a comparison, the freqency responses of the conventional IIR PRFB are shown with dotted lines in Fig. 6(c) and (d). Higher order FBs can be obtained by cascading connection of the proposed order-1 and 2 building blocks. For example, Fig. 6(e) and (f) are the freqency responses of the order-3 IIR PRFB by cascading $\mathbf{E}_1(z)$ and $\mathbf{E}_2(z)$. The frequency responses of the proposed method have better stopband attenuations than the conventional method. Table 2 compares stopband attenuation of the FBs. Each value denotes the largest value of $|H_m(z)|$ and $|F_m(z)|$ in stopband domain respectively. Clearly we can obtain better performance by the proposed method. Furthurmore, by using

Table 1 TFs form.

Filter order	analysis filter	synthesis filter
prop. order-1	$\frac{\times+\times z^{-1}}{\times+\times z^{-1}}$	$\times + \times z$
prop. order-2	$\frac{\times+\times z^{-1}+\times z^{-2}}{\times+\times z^{-1}+\times z^{-2}}$	$\times + \times z$
conv. order-1	$\frac{\times+\times z^{-1}}{\times+\times z^{-1}}$	$\frac{\times+\times z^{-1}}{\times+\times z^{-1}}$

Table 2 Comparison of stopband attenuations of 4-channel filterbanks.

	conv. FB in [9] (order-2)		prop. FB (order-1)		prop. FB (order-2)		prop. FB (order-3)	
k	$H_k(z)$	$F_k(z)$	$H_k(z)$	$F_k(z)$	$H_k(z)$	$F_k(z)$	$H_k(z)$	$F_k(z)$
0	−18.5 dB	−16.3 dB	−21.8 dB	−23.4 dB	−22.5 dB	−25.6 dB	−29.3 dB	−28.3 dB
1	−14.0 dB	−13.9 dB	−16.1 dB	−16.2 dB	−23.5 dB	−24.6 dB	−27.4 dB	−29.3 dB
2	−16.0 dB	−13.4 dB	−20.4 dB	−19.5 dB	−27.0 dB	−27.0 dB	−28.6 dB	−28.6 dB
3	−18.1 dB	−17.9 dB	−19.8 dB	−19.6 dB	−23.8 dB	−20.6 dB	−28.3 dB	−30.8 dB

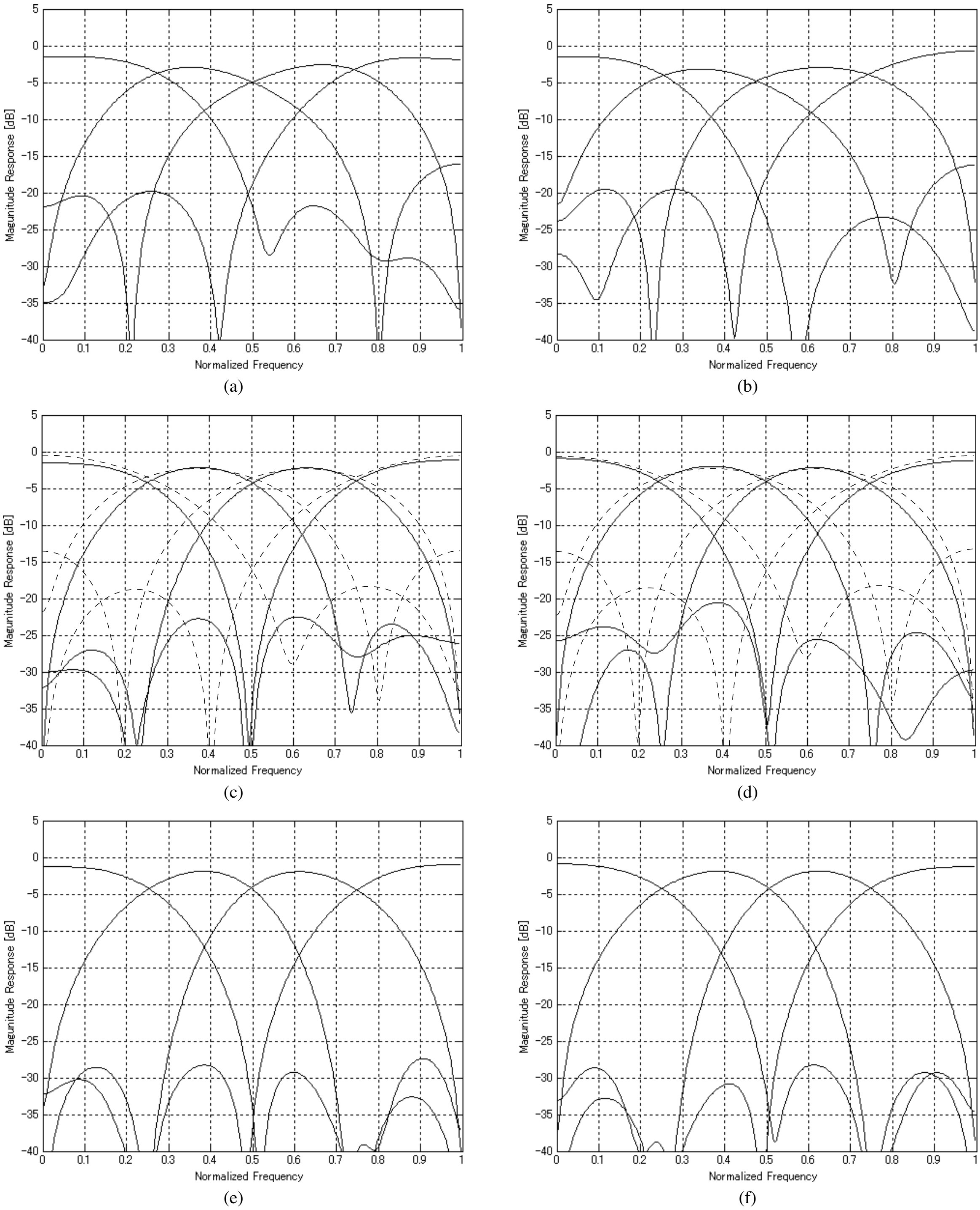
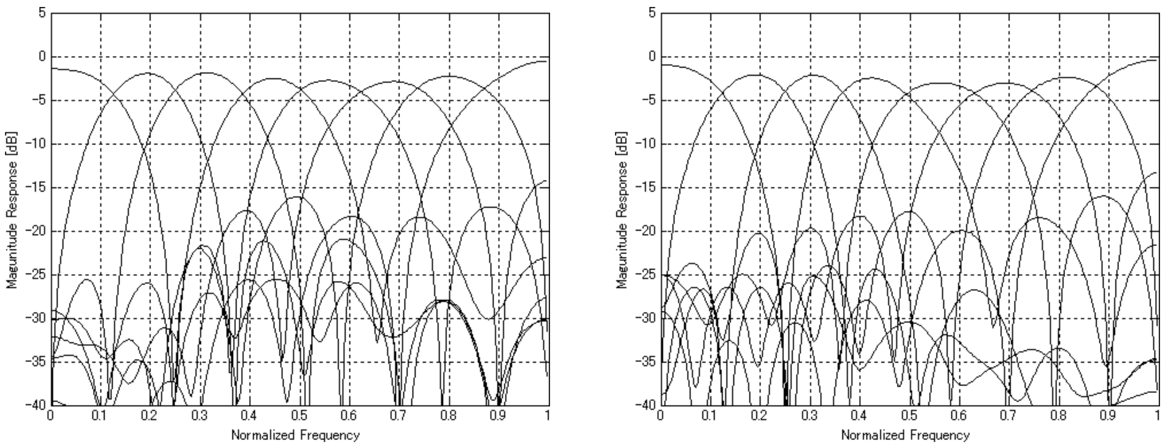


Fig. 6 Frequency responses of 4-channel order-1 prop. FB: (a)(b) analysis and synthesis bank. 4-channel order-2 prop. FB: (c)(d) analysis and synthesis bank (dotted line: order-2 conv. FB). 4-channel order-3 prop. FB: (e)(f) analysis and synthesis bank. 8-channel order-2 prop. FB: (g)(h) analysis and synthesis bank.

same structure of proposed building blocks, we can easily expand the number of channels. For example, Fig. 6(g) and (h) show frequency responses of 8-channel order-2 proposed IIR PRFB.

5. Conclusion

In this paper, new design approaches of *M*-channel IIR PRFBs based on the IIR-FIR hybrid building blocks are presented. We impose the condition on the building blocks with



(g) (h)

Fig. 6 Continued.

state space representation to keep the order despite cascading of building blocks. Since the new method has more free parameters than conventional one, we obtain better stopband attenuations. As a feature, our IIR PRFBs have IIR analysis filters and FIR synthesis filters and the order of the synthesis filters are consistently one.

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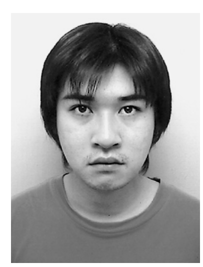

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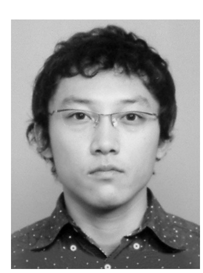

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