TWO DIMENSIONAL NON-SEPARABLE ADAPTIVE DIRECTIONAL LIFTING STRUCTURE OF DISCRETE WAVELET TRANSFORM

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ABSTRACT

In this paper, we propose a two dimensional (2D) non-separable adaptive directional lifting (ADL) structure of discrete wavelet transform (DWT) and its image coding application. Conventionally, we have proposed a polyphase representation of a 2D non-separable lifting structure of DWT. We generalize the polyphase representation in this paper and one structure of the class has been proven to reduce errors due to the rounding operations and improve a compatibility of the irreversible 9/7 DWT, compared with a 2D separable lifting structure of DWT. Adding the adaptive directional transforming property to the generalized structure, our proposed method improves the lossy image coding performance with maintaining the compatibility.

Index Terms— two dimensional non-separable lifting structure, discrete wavelet transform, adaptive directional transform, lossy to lossless image coding.

1. INTRODUCTION

The discrete wavelet transform (DWT) has been a fundamental tool for image and video processing for the last few decades. It is applied to the JPEG 2000 international image coding standard [1]

The irreversible 9/7 DWT utilized by the JPEG 2000 is efficient for the lossy image coding. However, it can not be applied to the lossless image coding. For the lossless image coding, the lifting structure of 9/7 DWT has been proposed [2]. It realizes the integer-to-integer transform, but it introduces a mismatching error due to the rounding operations when the 9/7 DWT is connected to the lifting structure of 9/7 DWT. Iwahashi $et\ al.$ have proven that the two dimensional (2D) non-separable lifting structure of 9/7 DWT requires less rounding operators than the conventional one and improves the compatibility.

These structures of DWT are efficient for the image coding application. However, since they can only transform images along the vertical and horizontal direction, they fail to provide sparse representation of transforming images when images contain rich directional high frequency component, such as edges and contours, and the coding efficiency is degraded. To avoid this degradation, an adaptive directional transform has been proposed [3–5]. The adaptive directional transform can flexibly switch the filtering direction according to the direction of edges and contours and compression efficiency is improved. Especially, one dimensional (1D) adaptive directional lifting (ADL) based wavelet transform [4] achieves high compression efficiency for the lossy image coding. However, since 1D ADL has to be applied twice for the separately implementation of 2D signals, it is redundant and provides a degradation of desired filtering directions due to the downsampling between the first and second filtering. Therefore, we have proposed the 2D ADL [5]. The 2D ADL is simpler than the 1D one and can select the filtering directions which are different from 1D ADL. It is efficient for the lossless image coding, but it is only 5/3 DWT.

In this paper, we propose a 2D non-separable ADL structure of DWT. We generalize the polyphase representation of 2D non-separable lifting structure of DWT. For the adaptive directional transform, the proposed method is realized by changing the sampling matrix by sub-regions of images. Therefore, with advantages of the non-separable structure and the adaptive directional transform, the proposed structure improves the performance of the lossy-to-lossless image coding application. Finally, lossy and lossless image coding results of the proposed structure are shown to validate the advantage of the proposed structure.

Notations : $\mathbf{A}^T$ denotes the transpose of the matrix $\mathbf{A}$. $\mathbf{I}$ and $\mathbf{0}$ are identity and the null matrices, respectively. $\mathbf{M}$ is an 2×2 nonsingular integer matrix. $\downarrow \mathbf{M}$ and $\uparrow \mathbf{M}$ also represent the down- and up-samplers with $\mathbf{M}$, respectively. Using vectors $\mathbf{z} = [z_x, z_y]^T$ and $\mathbf{k} = [k_x, k_y]^T$, a multiplication of vectors is defined as $\mathbf{z}^{\mathbf{k}} = z_x^{k_x} z_y^{k_y}$. $\mathrm{diag}(\cdot)$ denotes the block diagonal matrix.

2. REVIEW

The polyphase representation of 1D lifting structure of DWT can be expressed as follows [2], [6].

$$\begin{aligned}\begin{bmatrix} H_L(z) \\ H_H(z) \end{bmatrix} &= \begin{bmatrix} s & 0 \\ 0 & 1/s \end{bmatrix} \prod_{i=N-1}^{0} \left\{ \begin{bmatrix} 1 & U_i(z^2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ P_i(z^2) & 1 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ z^{-1} \end{bmatrix} \\ &= \mathbf{E}_{1\mathrm{D}}(z^2)\mathbf{d}_2(z)\end{aligned} \tag{1}$$

According to the separable implementation, the polyphase representation of the 2D separable lifting structure of DWT can be given by

$$\begin{aligned}\begin{bmatrix} H_{LL}(\mathbf{z}) \\ H_{HL}(\mathbf{z}) \\ H_{LH}(\mathbf{z}) \\ H_{HH}(\mathbf{z}) \end{bmatrix} &= \begin{bmatrix} H_L(z_x)H_L(z_y) \\ H_L(z_x)H_H(z_y) \\ H_H(z_x)H_L(z_y) \\ H_H(z_x)H_H(z_y) \end{bmatrix} = \begin{bmatrix} H_L(z_y) & 0 \\ H_H(z_y) & 0 \\ 0 & H_L(z_y) \\ 0 & H_H(z_y) \end{bmatrix} \begin{bmatrix} H_L(z_x) \\ H_H(z_x) \end{bmatrix} \\ &= \mathbf{E}(\mathbf{z}^{\mathbf{M}})\mathbf{d}_{\mathbf{M}}(\mathbf{z})\end{aligned} \tag{2}$$

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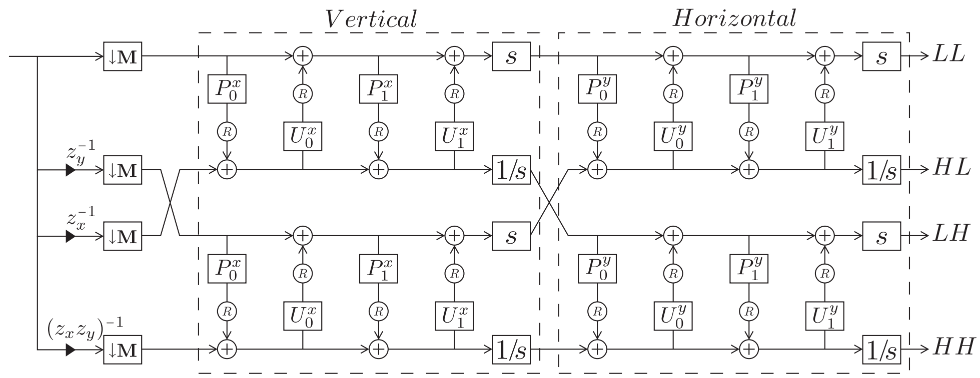


Fig. 1. 2D separable lifting structure of 9/7 DWT

where

$$\begin{aligned}\mathbf{M} &= \mathrm{diag}(2,2),\\ \mathbf{d_M}(\mathbf{z}) &= [\mathbf{z}^{-\mathbf{k}_0}, \mathbf{z}^{-\mathbf{k}_1}, \mathbf{z}^{-\mathbf{k}_2}, \mathbf{z}^{-\mathbf{k}_3}]^T,\\ \mathbf{z}^{\mathbf{M}} &= [z_x^{m_{0,0}} z_y^{m_{1,0}}, z_x^{m_{0,1}} z_y^{m_{1,1}}]^T,\end{aligned}$$

$\mathbf{z} = [z_x, z_y]^T$, z_x and z_y are the vertical and horizontal delay elements, respectively, $\mathbf{k}_i$ $(i = 0,1,2,3)$ is an integer vector within $\Xi(\mathbf{M})$, called as a delay vector, $\mathbf{k}_0$ is restricted to be $[0,0]^T$, $\Xi(\mathbf{M})$ is a set of integer vectors, which is defined as

$$\Xi(\mathbf{M}) = \{\mathbf{Mx} \mid \mathbf{x} \in [0,1)\},$$

$\mathbf{x} \in [0,1)$ denotes a set of 2×1 real vectors $\mathbf{x}$ whose the ith component satisfies $0 \le x_i < 1$ and $m_{i,j}$ denotes the (i,j) element of $\mathbf{M}$. The polyphase matrix $\mathbf{E}(\mathbf{z})$ is factorized as

$$\mathbf{E}(\mathbf{z}) = \begin{bmatrix}\mathbf{E}^y_{1\mathrm{D}} & \mathbf{0}\\ \mathbf{0} & \mathbf{E}^y_{1\mathrm{D}}\end{bmatrix} \begin{bmatrix} s\mathbf{I} & \mathbf{0}\\ \mathbf{0} & 1/s\mathbf{I}\end{bmatrix} \prod_{i=N-1}^{0} \left\{ \begin{bmatrix}\mathbf{I} & U_i^x\mathbf{I}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ P_i^x\mathbf{I} & \mathbf{I}\end{bmatrix}\right\} \tag{3}$$

where $\mathbf{E}^y_{1\mathrm{D}}$, P_i^j and U_i^j $(j = x, y)$ denote $\mathbf{E}_{1\mathrm{D}}(z_y)$, $P_i(z_j)$ and $U_i(z_j)$. According to the polyphase matrix, the delay vectors are decided as

$$\mathbf{d_M}(\mathbf{z}) = [1, z_y^{-1}, z_x^{-1}, z_x^{-1} z_y^{-1}]^T$$

For example, fig.1 shows the 2D separable lifting structure of 9/7 DWT with rounding operators R in lifting steps. Fig.1 shows that images are divided into four sets and transformed with maintaining an integer.

The pair of scaling factors can be realized as a lifting structure [2]. The scaling is factorized as

$$\begin{bmatrix} s & 0\\ 0 & 1/s\end{bmatrix} = \begin{bmatrix}1 & s-s^2\\ 0 & 1\end{bmatrix} \begin{bmatrix}1 & 0\\ -1/s & 1\end{bmatrix} \begin{bmatrix}1 & s-1\\ & 1\end{bmatrix} \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}. \tag{4}$$

For simplicity, scaling is not represented as the lifting structure in this paper.

3. 2D NON-SEPARABLE ADL STRUCTURE OF DWT

3.1. Basic structure

In [5], we have proposed the polyphase representation of the 2D non-separable lifting structure of 5/3 DWT by factorizing the separable one in (2). In this paper, we introduce the generalized polyphase representation. Here, some matrices are defined as follows.

$$\begin{aligned}
\mathbf{S} &= \mathrm{diag}\left(\begin{bmatrix}s & 0\\ 0 & 1/s\end{bmatrix}, \begin{bmatrix}s & 0\\ 0 & 1/s\end{bmatrix}\right), & \hat{\mathbf{S}} &= \begin{bmatrix}s\mathbf{I} & \mathbf{0}\\ \mathbf{0} & 1/s\mathbf{I}\end{bmatrix},\\
\mathbf{U}_i^j &= \mathrm{diag}\left(\begin{bmatrix}1 & U_i^j\\ 0 & 1\end{bmatrix}, \begin{bmatrix}1 & U_i^j\\ 0 & 1\end{bmatrix}\right), & \hat{\mathbf{U}}_i^j &= \begin{bmatrix}\mathbf{I} & U_i^j\mathbf{I}\\ \mathbf{0} & \mathbf{I}\end{bmatrix},\\
\mathbf{P}_i^j &= \mathrm{diag}\left(\begin{bmatrix}1 & 0\\ P_i^j & 1\end{bmatrix}, \begin{bmatrix}1 & 0\\ P_i^j & 1\end{bmatrix}\right), & \hat{\mathbf{P}}_i^j &= \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ P_i^j\mathbf{I} & \mathbf{I}\end{bmatrix}.
\end{aligned} \tag{5}$$

Using these matrices, (3) is rewritten as

$$\mathbf{E}(\mathbf{z}) = \mathbf{S} \prod_{i=N-1}^{0} \{\mathbf{U}_i^y \mathbf{P}_i^y\} \hat{\mathbf{S}} \prod_{i=N-1}^{0} \left\{\hat{\mathbf{U}}_i^x \hat{\mathbf{P}}_i^x\right\}. \tag{6}$$

$\hat{\mathbf{S}}$ and $\hat{\mathbf{U}}_i^x \hat{\mathbf{P}}_i^x$ can be moved as follows.

$$\mathbf{U}_i^y \mathbf{P}_i^y \hat{\mathbf{S}} = \hat{\mathbf{S}} \mathbf{U}_i^y \mathbf{P}_i^y, \quad \mathbf{U}_p^y \mathbf{P}_p^y \hat{\mathbf{U}}_q^x \hat{\mathbf{P}}_q^x = \hat{\mathbf{U}}_q^x \hat{\mathbf{P}}_q^x \mathbf{U}_p^y \mathbf{P}_p^y. \tag{7}$$

where $p, q = 0, 1, \ldots, N-1$. We have factorized $\mathbf{U}_p^y \mathbf{P}_p^y \hat{\mathbf{U}}_q^x \hat{\mathbf{P}}_q^x$ as $\hat{\mathbf{E}}_{p,q}$ in [5], where

$$\hat{\mathbf{E}}_{p,q} = \begin{bmatrix}1 & U_p^y & U_q^x & -U_p^y U_q^x\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}1 & 0 & 0 & 0\\ P_p^y & 1 & 0 & U_q^x\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ P_q^x & 0 & 1 & U_p^y\\ 0 & 0 & 0 & 1\end{bmatrix} \times \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ P_p^y P_q^x & P_q^x & P_p^y & 1\end{bmatrix} \tag{8}$$

Using (7) and (8), the any polyphase matrix $\mathbf{E}(\mathbf{z})$ of the 2D non-separable lifting structure of DWT can be realized. For example, in the case of 9/7 DWT, $\mathbf{E}(\mathbf{z})$ is designed as

$$\mathbf{E}(\mathbf{z}) = \tilde{\mathbf{S}} \mathbf{U}_1^y \mathbf{P}_1^y \hat{\mathbf{E}}_{0,1} \hat{\mathbf{U}}_0^x \hat{\mathbf{P}}_0^x \tag{9}$$

where $\tilde{\mathbf{S}} = \mathbf{S}\hat{\mathbf{S}} = \mathrm{diag}(s^2, 1, 1, 1/s^2)$. This structure is shown in fig.2. Compared with fig.1, it is shown to require less rounding operators. Therefore, the non-separable structure has better compatibilities to the irreversible 9/7 DWT in the JPEG 2000 than the separable structure [7].

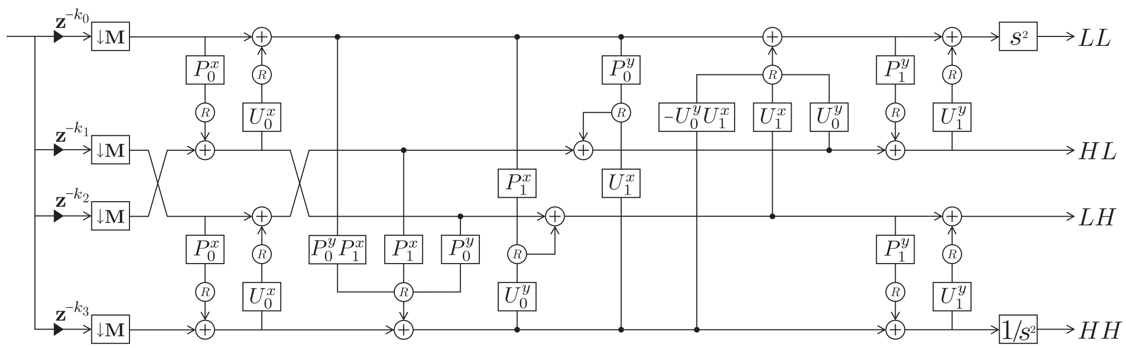


Fig. 2. 2D non-separable lifting structure of 9/7 DWT

3.2. Directional sampling

By changing the sampling matrix $\mathbf{M}$ by sub-regions of images, 2D non-separable ADL structure of DWT is realized. The arbitrary sampling matrix whose absolute determinant is 4 can be selected. The following sampling matrices $\mathbf{M}_d$ ($d = 0, 1, \ldots, 6$) are used in this paper.

$$\left\{\begin{bmatrix}2&0\\0&2\end{bmatrix}, \begin{bmatrix}2&2\\0&2\end{bmatrix}, \begin{bmatrix}2&-2\\0&2\end{bmatrix}, \begin{bmatrix}2&0\\2&2\end{bmatrix}, \begin{bmatrix}2&0\\-2&2\end{bmatrix}, \begin{bmatrix}2&4\\0&2\end{bmatrix}, \begin{bmatrix}2&-4\\0&2\end{bmatrix}\right\}$$

With changing the sampling matrix, the delay vectors have to be changed. For maintaining the linear phase property of filters, we define the delay vectors to be symmetry with respect to a point which is a center of $\mathbf{M}_d$. Table 1 shows the delay vectors corresponded to $\mathbf{M}_d$ and the filtering directions are represented in fig.3. It shows that the proposed structure can transform along with various directions besides vertically and horizontally shown in fig.3(a).

3.3. Optimal direction decision

For the image coding application, the optimal directions are decided according to the energy of highpass subband. Especially, we focus on the energy of "HH" subband in this paper. For all the sampling matrix $\mathbf{M}_d$, the HH subband components are described as h_d. The optimal direction $Dir(m,n)$ is chosen such that it minimize the energy of $h_d(m,n)$,

$$Dir(m,n) = \min_d |h_d(m,n)|. \tag{10}$$

Practically, the direction information is assigned to not the pixels but the blocks divided by the quad tree decomposition in order to reduce the side information. For this purpose, R-D optimization is processed as in the following. The full quad tree T is constructed by applying the quad tree decomposition to the image until reaching the predefined block size. B, $D(B)$ and R(B) are defined as an arbitrary subtree, a distortion and a rate of bits. The most suitable subtree B^* with optimal direction is provided by minimizing the cost function $J(B)$ expressed by

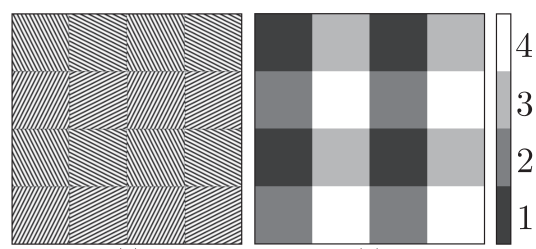
$$B^* = \min_B J(B) = \min_B (D(B) + \lambda R(B)) \tag{11}$$

where

$$D(B) = \sum_{\tau \in B} \sum_{m,n} |h_{\tau,d}(m,n)|,$$
$$R(B) = \sum_{\tau \in B} r_C(\tau) + \sum_{v \in B} r_T(v),$$

Table 1. Delay vectors

d	k_0	k_1	k_2	k_3
0	$[0,0]^T$	$[0,1]^T$	$[1,0]^T$	$[1,1]^T$
1	$[0,0]^T$	$[1,1]^T$	$[1,0]^T$	$[2,1]^T$
2	$[0,0]^T$	$[-1,1]^T$	$[1,0]^T$	$[0,1]^T$
3	$[0,0]^T$	$[0,1]^T$	$[1,1]^T$	$[1,2]^T$
4	$[0,0]^T$	$[0,1]^T$	$[1,-1]^T$	$[1,0]^T$
5	$[0,0]^T$	$[2,1]^T$	$[1,0]^T$	$[3,1]^T$
6	$[0,0]^T$	$[-2,1]^T$	$[1,0]^T$	$[-1,1]^T$



(a) (b)

Fig. 4. Simulation of directional selectivity

τ denotes a node of B, $r_C(\tau)$ and $r_T(v)$ are defined as the rate of the entropy encoding high pass coefficients in τ and coding the side information in node in v, respectively. Applying the automatical decision of optimal directions to a simulation image is shown in fig.4. Along directions of stripes, the filtering directions are selected correctly.

4. SIMULATION

Here, we apply the proposed structure shown in fig.2 into the lossy-to-lossless image coding application and compare the coding efficiency with the conventional one shown in fig.1. The embedded zerotree wavelet based on intraband partitioning (EZW-IP) [8] is used as the encoder. Since transformed coefficients are integer and encoded images are lossless bit streams, lossy bit streams are produced by discarding bits in the lower bit plane of the bit streams. The side information is encoded by the arithmetic coding algorithm [9]. At

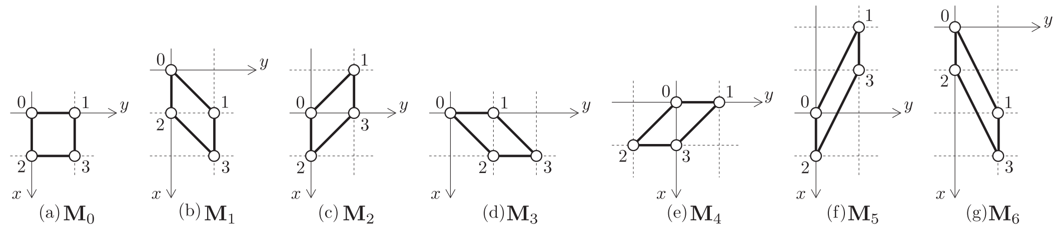


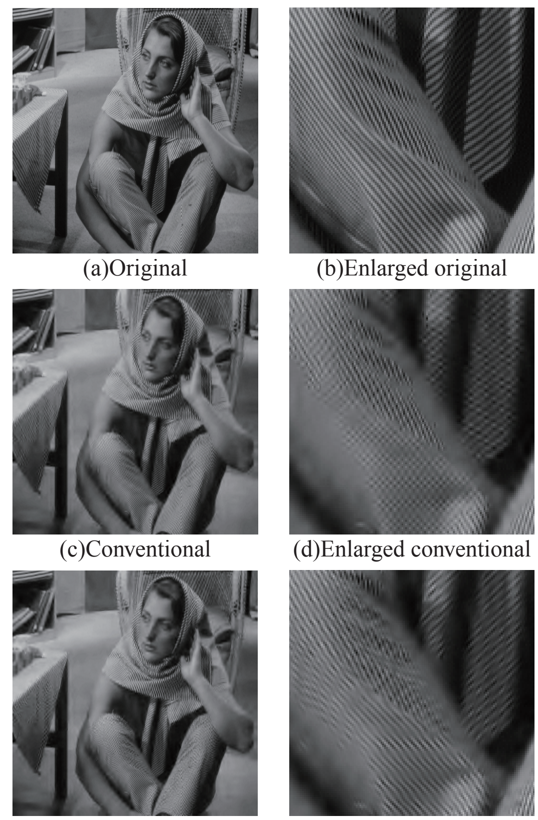
Fig. 3. Sampling matrices

Table 2. Image coding results

Lossless [bpp]		
	Conventional	Proposed
Entropy	4.81	4.76
Lossy [dB]		
Rate [bpp]	Conventional	Proposed
0.25	27.23	27.47
0.5	30.45	30.78
1	34.84	35.16

the lossy image coding application, the peak signal-to-noise ratio (PSNR) is used as a objective function measuring an image quality of reconstructed images. As a test image, $Barbara$ is used.

Table 2 indicates results of the lossless and lossy image coding application. From the table, the proposed structure improves the lossy-to-lossless image coding application compared with the conventional structure, because of the adaptive directional transform. In fig.5, the original and reconstructed images are represented, where the compression rate is 0.25 [bpp]. Compared with the conventional structure, the proposed structure can represent various direction besides vertically and horizontally. Especially, stripe textures in fig.5(f) are represented clearly.



(a)Original (b)Enlarged original
(c)Conventional (d)Enlarged conventional
(e)Proposed (f)Enlarged proposed

Fig. 5. Original images and reconstructed images

5. CONCLUSION

In this paper, we generalize the polyphase representation of the 2D non-separable lifting structure of DWT and propose ADL of the structure. With maintaining the advantage of the 2D non-separable and the adaptive directional transform, the proposed structure achieves better image coding results than the conventional one.

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