Integer DCT Using Direct-Lifting of Fast Hartley Transform

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***Abstract*— This paper presents an integer discrete cosine transforms (IntDCTs) based on *direct-lifting* of pre-processing block of discrete cosine transform (DCT) for lossy-to-lossless image coding which has image quality scalability from lossy data to lossless data. Many IntDCTs with lifting structures have already been presented to achieve lossy-to-lossless image coding. Recently, an IntDCT based on *direct-lifting* of DCT/IDCT, which means direct use of DCT and inverse DCT (IDCT) to lifting blocks, has been proposed. Although the IntDCT shows more efficient coding performance than any conventional IntDCT, it requires a side information which causes its handling trouble. On the other hand, the conventional IntDCTs without any additional information should be improved for efficient coding performance. As a result, the proposed IntDCT shows better coding performance than the conventional IntDCTs without any additional information by applying a *direct-lifting* to pre-processing block of DCT.**

Index Terms — **Direct-lifting, fast Hartley transform (FHT), integer discrete cosine transform (IntDCT), lossy-to-lossless image coding**

I. Introduction

Discrete cosine transform (DCT) [1] has several types, and DCT type-II (DCT-II) and -III (DCT-III) are commonly called DCT and inverse DCT (IDCT). They are applied to lossy image/video compression (coding) such as the international standard JPEG and H.26x family, because DCT has excellent energy compaction capability, many fast implementations [2–5] and numerous applications on JPEG devices. Since it is compatible with the lossy JPEG, DCT is especially promising.

Recently, with the rapid development of multimedia devices such as media tablets and smartphones, and the continual expansion of broadband, lossy-to-lossless image coding, which has image quality scalability from lossy data to lossless data, is demanded to respond to the various needs. To achieve it, many integer DCTs (IntDCTs) based on lifting structures [6–8] have been proposed [9], [10]. We also have proposed a *direct-lifting* which is a so effective structure for lossy-to-lossless image coding [11]. The IntDCT, however, requires a side information because both of a transform and its inverse transform are required, i.e., both of DCT and IDCT must be simultaneously implemented. As the solve, DCT is separated to pre- and post-processing block as the first stage. If at least one of the pre-/post-processing block is a symmetric orthogonal matrix, the DCT may perform efficient lossy-to-lossless image coding due to application of a *direct-lifting*. Although our previous work in [12] has such structure, which has a *direct-lifting* of Walsh-Hadamard transform (WHT) as the pre-processing block, it should be improved for more efficient coding performance because there are many rounding operations in the post-processing block.

On the other hand, faster DCT algorithms without multipliers are widely researched for real time encode/decode implementations [12–14]. We also have proposed multiplierless fast DCT based on fast Hartley transform (FHT) [15]. The structure has a FHT and the residue orthogonal matrix with $(M-2)/2$ rotation matrices as the pre- and post-processing block, respectively.

In this paper, we adopt a *direct-lifting* for FHT as pre-processing block of DCT for lossy-to-lossless image coding. We can implement it without considering extra things because FHT is a symmetric orthogonal matrix similar to WHT in [12]. Also, two-dimensional (2-D) separable block transform is considered to achieve a more efficient transform. The resulting simple structure avoids a propagation of rounding error generated by cascading rounding operations. As a result, the proposed IntDCT not only has an arbitrary size $M = 2^n$, but also shows better coding performance than the conventional IntDCT without any additional information which causes its handling trouble.

Notations: $\mathbf{I}$, $\mathbf{J}$, $\{\cdot\}_{[N]}$, $\{\cdot\}^T$ and $\mathrm{diag}\{\mathbf{M}, \mathbf{N}\}$ are an identity matrix, a reversal identity matrix, $N \times N$ matrix, transpose of a matrix and a block diagonal matrix of $\mathbf{M}$ and $\mathbf{N}$, respectively.

II. Review

A. Discrete Cosine Transform (DCT)

Not only is DCT [1] often used for the international standard JPEG and H.26x family, but also numerous devices for DCT have been developed. The (m,n)-elements of M-channel DCT-II matrix $\mathcal{C}$, DCT-III matrix $\mathcal{D}$ and DCT-IV matrix $\mathcal{E}$ are defined as

$$[\mathcal{C}]_{m,n} = \sqrt{\frac{2}{M}} c_m \cos\left(\frac{m\,(n+1/2)\,\pi}{M}\right)$$
$$[\mathcal{D}]_{m,n} = \sqrt{\frac{2}{M}} c_n \cos\left(\frac{(m+1/2)\,n\pi}{M}\right)$$
$$[\mathcal{E}]_{m,n} = \sqrt{\frac{2}{M}} \cos\left(\frac{(m+1/2)\,(n+1/2)\,\pi}{M}\right)$$

where $\mathcal{D} = \mathcal{C}^{-1} = \mathcal{C}^T$, $\mathcal{E}^{-1} = \mathcal{E}^T = \mathcal{E}$, $0 \le m, n \le M-1$ and

$$c_k = \begin{cases} \frac{1}{\sqrt{2}} & (k=0) \\ 1 & (k \neq 0) \end{cases},$$

respectively. For simplicity, let us define $M = 2^n$ $(n \in \mathbb{N})$. Also, many fast implementations of DCT have been widely researched [2–5]. For example, Chen *et al.* have proposed a fast implementation of M-channel DCT $\mathcal{C}_{[M]}$ in [2] which is the most popular structure as

$$\mathcal{C}_{[M]} = \frac{1}{\sqrt{2}} \mathbf{P}_{Chen} \begin{bmatrix} \mathcal{C}_{[M/2]} & \mathbf{0} \\ \mathbf{0} & \mathcal{E}_{[M/2]} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{J} \\ \mathbf{J} & -\mathbf{I} \end{bmatrix} \tag{1}$$

where $\mathbf{P}_{Chen}$ is an $M \times M$ permutation matrix. In case of $M = 8$, it is simplified by 4 rotation matrices with $\pi/4$ angles and 4-channel DCT-II/-IV as shown in Fig. 1(a). Note that any coding system by DCT is limited to operation in only lossy image coding because distortion of the decoded image is unavoidable with these lossy algorithms.

B. Fast Hartley Transform (FHT)

The FHT is a tool for the frequency analysis, design and implementation of digital signal processing algorithms and systems. It is strictly symmetrical concerning the transformation and its inverse,

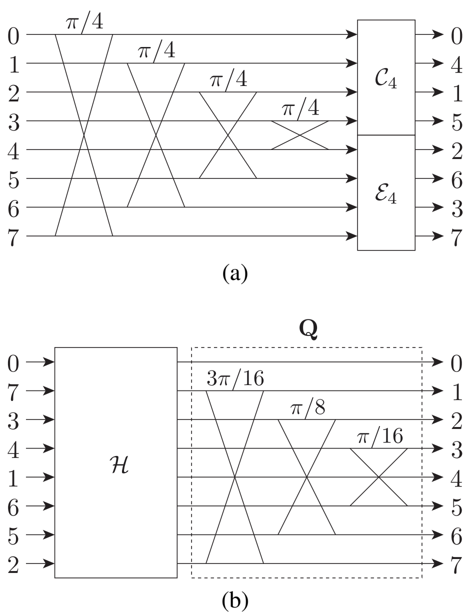

Fig. 1. 8-channel DCT $\mathcal{C}$: (a) Chen's DCT in [2], (b) DCT based on FHT in [15].

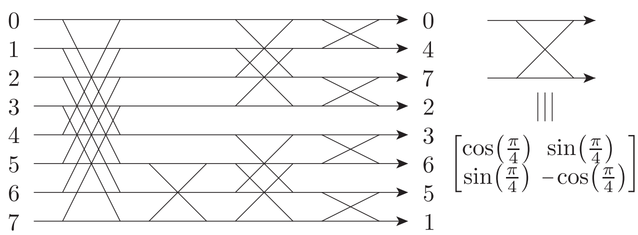

Fig. 2. 8-channel FHT $\mathbf{P}\mathcal{H}$.

i.e., a symmetric orthogonal matrix. The (m, n)-element of M-channel FHT matrix $\mathcal{H}$ is defined as [16]

$$[\mathcal{H}]_{m,n} = \frac{1}{\sqrt{M}} \operatorname{cas}\left(\frac{2mn\pi}{M}\right)$$

where $\operatorname{cas}\theta = \cos\theta + \sin\theta$ and $\mathcal{H}^{-1} = \mathcal{H}^T = \mathcal{H}$. Since $\mathcal{H}$ is not according to the frequency band order, let $\mathcal{H}$ with the correct order be $\mathbf{P}\mathcal{H}$ where $\mathbf{P}$ is a permutation matrix. In case of $M = 8$, it is simplified by 13 rotation matrices with $\pi/4$ angles as shown in Fig. 2.

C. Lifting Structure

Lifting structure, also known as a ladder structure, is an implementation method of wavelet transforms originally proposed [6–8] by Sweldens. It is a special type of lattice structure: a cascading construction using only elementary matrices, that is, identity matrices with a single nonzero off-diagonal element. Fig. 3(a) shows a standard lifting structure. In this case, the lifting matrix and its inverse matrix are as follows:

$$\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -T \\ 0 & 1 \end{bmatrix}.$$

Then, they are represented by

$$\begin{matrix} y_i = x_i + \operatorname{round}\{Tx_j\} \\ y_j = x_j \end{matrix} \rightarrow \begin{matrix} z_i = y_i - \operatorname{round}\{Ty_j\} = x_i \\ z_j = y_j = x_j \end{matrix}$$

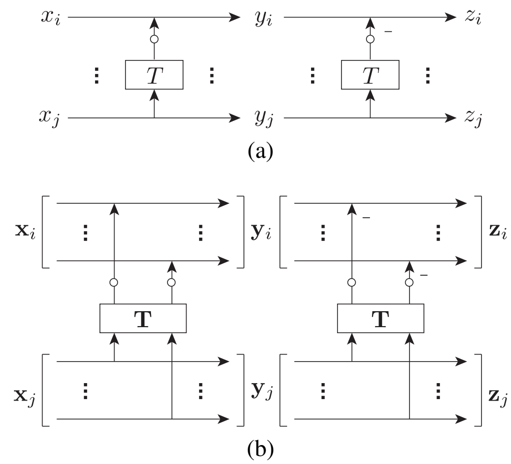

Fig. 3. A lifting structure (white circles mean rounding operations): (a) a standard lifting, (b) a *block-lifting*.

where T and $\operatorname{round}\{.\}$ are a lifting coefficient and a rounding operation, respectively. Thus the lifting structures with rounding operation can achieve lossy-to-lossless image coding.

To obtain more efficient transform performance than a transform by standard lifting, *block-lifting* as a class of lifting structure has been proposed [17]. Instead of a lifting coefficient T, we adopt a lifting matrix (block) $\mathbf{T}$. Such *block-lifting* can be expressed as follows:

$$\begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \begin{bmatrix} \mathbf{I} & \mathbf{T} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{I} & -\mathbf{T} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

and

$$\begin{matrix} \mathbf{y}_i = \mathbf{x}_i + \operatorname{round}\{\mathbf{T}\mathbf{x}_j\} \\ \mathbf{y}_j = \mathbf{x}_j \end{matrix} \rightarrow \begin{matrix} \mathbf{z}_i = \mathbf{y}_i - \operatorname{round}\{\mathbf{T}\mathbf{y}_j\} = \mathbf{x}_i \\ \mathbf{z}_j = \mathbf{y}_j = \mathbf{x}_j \end{matrix},$$

where $\mathbf{x}_i$, $\mathbf{x}_j$, $\mathbf{y}_i$, $\mathbf{y}_j$, $\mathbf{z}_i$ and $\mathbf{z}_j$ are the input and output signal matrices, as shown in Fig. 3(b). Similar to standard lifting, it is clear that the reconstructed signals $\mathbf{z}_i$ and $\mathbf{z}_j$ are exactly the same value as $\mathbf{x}_i$ and $\mathbf{x}_j$. The *block-lifting* would be efficient for lossy-to-lossless image coding due to having fewer number of rounding operations.[1]

III. IntDCT using Direct-Lifting of FHT

A. DCT based on FHT

A fast DCT based on FHT [15] is definitely different from the most popular Chen's structure in (1). The structure is separated to a permutation matrix $\mathbf{P}^T$, FHT $\mathcal{H}$ as the pre-processing block and the residue orthogonal matrix $\mathbf{Q}$ as the post-processing block as follows:

$$\mathcal{C} = \mathbf{Q}\mathcal{H}\mathbf{P}^T = \begin{bmatrix} 1 & \mathbf{0} & 0 & \mathbf{0} \\ \mathbf{0} & \mathbf{JCJ} & \mathbf{0} & \mathbf{JS} \\ \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{SJ} & \mathbf{0} & -\mathbf{C} \end{bmatrix} \mathcal{H}\mathbf{P}^T \tag{2}$$

where

$$[\mathbf{C}]_{k,k} = \cos\left(\frac{(k+1)\pi}{2M}\right) \quad \text{and} \quad [\mathbf{S}]_{k,k} = \sin\left(\frac{(k+1)\pi}{2M}\right)$$

for $0 \leq k \leq M/2 - 2$ as shown in Fig.1(b). Note that this structure can be easily expanded to a larger size than $M = 8$. However, if this DCT is factorized into the lifting part and the scaling part according to [15], it is inapplicable to lossy-to-lossless image coding due to M multipliers as the generated scaling part.

[1] When the number of rounding operations increases, the quantization error simultaneously increased affects the subband energy compaction.

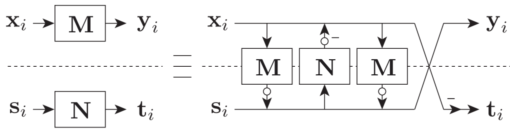

Fig. 4. A *direct-lifting* structure (white circles mean rounding operations).

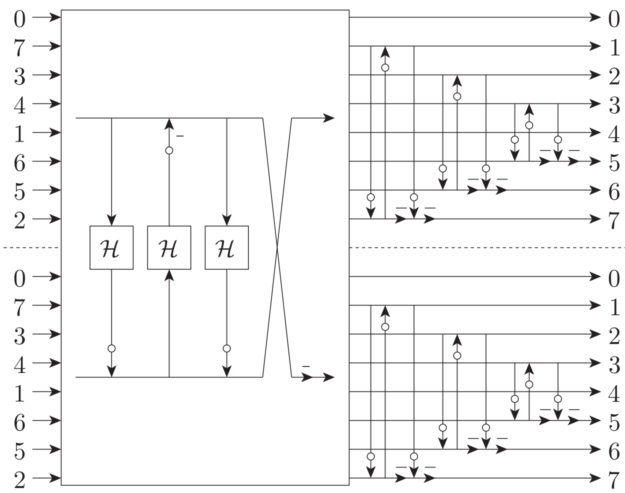

Fig. 5. 8-channel IntDCT using *direct-lifting* of FHT (white circles mean rounding operations).

B. Lifting Factorization of FHT and the Residue Orthogonal Matrix

To achieve lossy-to-lossless image coding, we apply a *direct-lifting* and standard liftings to FHT and the residue rotation matrices, respectively.

we have proposed *direct-lifting* as a class of *block-lifting* [11]. First, we process the two individual signals $\mathbf{x}_i$ and $\mathbf{s}_i$ by a matrix $\mathbf{M}$ and its inverse matrix $\mathbf{N}$, i.e.,

$$\begin{bmatrix}\mathbf{y}_i\\ \mathbf{t}_i\end{bmatrix} = \begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{N}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i\\ \mathbf{s}_i\end{bmatrix}$$

where $\mathbf{MN} = \mathbf{NM} = \mathbf{I}$, and $\mathbf{y}_i$ and $\mathbf{t}_i$ are the output signals of $\mathbf{x}_i$ and $\mathbf{s}_i$, respectively, as shown on the left side of Fig. 4. Then, the matrix $\mathrm{diag}\{\mathbf{M}, \mathbf{N}\}$ can be redefined to lifting structures as

$$\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{N}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{M} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathbf{N}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{M} & \mathbf{I}\end{bmatrix}.$$

This is called *direct-lifting* because two matrices $\mathbf{M}$ and $\mathbf{N}$ are directly used to lifting blocks. To obtain lossless data, a rounding operation is applied to each lifting step because it achieves an integer-to-integer transform without any distortion in the quantization part. Since the inverse of FHT $\mathcal{H}$ is itself, $\mathcal{H}\mathcal{H} = \mathbf{I}$, a *direct-lifting* of FHT is achieved as

$$\begin{bmatrix}\mathcal{H} & \mathbf{0}\\ \mathbf{0} & \mathcal{H}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathcal{H} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathcal{H}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathcal{H} & \mathbf{I}\end{bmatrix} \tag{3}$$

as shown in the left side of Fig.5.

Then, we use a standard lifting factorization of rotation matrix as

$$\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix} = \begin{bmatrix}1 & 0\\ \frac{1-\cos\theta}{\sin\theta} & 1\end{bmatrix}\begin{bmatrix}1 & \sin\theta\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{\cos\theta-1}{\sin\theta} & 1\end{bmatrix}$$

to the residue orthogonal matrix composed of $M/2-1$ rotation matrices.

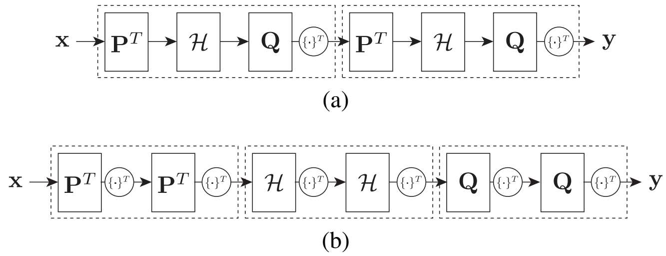

Fig. 6. A 2-D separable block transform : (a) a standard transform, (b) an ordered transform to each block.

TABLE I

THE NUMBER OF ROUNDING OPERATIONS IN EACH IntDCT.

IntDCT				
[9]	[10]	[11]	[12]	Prop.
21	**8**	12	21	15

C. Application of Two-Dimensional (2-D) Separable Block Transform

A DCT $\mathcal{C}$ is commonly applied into an $M \times M$ input signal $\mathbf{x}$ in column- and row-wise (vertical and horizontal direction) separately. The output signal $\mathbf{y}$ is expressed by

$$\mathbf{y} = \left(\mathcal{C}\left(\mathcal{C}\mathbf{x}\right)^T\right)^T = \mathcal{C}\mathbf{x}\mathcal{C}^T. \tag{4}$$

We call it a 2-D separable block transform [11]. Since the DCT $\mathcal{C}$ can be factorized as $\mathcal{C} = \mathbf{DQ}\mathcal{H}\mathbf{P}^T$ in (2), (4) is represented by

$$\mathbf{y} = \mathbf{Q}\mathcal{H}\mathbf{P}^T\mathbf{x}(\mathbf{Q}\mathcal{H}\mathbf{P}^T)^T = \mathbf{Q}\mathcal{H}\mathbf{P}^T\mathbf{x}\mathbf{P}\mathcal{H}^T\mathbf{Q}^T.$$

This equation means 2-D separable block transforms are implemented according to the following order as shown in Fig.6.

(a) A 2-D separable block transform by $\mathbf{P}^T$ is implemented.
(b) A 2-D separable block transform by $\mathcal{H}$ is implemented after (a) is implemented.
(c) A 2-D separable block transform by $\mathbf{Q}$ is implemented after (b) is implemented.

Also, let 2-D separable block transforms of $\mathbf{x}$ by $\mathcal{H}$ be defined as

$$\mathcal{H}\mathbf{x}\mathcal{H}^T \triangleq \mathcal{H}_{2D}(\mathbf{x}).$$

Therefore, (3) are represented by

$$\begin{bmatrix}\mathcal{H}_{2D}(\mathbf{x}) & \mathbf{0}\\ \mathbf{0} & \mathcal{H}_{2D}(\mathbf{x})\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathcal{H}_{2D}(\mathbf{x}) & \mathbf{I}\end{bmatrix} \times \begin{bmatrix}\mathbf{I} & -\mathcal{H}_{2D}(\mathbf{x})\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathcal{H}_{2D}(\mathbf{x}) & \mathbf{I}\end{bmatrix}$$

as shown in Fig. 5(b). Such lifting matrices are more efficient for lossy-to-lossless image coding than lifting matrices without considering 2-D separable block transform, because the number of rounding operations per an one-dimensional (1-D) transform of $M \times 1$ signals is reduced from $3M/2$ to $3M/4$ in each block and rounding error is also reduced much more. Table I shows the number of rounding operations in each IntDCT.

IV. Comparison of Lossy-to-Lossless Image Coding

The resulting IntDCT based on *direct-lifting* of pre-processing block of DCT and the conventional IntDCTs in [9–12] are applied to lossy-to-lossless image coding. An integer-to-integer transform can be obtained by using a rounding operation at each lifting step. To evaluate transform performance fairly, after the images were

TABLE II

LOSSY IMAGE CODING RESULTS (PSNR [DB]).

Test Image	bit rate [bpp]	IntDCT [9]	[10]	[11]	[12]	Prop.
	0.25	**30.70**	30.42	**30.70**	**30.70**	**30.70**
Airplane	0.50	34.76	34.52	**34.78**	34.76	34.76
	1.00	39.28	39.38	**39.49**	39.37	39.42
	0.25	26.94	26.73	**26.95**	26.94	26.94
Barbara	0.50	30.67	30.40	**30.68**	**30.68**	**30.68**
	1.00	35.97	35.84	**36.03**	35.98	35.99
	0.25	22.54	22.21	**22.55**	**22.55**	**22.55**
Finger	0.50	25.55	25.36	**25.56**	**25.56**	**25.56**
	1.00	29.66	29.47	**29.70**	29.68	29.68
	0.25	29.36	29.13	**29.37**	**29.37**	**29.37**
Goldhill	0.50	31.94	31.80	**31.97**	31.96	31.96
	1.00	35.27	35.19	**35.36**	35.31	35.33
	0.25	31.87	31.86	**31.89**	31.88	31.87
Lena	0.50	35.53	35.49	**35.59**	35.57	35.55
	1.00	38.95	38.93	**39.13**	38.99	39.05
	0.25	31.40	**31.43**	31.42	31.41	31.41
Pepper	0.50	34.41	34.33	**34.46**	34.41	34.43
	1.00	36.83	36.71	**36.97**	36.85	36.89

transformed, they were encoded by the set partitioning in hierarchical trees (SPIHT) progressive image transmission algorithm with PSNR scalability [18]. Test images are several 512×512 grayscale standard images such as *Barbara* and *Lena*, respectively. In lossy image coding, we compare their peak signal-to-noise ratios (PSNRs):

$$\mathrm{PSNR}\ [\mathrm{dB}] = 10 \log_{10} \left(\frac{255^2}{\mathrm{MSE}} \right),$$

where MSE is the mean squared error, for three standard images as shown in Table II. In lossless image coding, we compare their lossless bit rates (LBRs) [bpp]:

$$\mathrm{LBR}\ [\mathrm{bpp}] = \frac{\text{Total number of bits [bit]}}{\text{Total number of pixels [pixel]}}$$

as shown in Table III.

Our previous work in [11] shows the best performance with a side information which causes its handling trouble. It is, however, clear that the proposed IntDCT is better than the conventional methods without any additional information in both lossy and lossless image coding, because the proposed IntDCT has a few number of rounding operations as shown in Table I and the simple structure avoids a propagation of rounding error generated by cascading rounding operations. Note that Chokchaitam's IntDCT in [10] with the fewest rounding operations in case of $M = 8$ cannot be performed enough due to several removed scaling factors.

V. CONCLUSION

This paper presented an integer discrete cosine transforms (IntDCTs) based on *direct-lifting* of pre-processing block of discrete cosine transform (DCT) for lossy-to-lossless image coding. It was obtained by applying a *direct-lifting* and a standard lifting to FHT and the residue orthogonal matrix as pre- and post-processing block of DCT, respectively, and considering a two-dimensional (2-D) separable block transform. The resulting IntDCT shows better coding performance than the conventional IntDCT without any additional information, which causes its handling trouble, due to a few rounding operations.

TABLE III

LOSSLESS IMAGE CODING RESULTS (LBR [BPP]).

Test Image	IntDCT [9]	[10]	[11]	[12]	Prop.
Airplane	4.44	4.40	**4.36**	4.40	4.39
Baboon	**6.27**	6.28	**6.27**	**6.27**	**6.27**
Barbara	4.98	4.97	**4.94**	4.96	4.96
Boat	5.20	5.19	**5.18**	5.20	5.19
Bridge	6.00	6.00	**5.99**	6.00	6.00
Elaine	5.23	5.25	**5.22**	5.23	5.23
Finger	6.07	**6.06**	**6.06**	**6.06**	**6.06**
Goldhill	5.17	5.16	**5.14**	5.16	5.15
Grass	**6.17**	6.18	**6.17**	**6.17**	**6.17**
Lena	4.65	4.63	**4.61**	4.63	4.63
Pepper	4.96	4.96	**4.94**	4.96	4.95
Tank	5.20	5.20	**5.19**	5.20	**5.19**

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