BLOCK-LIFTING FACTORIZATION OF M-CHANNEL BIORTHOGONAL FILTER BANKS WITH AN ARBITRARY MCMILLAN DEGREE

Taizo Suzuki[1], *Masaaki Ikehara*[2], *and Truong Q. Nguyen*[3]

1: College of Eng., Nihon University, Koriyama, Fukushima 963-8642, Japan
2: EEE Dept., Keio University, Yokohama, Kanagawa 223-8522, Japan
3: ECE Dept., University of California, San Diego, La Jolla, CA 92093, USA

ABSTRACT

A block-lifting factorization of M-channel biorthogonal filter banks (BOFBs) with *degree-N* building blocks and even/odd M ($M \geq 2$) for lossless-to-lossy image coding is introduced in this paper. In the previous work, block-lifting factorization of M-channel BOFBs has been proposed. Since the block-lifting structure does not require the restriction of determinant of each building block, it achieves better coding performance than the conventional methods. However, the factorization is not completed because McMillan degree in each building block is fixed $M/2$ (M is even). This paper proposes a block-lifting factorization without restrictions for a fixed degree and even block size. Our proposal is validated by several filter designs and their application to lossless-to-lossy image coding.

***Index Terms*—** Biorthogonal filter banks (BOFBs), block-lifting, *degree-N* building block, lossless-to-lossy image coding

1. INTRODUCTION

For a few decades, filter bank (FB) has been considered one of the most efficient techniques to compress multimedia signals [1]. FBs are adopted to audio, image, and video coding standards as JPEG [2] and MPEG [3]. Fig. 1 illustrates an uniform maximally decimated M-channel ($M \geq 2$) FB where $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are the type-I and -II polyphase matrices, respectively. If perfect reconstruction (PR) is achieved, the synthesis polyphase matrix $\mathbf{R}(z)$ can be chosen as the inverse of $\mathbf{E}(z)$, i.e., $\mathbf{R}(z) = \mathbf{E}^{\dagger}(z^{-1})$. The obtained FB is called a biorthogonal FB (BOFB).

On the other hand, Sweldens has introduced lifting factorization of discrete wavelet transforms (DWTs) in [4]. When a rounding operation is performed before addition, this lifting step is integer-to-integer transform. Therefore, FBs factorized into cascading lifting structures can be regarded as integer-to-integer transforms. The problem is how to factorize these FBs into pure lifting structure with fewer constraints. 2-channel lifting based 5/3-tap DWT and an 4×8 hierarchical lapped biorthogonal transform (HLBT) has been proposed and adopted to JPEG2000 lossless compression standard [5] and JPEG XR [6], respectively. Also, several lifting factorizations have been proposed [7–10]. However, they still have a strong restriction, i.e., paraunitary, fixed determinant or symmetry. Then, the authors have proposed a block-lifting structure which does NOT need the above restrictions [11].

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Email: taizo@ee.ce.nihon-u.ac.jp

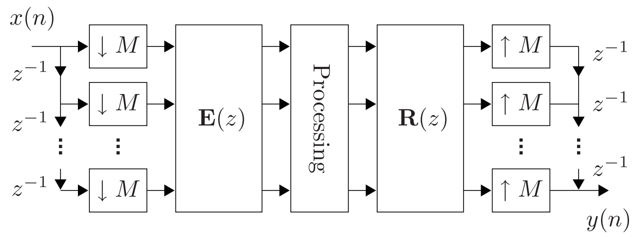

Fig. 1. M-band filter bank: (a) regular and (b) polyphase structure.

In this paper, a block-lifting factorization for BOFBs with *degree-N* building blocks and even/odd M for lossless-to-lossy image coding is proposed. Although McMillan degree in each building block can be set freely, the previous work [11] limits the degree to $M/2$ (M is even). This paper considers the general case without restriction on the degree or block size. Several designs using the proposed structure is implemented in the application of lossless-to-lossy image coding.

Notations: $\mathbf{I}_M$, $\mathbf{M}^T$, and $\mathbf{M}^{\dagger}$ denote an $M \times M$ identity matrix, a transpose of a matrix $\mathbf{M}$, and a conjugate transopose of a matrix $\mathbf{M}$ respectively.

2. REVIEW

2.1. Lattice Structure

A class of causal M-channel maximally decimated FIR BOFBs with filter length KM ($K \in \mathbb{N}$) are factorized as follows [11]:

$$\begin{aligned} \mathbf{E}(z) &= \mathbf{E}_{K-1}(z)\mathbf{E}_{K-2}(z)\cdots\mathbf{E}_1(z)\mathbf{X}_0 \qquad (1) \\ \mathbf{E}_k(z) &= \mathbf{X}_k\mathbf{\Lambda}_k(z)\mathbf{X}_k^{-1} \\ \mathbf{\Lambda}_k(z) &= \mathrm{diag}\{\mathbf{I}_{\delta_k}, z^{-1}\mathbf{I}_{\gamma_k}\} \end{aligned}$$

where $\mathbf{X}_k$ ($0 \leq k \leq K-1$) is an $M \times M$ nonsingular matrix and $\delta_k = M - \gamma_k$, respectively. γ_k is called the McMillan degree of $\mathbf{E}_k(z)$. When $\gamma_k = N$, $\mathbf{E}_k(z)$s are called *degree-N* building blocks. Especially, when $\gamma_k = M/2$ (M is even) or $(M-1)/2$ (M is odd), $\mathbf{E}_k(z)$s are called *order-1* building blocks. Hence, *order-1* structure is considered as a subset of *degree-N* structure. This paper proposes block-lifting factorization of *degree-N* structure which is more general case than *order-1* structure in the previous paper [11]. Hereafter, $\mathbf{\Lambda}_k(z)$ with $\gamma_k = N$ is written as $\mathbf{\Lambda}(z)$.

Moreover, let $\mathbf{X}_k$ and $\mathbf{X}_k^{-1}$ be

$$\mathbf{X}_k = \begin{bmatrix} \times & \mathbf{u}_{k0} \\ \times & \mathbf{u}_{k1} \end{bmatrix} \quad \text{and} \quad \mathbf{X}_k^{-1} = \begin{bmatrix} \times & \times \\ \mathbf{v}_{k0}^T & \mathbf{v}_{k1}^T \end{bmatrix}$$

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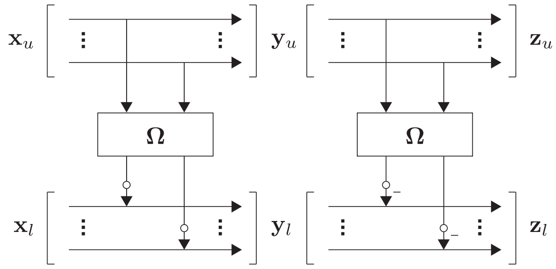

Fig. 2. A block-lifting structure (white circles represent rounding operations).

where $\mathbf{u}_{k0}$ and $\mathbf{v}_{k0}$ are $\delta_k \times \gamma_k$ matrices, and $\mathbf{u}_{k1}$ and $\mathbf{v}_{k1}$ are $\gamma_k \times \gamma_k$ square matrices, respectively. Since $\mathbf{X}_k\mathbf{X}_k^{-1} = \mathbf{I}_M$, a *degree-N* building block $\mathbf{E}_k(z)$ is finally represented as

$$\mathbf{E}_k(z) = \mathbf{I}_M - \mathcal{U}_k\mathcal{V}_k^T + z^{-1}\mathcal{U}_k\mathcal{V}_k^T$$

where

$$\mathcal{U}_k = \begin{bmatrix}\mathbf{u}_{k0}\\ \mathbf{u}_{k1}\end{bmatrix},\ \mathcal{V}_k = \begin{bmatrix}\mathbf{v}_{k0}\\ \mathbf{v}_{k1}\end{bmatrix} \quad \text{and} \quad \mathcal{V}_k^T\mathcal{U}_k = \mathbf{I}_{\gamma_k}. \qquad (2)$$

2.2. Block-Lifting Structure

The lifting structure is a special factorization of lattice structure. It is a cascade of elementary matrices which are identity matrices with one single nonzero off-diagonal element [4]. Block-lifting structure is a more efficient lifting structure [11]. A block-lifting structure is shown in Fig. 2. If $M = \delta_k + \gamma_k$ $(\delta_k, \gamma_k \leq 1)$, $\mathbf{x}_u, \mathbf{x}_l, \mathbf{y}_u, \mathbf{y}_l, \mathbf{z}_u, \mathbf{z}_l$ and $\mathbf{\Omega}$ are an $\delta_k \times 1$ analysis input signal vector, an $\gamma_k \times 1$ analysis input signal vector, an $\delta_k \times 1$ analysis output signal vector (synthesis input signal vector), an $\gamma_k \times 1$ analysis output signal vector (synthesis input signal vector), an $\delta_k \times 1$ synthesis output signal vector, an $\gamma_k \times 1$ synthesis output signal vector and an $\gamma_k \times \delta_k$ lifting block, respectively. The lifting structure can be expressed as follows:

$$\mathbf{y}_u = \mathbf{x}_u, \quad \mathbf{y}_l = \mathbf{x}_l + \mathrm{round}[\mathbf{\Omega}\mathbf{x}_u]$$
$$\mathbf{z}_u = \mathbf{y}_u, \quad \mathbf{z}_l = \mathbf{y}_l - \mathrm{round}[\mathbf{\Omega}\mathbf{x}_u]$$

It is clear that the reconstructed signals $\mathbf{z}_u$ and $\mathbf{z}_l$ are exactly the same value as $\mathbf{x}_u$ and $\mathbf{x}_l$. Therefore, block-lifting structure would be efficient for designing BOFBs for lossless-to-lossy image coding due to having fewer number of rounding operations.

3. BLOCK-LIFTING FACTORIZATION OF BOFBS

3.1. Building Blocks

In this section, we introduce a block-lifting factorization of *degree-N* building blocks in BOFB according to [11]. We start with the factorization of $\mathbf{E}_k(z)$ into a block-lifting structure. Assume that two block lifting matrices are multiplied from the left and right sides of $\mathbf{E}_k(z)$. If they can be represented as a block-lifting structure, $\mathbf{E}_k(z)$ will be factorized into the product of some block-lifting matrices. Let $\mathbf{S}_k$ and $\mathbf{T}_k$ be $M \times M$ block-lifting matrices as

$$\mathbf{S}_k = \begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{0}\\ \bar{\mathbf{S}}_k & \mathbf{I}_{\gamma_k}\end{bmatrix} \quad \text{and} \quad \mathbf{T}_k = \begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{0}\\ \bar{\mathbf{T}}_k & \mathbf{I}_{\gamma_k}\end{bmatrix}$$

where $\bar{\mathbf{S}}_k$ and $\bar{\mathbf{T}}_k$ are $\gamma_k \times \delta_k$ matrices. First, $\mathbf{S}_k$ is multiplied from the right side of $\mathbf{E}_k(z)$.

$$\mathbf{E}_k(z)\mathbf{S}_k = \begin{bmatrix}\mathbf{S}_{ka}(z) & \mathbf{S}_{kb}(z)\\ \mathbf{S}_{kc}(z) & \mathbf{S}_{kd}(z)\end{bmatrix} \triangleq \mathbf{S}_k(z)$$

where

$$\begin{aligned}
\mathbf{S}_{ka}(z) &= \mathbf{I}_{\delta_k} + (z^{-1}-1)\mathbf{u}_{k0}\mathbf{v}_{k0}^T + (z^{-1}-1)\mathbf{u}_{k0}\mathbf{v}_{k1}^T\bar{\mathbf{S}}\\
\mathbf{S}_{kb}(z) &= (z^{-1}-1)\mathbf{u}_{k0}\mathbf{v}_{k1}^T\\
\mathbf{S}_{kc}(z) &= (z^{-1}-1)\mathbf{u}_{k1}\mathbf{v}_{k0}^T + (\mathbf{I}_{\gamma_k} + (z^{-1}-1)\mathbf{u}_{k1}\mathbf{v}_{k1}^T)\bar{\mathbf{S}}\\
\mathbf{S}_{kd}(z) &= \mathbf{I}_{\gamma_k} + (z^{-1}-1)\mathbf{u}_{k1}\mathbf{v}_{k1}^T
\end{aligned}$$

Next, $\mathbf{T}_k$ is multiplied from the left side of $\mathbf{S}_k(z)$. Finally, the product is represented as follows:

$$\begin{aligned}
&\mathbf{T}_k\mathbf{S}_k(z)\\
&= \begin{bmatrix}\mathbf{S}_{ka}(z)\mathbf{S}_{kb}(z) & \\ \bar{\mathbf{T}}_k\mathbf{S}_{ka}(z) + \mathbf{S}_{kc}(z) & \bar{\mathbf{T}}_k\mathbf{S}_{kb}(z) + \mathbf{S}_{kd}(z)\end{bmatrix} \triangleq \mathbf{T}_k(z)
\end{aligned}$$

As mentioned above, if the product $\mathbf{T}_k(z)$ has a pure block-lifting structure, $\mathbf{E}_k(z)$ can be factorized into a number of block-lifting matrices such as $\mathbf{E}_k(z) = \mathbf{T}_k^{-1}\mathbf{T}_k(z)\mathbf{S}_k^{-1}$.

The conditions that $\mathbf{T}_k(z)$ has a block-lifting structure are obtained as follows:[1]

$$\mathbf{S}_{ka}(z) = \mathbf{I}_{\delta_k} \qquad (3)$$
$$\bar{\mathbf{T}}_k\mathbf{S}_{ka}(z) + \mathbf{S}_{kc}(z) = \mathbf{0} \qquad (4)$$
$$\bar{\mathbf{T}}_k\mathbf{S}_{kb}(z) + \mathbf{S}_{kd}(z) = \mathbf{I}_{\gamma_k}. \qquad (5)$$

From (3) and (4), the condition of $\bar{\mathbf{S}}_k$ and $\bar{\mathbf{T}}_k$ are obtained as

$$\bar{\mathbf{S}}_k = -\mathbf{v}_{k1}^{-T}\mathbf{v}_{k0}^T \quad \text{and} \quad \bar{\mathbf{T}}_k = \mathbf{v}_{k1}^{-T}\mathbf{v}_{k0}^T. \qquad (6)$$

Under the conditions (6) and $\mathbf{v}_{k0}^T\mathbf{u}_{k0} + \mathbf{v}_{k1}^T\mathbf{u}_{k1} = \mathbf{I}_{\gamma_k}$ in (2), (5) can be rewritten as

$$\bar{\mathbf{T}}_k\mathbf{S}_{kb}(z) + \mathbf{S}_{kd}(z) = z^{-1}\mathbf{I}_{\gamma_k}.$$

Consequently, $\mathbf{T}_k(z)$ is formulated as

$$\mathbf{T}_k\mathbf{E}_k(z)\mathbf{S}_k = \begin{bmatrix}\mathbf{I}_{\delta_k} & (z^{-1}-1)\mathbf{u}_{k0}\mathbf{v}_{k1}^T\\ \mathbf{0} & z^{-1}\mathbf{I}_{\gamma_k}\end{bmatrix}.$$

Therefore, a *degree-N* building block of BOFB $\mathbf{E}_k(z) = \mathbf{I}_M - \mathcal{U}_k\mathcal{V}_k^T + z^{-1}\mathcal{U}_k\mathcal{V}_k^T$ in (1) can be factorized into

$$\begin{aligned}
\mathbf{E}_k(z) &= \begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{0}\\ -\mathbf{v}_{k1}^{-T}\mathbf{v}_{k0}^T & \mathbf{I}_{\gamma_k}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{u}_{k0}\mathbf{v}_{k1}^T\\ \mathbf{0} & \mathbf{I}_{\gamma_k}\end{bmatrix}\mathbf{\Lambda}_k(z)\\
&\times \begin{bmatrix}\mathbf{I}_{\delta_k} & -\mathbf{u}_{k0}\mathbf{v}_{k1}^T\\ \mathbf{0} & \mathbf{I}_{\gamma_k}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{0}\\ \mathbf{v}_{k1}^{-T}\mathbf{v}_{k0}^T & \mathbf{I}_{\gamma_k}\end{bmatrix}. \qquad (7)
\end{aligned}$$

The design parameters of *degree-N* building block $\mathbf{E}_k(z)$ are utilized for $\mathbf{u}_{k0}$, $\mathbf{v}_{k0}$ and $\mathbf{v}_{k1}$ in (2). For simplicity, since $\mathbf{v}_{k1}^{-T}\mathbf{v}_{k0}^T$ and $\mathbf{u}_{k0}\mathbf{v}_{k1}^T$ can be any $\gamma_k \times \delta_k$ and $\delta_k \times \gamma_k$ matrices, respectively, (7) can be represented by

$$\mathbf{E}_k(z) = \begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{0}\\ -\mathbf{L}_k & \mathbf{I}_{\gamma_k}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{U}_k\\ \mathbf{0} & \mathbf{I}_{\gamma_k}\end{bmatrix}\mathbf{\Lambda}_k(z)\begin{bmatrix}\mathbf{I}_{\delta_k} & -\mathbf{U}_k\\ \mathbf{0} & \mathbf{I}_{\gamma_k}\end{bmatrix}\begin{bmatrix}\mathbf{I}_{\delta_k} & \mathbf{0}\\ \mathbf{L}_k & \mathbf{I}_{\gamma_k}\end{bmatrix}$$

where $\mathbf{L}_k$ and $\mathbf{U}_k$ are $\gamma_k \times \delta_k$ and $\delta_k \times \gamma_k$ any matrices, respectively, as shown in Fig. 3. The number of design parameters is $2\delta_k\gamma_k$. Note that the restriction $\det(\mathbf{E}_k(z)) = \pm 1$, which has been conventionally required for lifting factorization, is not required by using the proposed block-lifting structure. As a result, more efficient FBs can be designed comparing to the conventional lifting based FBs.

[1] Strictly speaking, to obtain an integer-to-integer transform, (3) and (5) do not have to be $\mathbf{I}$. $z^{-n}\mathbf{I}$ is also allowed.

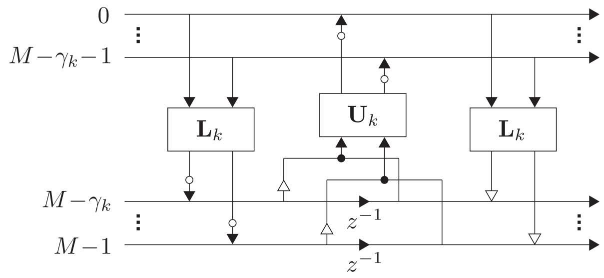

Fig. 3. A block-lifting structure of *degree-N* building blocks of BOFB (white circles, black circles and white triangles represent rounding, adder and -1 operations, respectively).

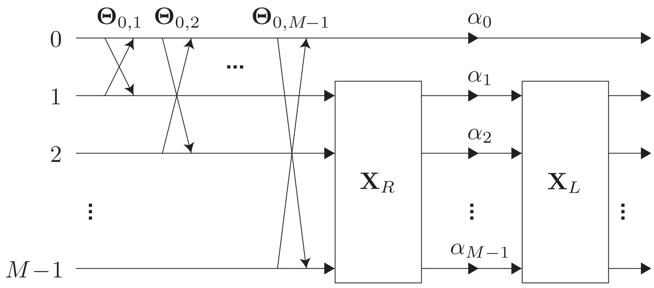

Fig. 4. The parameterization of the initial block $\mathbf{X}_0$.

3.2. Initial Block

In FB theory, the regularity is an important property for image compression. Therefore, we investigated lattice structures of FB with one degree of regularity. This paper uses initial block $\mathbf{X}_0$ with structural $(1,1)$-regularity as [12]

$$\mathbf{X}_0 = \left[\begin{array}{c|c} 1 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{X}_L \end{array}\right] \mathrm{diag}\{\alpha_0, \alpha_1, \cdots, \alpha_{M-1}\} \times \left[\begin{array}{c|c} 1 & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{X}_R \end{array}\right] \mathbf{\Theta}_{M-1}\mathbf{\Theta}_{M-2}\cdots\mathbf{\Theta}_1$$

where $\mathbf{X}_L$ and $\mathbf{X}_R$ are $(M-1)\times(M-1)$ orthogonal matrices, and

$$[\mathbf{\Theta}_j]_{k,l} = \begin{cases} 1 & (k = l \neq 0 \text{ or } j) \\ \cos(\arctan\frac{1}{\sqrt{k}}) & (k = l = 0 \text{ or } j) \\ \sin(\arctan\frac{1}{\sqrt{k}}) & (k = 0 \text{ and } l = j) \\ -\sin(\arctan\frac{1}{\sqrt{k}}) & (k = j \text{ and } l = 0) \\ 0 & (\text{otherwise}) \end{cases}$$

as shown in Fig. 4.

Next, the initial block $\mathbf{X}_0$ must be factorized into lifting structures. However, since the proposed block-lifting factorization is not applicable, we use the single-row elementary reversible matrix (SERM) structures in [9]. SERM factorization of $\mathbf{X}_0$ is implemented by only $M+1$ rounding operations if $\det(\mathbf{X}_0) = \pm 1$. Since $\det(\mathbf{X}_L) = \det(\mathbf{X}_R) = 1$ is already satisfied, only $\mathbf{D}$ is restricted by $\alpha_{M-1} = \prod_{i=0}^{M-2}\alpha_i^{-1}$ to obtain the perfect lifting structures. As a result, $\mathbf{X}_0$ without the structural regularity is designed by M^2-1 parameters. It means that only one parameter is restricted to implement whole lifting factorization of a BOFB.

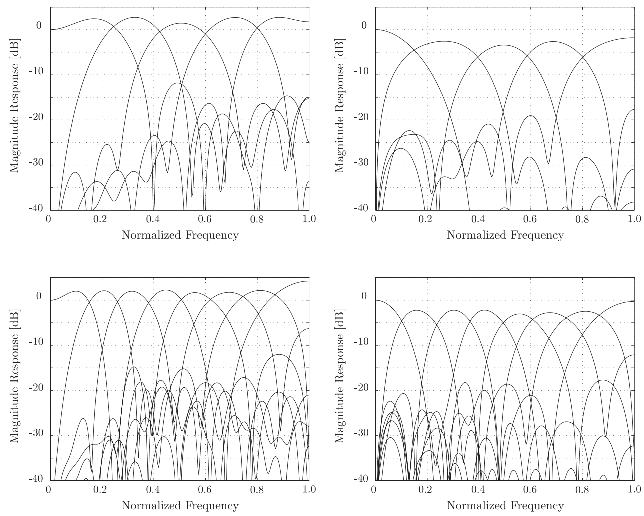

Fig. 5. Frequency responses of the proposed BOFBs ($N=2$): (top) 5×15 analysis and synthesis, (bottom) 8×24 analysis and synthesis.

4. DESIGN AND EXPERIMENTAL RESULTS

4.1. Design Examples

In this paper, we present several design examples of the proposed BOFBs. The cost function to design these FBs is a weighted linear combination of the coding gain C_{cdg} and the stopband attenuations of analysis and synthesis filters $C_{ana,stpb}$ and $C_{syn,stpb}$ as follows [12]:

$$\Phi = \sum_{i,j}(w_1 C_{ana,stpb} + w_2 C_{syn,stpb}) - w_3 C_{cdg}$$

The weighting coefficients $\{w_1, w_2, w_3\}$ have been set empirically. We optimized design parameters by cost function Φ using `fminunc.m` in Optimization ToolBox of MATLAB. The proposed method can design block-lifting based BOFBs even if M is odd or McMillan degree is N as shown in Fig. 5[2].

4.2. Lossless-to-Lossy Image Coding

8×24 BOFBs with *degree-2* and *degree-4* [i.e. *order-1*] building blocks are applied to lossless-to-lossy image coding. Based on the optimized parameters, lifting coefficients are determined. Integer-to-integer transform can be obtained by using a rounding operation at each lifting step. To evaluate transform performance fairly, a wavelet-based coder EZW-IP [13] was used in the simulation. Also, for image boundaries, the periodic extension was used in our proposals and [8], and the symmetric extension was used in 9/7 and 5/3-tap DWTs.

Since the proposed FBs are integer-to-integer transforms, we can obtain lossless reconstructed images at high bitrate. Lossless coding performance in lossless bitrate [bpp] is shown in Table 1. In Table 2, we also compare the lossy coding results in peak signal-to-noise ratio (PSNR) [dB] at 0.25, 0.50 and 1.00 bit per pixel (bpp) for several standard images. The results for the images contain rich textures show the effectiveness of the proposed method. Fig. 6 illustrates the comparison of the part of the enlarged images of *Barbara*. It

[2]BLBOFBs in Fig. 5 were designed by considering only the coding gain C_{cdg}.

Table 1. Comparison of lossless results (lossless bitrate [bpp]).

Images	5/3-tap DWT [5]	4 × 8 HLBT [6]	8 × 24 BOFBs ($N = 2$)	($N = 4$)
Barbara	4.87	4.81	4.78	**4.77**
Boat	5.10	5.13	5.10	**5.09**
Elaine	5.11	5.17	5.07	**5.04**
Finger1	5.84	5.71	**5.63**	5.65
Finger2	5.60	5.42	**5.37**	**5.37**
Grass	6.06	6.05	**6.04**	6.05

Table 2. Comparison of lossy results (PSNR [dB]).

bitrate [bpp]	9/7-tap DWT [5]	4 × 8 HLBT [6]	8 × 24 BOFBs ($N = 2$)	($N = 4$)
Image: *Barbara*				
0.25	27.28	26.85	27.94	**28.67**
0.50	30.55	30.43	31.60	**32.23**
1.00	35.16	35.05	36.01	**36.41**
Image: *Finger1*				
0.25	23.52	22.96	23.56	**23.85**
0.50	26.02	25.56	26.67	**26.95**
1.00	29.15	29.01	30.55	**30.81**
Image: *Grass*				
0.25	24.38	23.99	24.25	**24.51**
0.50	26.13	25.86	26.32	**26.62**
1.00	28.75	28.68	29.10	**29.44**

is obvious that the proposed FB achieves comparable or even better performance on perceptual visual quality of reconstructed image against the conventional methods. Our FBs show the effectiveness for the images with high frequency components in both lossless and lossy image coding.

5. CONCLUSION

In this paper, we introduced a block-lifting structure of M-channel biorthogonal filter banks (BOFBs) with *degree-N* building blocks. Since block-lifting structure with rounding operations can implement integer-to-integer transform, they can be easily used for lossless-to-lossy image coding scenario. Also, block-lifting structure of each building block of BOFBs does NOT require the restriction such as paraunitary, fixed determinant or symmetry. In addition, although the previous work is restricted to *order-1* structure and even M, the proposed method can achieve *degree-N* structure and even/odd M. As a result, the proposed method showed excellent coding performance due to a more efficient filter design for lossless-to-lossy image coding. The reduction of computational cost is a future work.

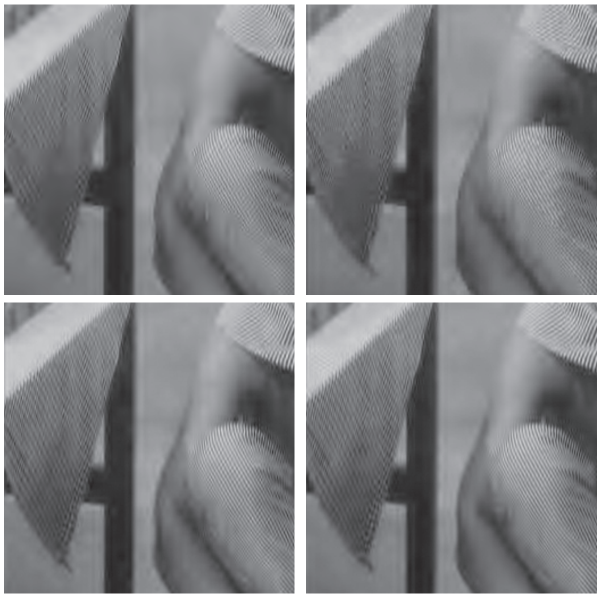

Fig. 6. Comparison of the part of the enlarged images of *Barbara* (bitrate: 0.25[bpp]): (top-left) 9/7-tap DWT (JPEG2000), (top-right) 4×8 HLBT (JPEG XR), (bottom-left) 8×24 BOFB ($N = 2$), (bottom-right) 8×24 BOFB ($N = 4$).

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