Structurally Regular Integer Discrete Cosine Transform for Low-Bit-Word-Length Coefficients

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***Abstract*— This paper presents an IntDCT with only dyadic values such as $k/2^n$ (k, $n \in \mathbb{N}$). Although some IntDCTs have been proposed, they are unsuitable for lossless-to-lossy image coding in low-bit-word-length (coefficients). First, the proposed M-channel lossless WHT (LWHT) can be constructed by only ($\log_2 M$)-bit-word-length and has structural regularity. Then, our 8-channel IntDCT keeps good coding performance even in low-bit-word-length because LWHT, which is main part of IntDCT, can be implemented by 3-bit-word-length. Finally, our method is validated in lossless-to-lossy image coding.**

Index Terms **— IntDCT, dyadic values, low-bit-word-length, LWHT, lossless-to-lossy image coding**

I. Introduction

Multimedia coding (compression) standards such as JPEG and MPEG use the discrete cosine transform (DCT) [1] which has excellent properties, several types and many fast algorithms. In JPEG, DCT type-II (DCT-II) and -III (DCT-III), which is the inverse transform of DCT-II, are applied to lossy image coding, and the differential pulse code modulation (DPCM) is applied to lossless image coding. We have to prepare lossy and lossless compressed data separately according to application.

Recently, high speed transmission of high quality data has become very desirable due to the advances in Internet and multimedia contents. Several integer DCTs (IntDCTs), which are constructed by lifting structures [2] and rounding operations based on DCT-II, have been proposed for lossless-to-lossy image coding [3–6]. However, the conventional IntDCTs generate a checker-board artifact in low-bit-word-length (coefficients).

In this paper, we propose an IntDCT via LWHT with structural regularity even if low-bit-word-length is used. First, IntDCT is separated to Walsh-Hadamard transform (WHT) as the pre-processing part and the others as the post-processing part [5]. Second, we present an M-channel lossless WHT (LWHT) which can be constructed by only ($\log_2 M$)-bit-word-length. It is achieved by using the scaling shift and considering the two-dimensional (2-D) separable transform of one-dimensional (1-D) WHT. The proposed IntDCT has regularity which is imposed structurally by LWHT even if low-bit-word-length is used. As result, we can achieve an IntDCT with low cost and fast implementation. Finally, the validity is shown by the comparison of our methods and the conventional methods in the results of lossless-to-lossy image coding.

Notations: $\mathbf{I}$ is an identity matrix, $\mathbf{M}^T$ is a transpose of matrix $\mathbf{M}$, and $\mathbf{M}^{[N]}$ is an $N \times N$ square matrix, respectively.

II. Review

A. Multiplierless Lifting Structure

The lifting structure [2], also known as the ladder structure, is a special type of lattice structure, a cascading construction using only elementary matrices - identity matrices with single nonzero off-diagonal element.

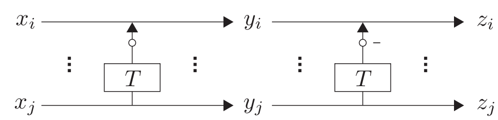


Fig. 1. A lifting structure (white circles: rounding operations).

Fig. 1 shows a basic lifting structure. In this case, the lifting matrix and its inverse matrix are as follows:

$$\begin{bmatrix}1 & T\\0 & 1\end{bmatrix}, \quad \begin{bmatrix}1 & T\\0 & 1\end{bmatrix}^{-1} = \begin{bmatrix}1 & -T\\0 & 1\end{bmatrix}$$

where T is lifting coefficient. Then, they are represented by

$$\begin{aligned}y_i &= x_i + round[Tx_j] \\ y_j &= x_j\end{aligned} \quad \rightarrow \quad \begin{aligned}z_i &= y_i - round[Ty_j] = x_i \\ z_j &= y_j = x_j\end{aligned}$$

where $round[.]$ is rounding operation. Thus the lifting structure with rounding operation can achieve lossless-to-lossy coding.

For high-speed implementation, lifting coefficients are required to approximate floating-point to hardware-friendly dyadic values such as $k/2^n$ (k, $n \in \mathbb{N}$) which can be implemented by only bit-shift and addition operations. It performs fast implementation in a real time software encoder and reduce the circuit size. The multiplications are expressed by $k/2^n$ (k, $n \in \mathbb{N}$) which is n-bit-word-length. For example, a coefficient $51/2^8 = 51/256$ can be operated as

$$\begin{aligned}\frac{51}{256} &= \frac{32}{256} + \frac{16}{256} + \frac{2}{256} + \frac{1}{256} \\ &= \frac{1}{8} + \frac{1}{16} + \frac{1}{128} + \frac{1}{256} \\ &= \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^7} + \frac{1}{2^8}.\end{aligned} \tag{1}$$

From (1), we can replace the multiplier by $51/256$ to sum results from 3 bit-shift, 4 bit-shift, 7 bit-shift, and 8 bit-shift operations as illustrated in Fig. 2. We can find that the perfect reconstruction in lifting structure is always kept even if lifting coefficients are approximated.

B. WHT

As well known, WHT has interesting relationships with other discrete transforms such as DCT [1] etc. It is expressed by

$$\mathbf{W}^{[1]} = \begin{bmatrix}1\end{bmatrix}, \quad \mathbf{W}^{[2]} = \begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$$

$$\mathbf{W}^{[M]} = \mathbf{W}^{[2]} \otimes \mathbf{W}^{[\frac{M}{2}]} = \begin{bmatrix}\mathbf{W}^{[\frac{M}{2}]} & \mathbf{W}^{[\frac{M}{2}]}\\ \mathbf{W}^{[\frac{M}{2}]} & -\mathbf{W}^{[\frac{M}{2}]}\end{bmatrix} \tag{2}$$

where $\otimes$ is Kronecker product. Note that (2) is still not normalized by the scaling factor $1/\sqrt{M}$.

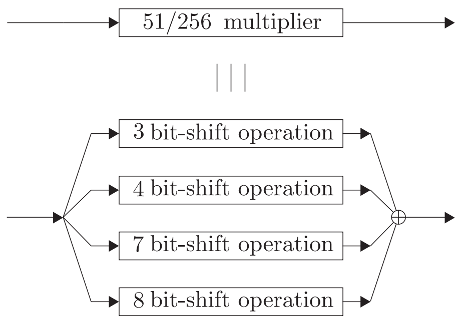


Fig. 2. An approximation from floating multiplication to bit-shift and addition operations.

C. IntDCT via WHT

Let us normalize $\mathbf{W}$ such that $\hat{\mathbf{W}} = (1/\sqrt{M})\mathbf{W}$ is an orthonormal matrix: $\hat{\mathbf{W}}^{-1} = \hat{\mathbf{W}}^T$. Then one can prove that [1]

$$\mathbf{C}_{II}^{[M]} = \hat{\mathbf{Q}}^{[M]}\hat{\mathbf{W}}^{[M]} = \mathbf{P}^{[M]^T}\mathbf{Q}^{[M]}\mathbf{P}^{[M]}\hat{\mathbf{W}}^{[M]} \tag{3}$$

where $\hat{\mathbf{W}}^{[M]}$ and $\hat{\mathbf{Q}}^{[M]}$ are the pre- and post-processing part, respectively. In case of $M = 8$,

$$\mathbf{P}^{[8]} = \mathbf{P}^{[8]^T} = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

and

$$\mathbf{Q}^{[8]} = \begin{bmatrix} \begin{bmatrix} \mathbf{I}^{[2]} & \mathbf{0}^{[2]} \\ \mathbf{0}^{[2]} & \begin{bmatrix} c_8^{\pi} & s_8^{\pi} \\ -s_8^{\pi} & c_8^{\pi} \end{bmatrix} \end{bmatrix} & \mathbf{0}^{[4]} \\ \mathbf{0}^{[4]} & \mathbf{S}^{[4]} \end{bmatrix}$$

where

$$\mathbf{S}^{[4]} = \begin{bmatrix} c_{16}^{7\pi} & 0 & 0 & -s_{16}^{7\pi} \\ 0 & c_{16}^{3\pi} & -s_{16}^{3\pi} & 0 \\ 0 & s_{16}^{3\pi} & c_{16}^{3\pi} & 0 \\ s_{16}^{7\pi} & 0 & 0 & c_{16}^{7\pi} \end{bmatrix} \times \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} c_8^{3\pi} & 0 & -s_8^{3\pi} & 0 \\ 0 & c_8^{3\pi} & 0 & -s_8^{3\pi} \\ s_8^{3\pi} & 0 & c_8^{3\pi} & 0 \\ 0 & s_8^{3\pi} & 0 & c_8^{3\pi} \end{bmatrix},$$

$c_b^a = \cos(a/b)$ and $s_b^a = \sin(a/b)$. To obtain IntDCT via WHT, $\mathbf{W}^{[8]}$ instead of $\hat{\mathbf{W}}^{[8]}$ and the lifting factorization of Givens rotation matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & \frac{\cos\theta - 1}{\sin\theta} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{\cos\theta - 1}{\sin\theta} \\ 0 & 1 \end{bmatrix} \tag{4}$$

are used [5]. In this paper, the n-bit-word-length, $k/2^n$ (k, $n \in \mathbb{N}$ and $3 \leq n \leq 6$), is applied to all transform coefficients α_0-α_9 and β_0-β_4 as shown in Fig. 3 and Table I.

III. Structurally Regular IntDCT via LWHT for Low-Bit-Word-Length

The desired conditions $\sharp 1$ and $\sharp 2$ for IntDCT in lossless-to-lossy image coding are as follows:

TABLE I
LIFTING COEFFICIENTS OF CHEN'S INTDCT [5].

	floating-point	$k/2^3$	$k/2^4$	$k/2^5$	$k/2^6$
α_0	0.1989123674...	1/4	3/16	3/16	13/64
β_0	-0.3826834324...	-3/8	-3/8	-3/8	-3/8
α_1	0.1989123674...	1/4	3/16	3/16	13/64
α_2	-0.6681786379...	-5/8	-11/16	-21/32	-43/64
β_1	0.9238795326...	7/8	15/16	15/16	59/64
α_3	-0.6681786379...	-5/8	-11/16	-21/32	-43/64
α_4	-0.6681786379...	-5/8	-11/16	-21/32	-43/64
β_2	0.9238795326...	7/8	15/16	15/16	59/64
α_5	-0.6681786379...	-5/8	-11/16	-21/32	-43/64
α_6	-0.8206787908...	-7/8	-13/16	-13/16	-53/64
β_3	0.9807852804...	1	1	31/32	63/64
α_7	-0.8206787908...	-7/8	-13/16	-13/16	-53/64
α_8	-0.3033466836...	-1/4	-5/16	-5/16	-19/64
β_4	0.5555702330...	1/2	9/16	9/16	9/16
α_9	-0.3033466836...	-1/4	-5/16	-5/16	-19/64

$\sharp 1$: The transform should not have DC leakage.
$\rightarrow$ If DC leakage is generated, the signal energy decomposed to frequency domain is not sufficiently concentrated. And if so, better compression ratio can not be achieved.
$\Rightarrow$ Regularity should be imposed.

$\sharp 2$: Dynamic range of the output signals should be as small as possible.
$\rightarrow$ If dynamic range is huge, to avoid redundant it, quantization part would be required after transform part. And if so, realization of lossless image coding is impossible due to its error.
$\Rightarrow$ The transform should be normalized without using quantization part.

Although the other conventional ones do not have regularity in low-bit-word-length, IntDCT in [5] satisfies the condition $\sharp 1$. However, it does not satisfies the condition $\sharp 2$ because the coefficients of the pre-processing part $\mathbf{W}^{[8]}$ are not normalized. Hence this paper solves the normalization problem of WHT: $\hat{\mathbf{W}}^{[8]} = (1/\sqrt{8})\mathbf{W}^{[8]}$, in IntDCT in [5] while keeping the other part as it is.

A. Two-Dimensional (2-D) Block Transform

When we apply a block transform matrix $\mathbf{F}$ into an 2-D input signal $\mathbf{x}$ in column- and row-wise separately, the 2-D output signal $\mathbf{y}$ is expressed by

$$\mathbf{y} = (\mathbf{F}(\mathbf{F}\mathbf{x})^T)^T = \mathbf{F}\mathbf{x}\mathbf{F}^T. \tag{5}$$

We call it 2-D separable block transform. If the transform $\mathbf{F}$ is factorized as $\mathbf{F} = \mathbf{F}_1\mathbf{F}_0$ where $\mathbf{F}_0$ and $\mathbf{F}_1$ are the pre- and post-processing part, (5) is represented by

$$\mathbf{y} = \mathbf{F}_1\mathbf{F}_0\mathbf{x}(\mathbf{F}_1\mathbf{F}_0)^T = \mathbf{F}_1\mathbf{F}_0\mathbf{x}\mathbf{F}_0^T\mathbf{F}_1^T.$$

This equation means that 2-D block transform by $\mathbf{F}_1$ is applied after 2-D block transform by $\mathbf{F}_0$. Then we regard $\mathbf{F}_0$ and $\mathbf{F}_1$ as $\hat{\mathbf{W}}$ and $\hat{\mathbf{Q}}$ in (3), respectively.

B. Structurally Regular LWHT for Low-Bit-Word-Length

In this subsection, we consider the normalization of $\mathbf{W}^{[M]}$ in (3) and present M-channel LWHT which has only ± 1 and $\pm 1/M$, $(\log_2 M)$-bit-word-length, and structural regularity.

According to the condition $\sharp 2$, if $\mathbf{W}$ normalized by the scaling factor $1/\sqrt{M}$ is used in transform part, (5) can be represented by

$$\mathbf{y} = \hat{\mathbf{W}}\mathbf{x}\hat{\mathbf{W}}^T = \left(\frac{1}{\sqrt{M}}\mathbf{W}\left(\frac{1}{\sqrt{M}}\mathbf{W}\mathbf{x}\right)^T\right)^T. \tag{6}$$

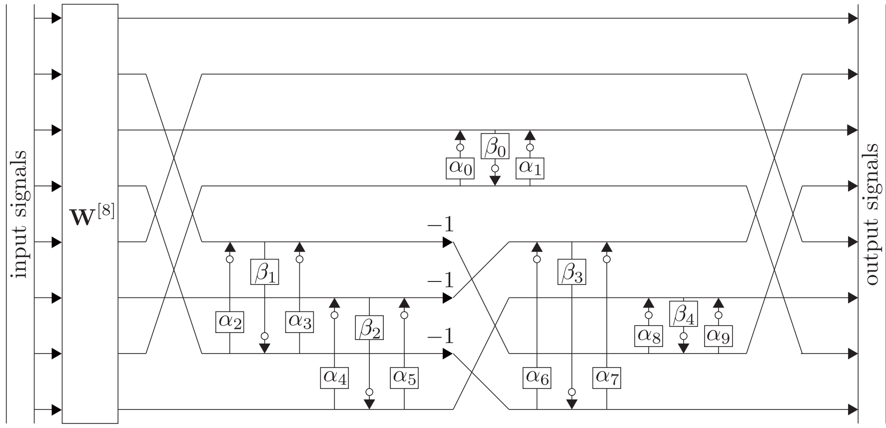


Fig. 3. Chen's IntDCT [5] (white circles: rounding operations).

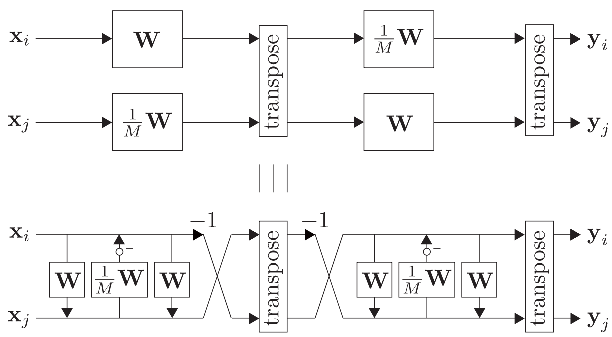


Fig. 4. LWHT with structural regularity for lower-bit-word-length (white circles: rounding operations).

In (6), one scaling factor $1/\sqrt{M}$ can be moved to another one as

$$\mathbf{y} = \left(\frac{1}{M}\mathbf{W}\left(\mathbf{W}\mathbf{x}\right)^T\right)^T \quad \text{or} \quad \mathbf{y} = \left(\mathbf{W}\left(\frac{1}{M}\mathbf{W}\mathbf{x}\right)^T\right)^T. \qquad (7)$$

Be careful that (7) can not be applied to lossless image coding as it is because real values generated by the merged scaling factor $1/M$ can not be eliminated. Now, we consider to transform two adjacent blocks simultaneously as

$$\begin{bmatrix}\mathbf{y}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{y}_j\end{bmatrix} = \left(\begin{bmatrix}\frac{1}{M}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}\end{bmatrix}\left(\begin{bmatrix}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \frac{1}{M}\mathbf{W}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{x}_j\end{bmatrix}\right)^T\right)^T \qquad (8)$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ are adjacent $M \times M$ blocks in the 2-D image and $\mathbf{y}_i$ and $\mathbf{y}_j$ are the output signals of them. (8) is illustrated as the upper half in Fig. 4. Next, we can obtain a lifting factorization as follows:

$$\begin{bmatrix}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \frac{1}{M}\mathbf{W}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\frac{1}{M}\mathbf{W}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}$$
$$\begin{bmatrix}\frac{1}{M}\mathbf{W} & \mathbf{0}\\ \mathbf{0} & \mathbf{W}\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\frac{1}{M}\mathbf{W}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{W} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{I} & \mathbf{0}\end{bmatrix}$$

where $(1/M)\mathbf{W} = \mathbf{W}^{-1}$. Consequently, LWHT can be constructed by only ± 1 and $\pm 1/M$. The proposed LWHT is illustrated as the lower half in Fig. 4. Thus the 2-D separable transform by $\hat{\mathbf{W}}$ can be implemented by efficient lifting structure without any scaling.

TABLE II
COMPARISON OF CODING GAIN.

Lifting coefficients	Coding gain [dB] F's [3]	T's [4]	C's [6]	Prop.
float	8.8259	6.2256	8.6960	8.8259
$k/2^3$	8.4829	6.1446	8.1177	8.7344
$k/2^4$	8.7724	6.2077	8.5560	8.8206
$k/2^5$	8.8044	6.2258	8.6830	8.8155
$k/2^6$	8.8257	6.2243	8.6774	8.8244

C. Expansion from LWHT to IntDCT

In this paper, we set $M = 8$ as the most popular block size. First, an original image is separated to upper half and lower half. Second, the 2-D separable block transform of the images is implemented by our 8-channel LWHT such as Fig. 4. This implementation requires only 3-bit-word-length. Next, $\hat{\mathbf{Q}}^{[8]}$ in (3) is implemented. This IntDCT has structural regularity because the proposed LWHT has structural regularity. If the floating-points in Table I are used and rounding operations are not used, our IntDCT shows performance same as the original DCT. But, if dyadic values $k/2^n$ ($k, n \in \mathbb{N}$) in Table I and rounding operations are used, our IntDCT can achieve lossless-to-lossy image coding by the lower cost implementation.

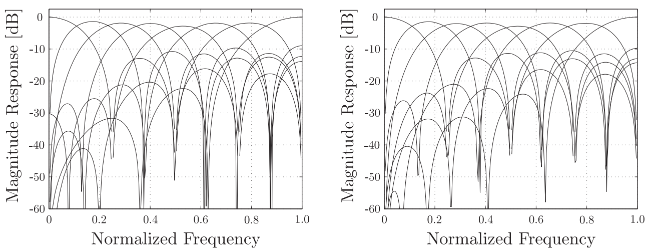
IV. RESULTS

A. Coding Gain and Frequency Response

Coding gain is one of the most important factors to be considered in compression applications. A transform with higher coding gain compacts more energy into a fewer number of coefficients. As a result, higher objective performances such as PSNR would be achieved after quantization. Since coding gain of the DCT approximates the optimal KLT closely, it is desired that the our IntDCT has similar coding gain to that of the original DCT. The biorthogonal coding gain is defined as [7]

$$\text{Coding gain [dB]} = 10\log_{10}\frac{\sigma_x^2}{\prod_{k=0}^{M-1}\sigma_{x_k}^2 \parallel f_k \parallel^2}$$

where σ_x^2 is the variance of the input signal, $\sigma_{x_k}^2$ is the variance of the k-th subbands and $\parallel f_k \parallel^2$ is the norm of the k-th synthesis filter. Table II shows the comparison of coding gain of the proposed IntDCT and the conventional ones. It is clear that coding gains of the conventional IntDCTs are not kept in low-bit-word-length, but the proposed IntDCT almost kept it even in 3-bit-word-length.



(a) (b)

Fig. 5. Frequency responses in 4-bit-word-length: (a)Fukuma's LDCT [3], (b)proposed IntDCT.

TABLE III
COMPARISON OF LOSSLESS IMAGE CODING IN 4-BIT-WORD-LENGTH.

Lossless bit rate [bpp]				
Image 512×512	F's [3]	T's [4]	C's [6]	Prop. IntDCT
Barbara	5.39	5.45	5.15	**5.02**
Lena	5.26	5.16	4.88	**4.68**
Pepper	5.48	5.47	5.14	**4.98**
Avg. of 10 images	5.79	5.93	5.59	**5.47**

Frequency responses of the proposed IntDCTs and the conventional ones are shown in Fig. 5. It is clear that [3] has DC leakage which may generate a checker-board artifact in low bit rate. The proposed IntDCT does not have DC leakage, because 8-channel LWHT as the pre-processing part can be constructed with only 3-bit-word-length and regularity can be always kept in more than 3-bit-word-length. Also, although [4] does not have DC leakage, it shows worse coding gain than ours.

B. Lossless-to-Lossy Image Coding

The proposed IntDCTs are applied to lossless-to-lossy image coding. 10 test images, which have 512×512 size, were used and [3], [4] and [6] were chosen for comparison. The set partitioning in hierarchical trees (SPIHT) progressive image transmission algorithm [8] was used to encode the transformed images. The comparison of

$$\text{Lossless bit rate [bpp]} = \frac{\text{Total number of bits [bit]}}{\text{Total number of pixels [pixel]}}$$

in lossless image coding is shown in Table III.

If lossy compressed data is required, it can be achieved by interrupting the obtained lossless bit stream. The comparison of

$$\text{PSNR [dB]} = 10 \log_{10} \left(\frac{255^2}{\text{MSE}} \right)$$

where MSE is the mean squared error in lossy image coding are shown in Table IV. Furthermore, Fig. 6 shows the enlarged images of 'Lena' which are lossy compressed images by the proposed IntDCT and the conventional ones in 4-bit-word-length when bit rate is 0.25[bpp].

In Table III, IV and Fig. 6, the proposed IntDCT presents better performance than the conventional ones in lossless-to-lossy image coding. Especially, in Fig. 6, we can find that a checker-board artifact is not generated in the proposed IntDCT due to structural regularity.

V. CONCLUSION

This paper presents an integer discrete cosine transform (IntDCT) via lossless Walsh-Hadamard transform (LWHT) with structural regularity even in low-bit-word-length (coefficients). It can be achieved by using a novel LWHT considering normalization as the pre-processing part in IntDCT. Our M-channel LWHT can be constructed with only $(\pm \log_2 M)$-bit-word-length and has structural regularity. Therefore, our IntDCTs are more suitable for lossless-to-lossy image coding and show better performance than the conventional IntDCTs when lower-bit-word-length is allocated to every lifting coefficient. Naturally, 4-channel our IntDCT for JPEG-XR can be also easily obtained.

TABLE IV
COMPARISON OF LOSSY IMAGE CODING IN 4-BIT-WORD-LENGTH.

		PSNR [dB]			
Image 512×512	bit rate [bpp]	F's [3]	T's [4]	C's [6]	Prop. IntDCT
Barbara	0.25	26.30	23.86	26.43	**26.92**
	0.50	28.95	27.27	29.91	**30.62**
	1.00	32.97	31.25	34.94	**35.78**
Lena	0.25	29.16	29.03	31.32	**31.80**
	0.50	31.57	32.34	34.12	**35.35**
	1.00	35.47	35.70	36.95	**38.54**
Pepper	0.25	29.01	28.57	31.02	**31.35**
	0.50	31.59	31.68	33.14	**34.28**
	1.00	34.71	34.43	35.62	**36.61**

(a) (b)

Fig. 6. Enlarged images of 'Lena' in 4-bit-word-length (bit rate: 0.25 [bpp]): (a)Fukuma's LDCT [3], (b)proposed IntDCT.

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