LIFTING FACTORIZATION BASED ON BLOCK PARALLEL SYSTEM OF *M*-CHANNEL PERFECT RECONSTRUCTION FILTER BANKS

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ABSTRACT

Several lifting-based filter banks (LBFBs) are proposed for lossless-to-lossy image coding. However, the conventional methods can not achieve good coding performance due to many rounding operations, and they may have wide dynamic range of lifting coefficients due to inverse matrix. This paper presents a more practical lifting factorization of M-channel perfect reconstruction filter bank (PRFB). Using the block parallel system of PRFBs with an imposed condition, we can factorize them into a novel lifting structure which is called the parallel lifting. The validity of our structures is shown by applying them to lossless-to-lossy image coding.

Index Terms — Perfect reconstruction filter bank (PRFB), block parallel system, parallel lifting, lossless-to-lossy image coding.

1. INTRODUCTION

Image coding has lossy and lossless mode. Lossy image coding is adopted to Internet contents and mobile etc., and lossless image coding is adopted to medical and art field etc. Then lossless and lossy compressed data are usually independent each other. For example, in the international standard JPEG [1], discrete cosine transform (DCT) [2] is applied to lossy image coding and DCM is applied to lossless image coding.

With the rapid development of Internet and multimedia technology, the unification of lossy and lossless compressed data is demanded to obtain higher quality and compression ratio such as JPEG2000 [3] with 9/7 and 5/3-tap discrete wavelet transform (DWT) for lossy and lossless image coding, respectively. Several lifting-based filter banks (LBFBs) which are filter banks (FBs) with the lifting structure [4] are widely researched for the unification of lossy and lossless data [5–7]. This coding is called lossless-to-lossy image coding. However, they are not practical because [5, 6] can not achieve good coding performance due to many rounding operations, and [7] may have wide dynamic range of lifting coefficients due to inverse matrix.

In this paper, using the block parallel system of perfect reconstruction FBs (PRFBs) with an imposed condition, we can factorize them into a novel lifting structure which is called the parallel lifting. The every building blocks have more practical structure which consists of three block lifting matrices with same lifting coefficient matrix without an inverse operation. Finally, the validity of our structures is shown by applying them to lossless-to-lossy image coding.

Notations: $\mathbf{I}$, $\mathbf{M}^T$ and $\mathbf{M}^{[M]}$ are an identity matrix, a transpose of a matrix $\mathbf{M}$ and an $M \times M$ matrix $\mathbf{M}$, respectively.

2. REVIEW

2.1 Perfect Reconstruction Filter Bank (PRFB)

Fig. 1 shows an M-channel FB which consists of parallel analysis filters $H_k(z)$, synthesis filters $F_k(z)$, decimaters and interpolators. Fig. 2 illustrates its equivalent polyphase representation where $\mathbf{E}(z)$ and $\mathbf{R}(z)$ are the polyphase matrices, respectively. The polyphase representation is formulated as follows [8]:

$$[H_0(z)\, H_1(z) \cdots H_{M-1}(z)]^T = \mathbf{E}(z^M)\mathbf{e}(z)^T$$
$$[F_0(z)\, F_1(z) \cdots F_{M-1}(z)] = \mathbf{e}(z)\mathbf{R}(z^M)$$

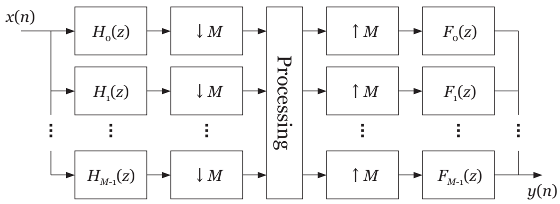

Figure 1: An M-channel filter bank.

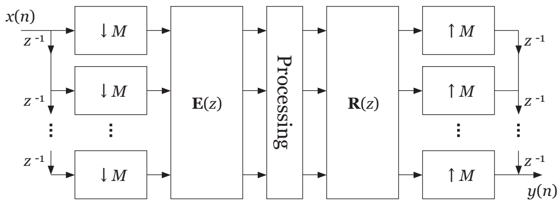

Figure 2: A polyphase structure of filter bank.

where $\mathbf{e}(z) = \left[1\; z^{-1}\; \cdots\; z^{-(M-1)}\right]$. If perfect reconstruction is achieved, the synthesis polyphase matrix $\mathbf{R}(z)$ can be chosen as the inverse of $\mathbf{E}(z)$. The obtained FB is called a perfect reconstruction FB (PRFB). If $\mathbf{E}^T(z^{-1})\mathbf{E}(z) = \mathbf{I}$ and $\mathbf{R}(z) = \mathbf{E}^T(z^{-1})$, the FB belongs to a special class of PRFBs called a paraunitary FB (PUFB). PRFB without PUFB is called a biorthogonal FB (BOFB).

Therefore, M-channel PRFBs can be implemented by the lattice structure that consists of the product of the first and several building blocks. In this paper, we assume that the filter length is $MK (K \in \mathbb{N})$. The polyphase matrix $\mathbf{E}(z)$ of the PRFB is expressed by [8]

$$\mathbf{E}(z) = \mathbf{W}_L \mathbf{\Lambda}_L(z) \cdots \mathbf{W}_1 \mathbf{\Lambda}_1(z) \mathbf{W}_0 \qquad (1)$$

where $L = K - 1$, $\mathbf{W}_k$ is arbitrary $M \times M$ nonsingular matrix and $\mathbf{\Lambda}_k(z)$ is expressed as

$$\mathbf{\Lambda}_k(z) = \begin{bmatrix} \mathbf{I}^{[M-\gamma_k]} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I}^{[\gamma_k]} \end{bmatrix}.$$

Although γ_k is arbitrary integer $1 \le \gamma_k < M$, we set $\gamma_k = M/2$ and $\mathbf{\Lambda}(z) \triangleq \mathbf{\Lambda}_k(z)$ for simplicity when M is even. Fig. 3 shows a lattice structure of PRFB. When $\mathbf{W}_k$ is an orthogonal matrix, $\mathbf{W}_k^{-1} = \mathbf{W}_k^T$, it is a PUFB.

2.2 Block Lifting Structure

Lifting structure [4] is researched widely. Since the structure achieves integer-to-integer transform, lossless-to-lossy image coding can be realized. Fig. 4 shows a lifting structure. In Fig. 4, analysis input signals x_i and x_j, analysis output and synthesis input signals y_i and y_j, synthesis output signals z_i and z_j and a lifting

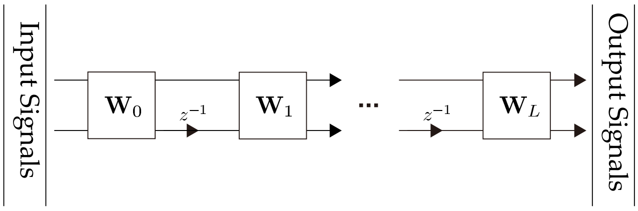

Figure 3: A lattice strcture of PRFB.

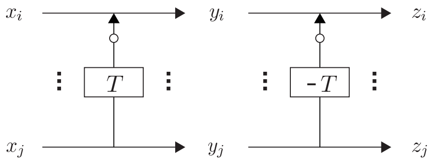

Figure 4: Lifting structure (White circles: rounding operations.)

coefficient T are represented as

$$\begin{aligned} y_i &= x_i + R[Tx_j] \\ y_j &= x_j \end{aligned} \quad \rightarrow \quad \begin{aligned} z_i &= y_i - R[Ty_j] = x_i \\ z_j &= y_j = x_j \end{aligned}$$

where $R[.]$ is rounding operation.

In other hand, the block lifting structure [7] is known as an efficient one. The structure achieves higher compression ratio because it can merge many rounding operations. Fig. 5(a) shows a block lifting structure before merging the rounding operations, and Fig. 5(b) shows one afrer merged them. It is clear that the number of rounding operations is reduced from N^2 to N when the size of the lifting coefficient matrix $\mathbf{T}$ is $N \times N$. In Fig. 5(b), analysis input signals vector $\mathbf{x}_i$ and $\mathbf{x}_j$, analysis output and synthesis input signals vector $\mathbf{y}_i$ and $\mathbf{y}_j$, synthesis output signals vector $\mathbf{z}_i$ and $\mathbf{z}_j$ and a lifting coefficient matrix $\mathbf{T}$ are represented as

$$\begin{aligned} \mathbf{y}_i &= \mathbf{x}_i + R[\mathbf{T}\mathbf{x}_j] \\ \mathbf{y}_j &= \mathbf{x}_j \end{aligned} \quad \rightarrow \quad \begin{aligned} \mathbf{z}_i &= \mathbf{y}_i - R[\mathbf{T}\mathbf{y}_j] = \mathbf{x}_i \\ \mathbf{z}_j &= \mathbf{y}_j = \mathbf{x}_j \end{aligned}$$

where $R[.]$ is rounding operation.

3. PARALLEL LIFTING-BASED PERFECT RECONSTRUCTION FILTER BANK (PLPRFB)

In this section, we propose an efficient lifting structure using the block parallel systemof PRFB.

3.1 Block Parallel System

First, the lattice structure can be expressed as a time-series processing such as Fig. 6 [9]. Two building blocks $\mathbf{W}_k$ framed by dashed line in Fig. 6 are expressed as

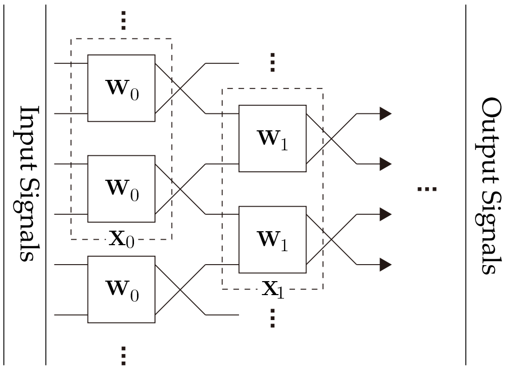

Figure 6: A time-series processing of lattice strcture of PRFB.

$$\mathbf{X}_k = \begin{bmatrix} \mathbf{W}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_k \end{bmatrix}. \tag{2}$$

A system in (2) is called the block parallel system. Then, if $\mathbf{X}_k$ in (2) can be factorized into lifting structure, we can achieve lossless-to-lossy image coding. In conclusion, a method of approach for lossless-to-lossy image coding is boiled down to how to factorize $\mathbf{X}_k$ into lifting structure.

3.2 Parallel Lifting Factorization

First, let define $\tilde{\mathbf{X}}_k$ as

$$\tilde{\mathbf{X}}_k \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{X}_k = \begin{bmatrix} \mathbf{0} & \mathbf{W}_k \\ \mathbf{W}_k & \mathbf{0} \end{bmatrix}. \tag{3}$$

Second, the block lifting matrix is multiplied from the right sides of (3) as

$$\tilde{\mathbf{X}}_k \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{W}_k^{-1} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{W}_k \\ \mathbf{W}_k & \mathbf{0} \end{bmatrix}. \tag{4}$$

Next, the other block lifting matrix is multiplied from the right sides of (4) as

$$\begin{bmatrix} \mathbf{I} & \mathbf{W}_k \\ \mathbf{W}_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\mathbf{W}_k \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{W}_k & -\mathbf{W}_k^2 \end{bmatrix}. \tag{5}$$

Therefore, the block lifting matrix which is same as (4) is multiplied from the right sides of (5) as

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{W}_k & -\mathbf{W}_k^2 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{W}_k^{-1} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{W}_k^2 \end{bmatrix}. \tag{6}$$

Consequntry, $\tilde{\mathbf{X}}_k$ in (3) can be factorized into the lifting structures using (4)-(6) as

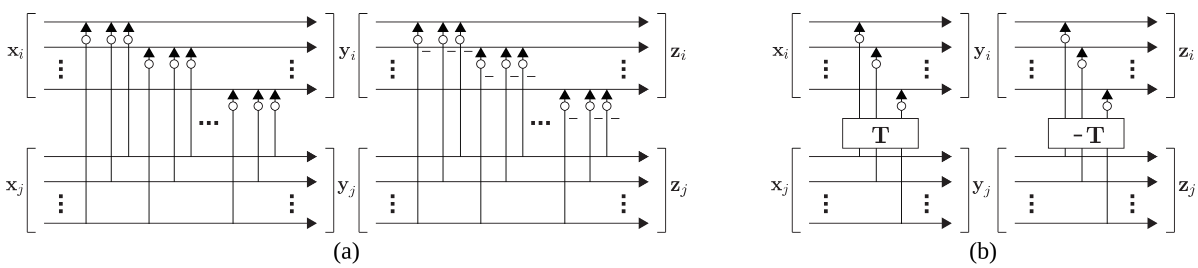

Figure 5: Block lifting structure: (a)before merging, (b)after merging rounding operations. (White circles: rounding operations.)

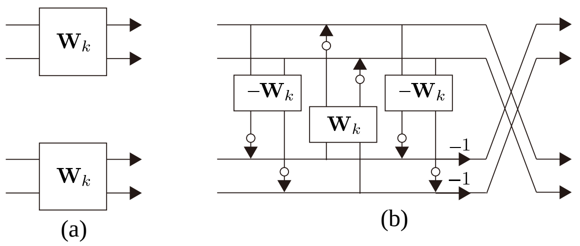

Figure 7: A block parallel system of PRFB: (a)basic structure, (b)parallel lifting structure. (White circles: rounding operations.)

Table 1: Comparison of the number of free parameters.

	[6]	[7]	PLPUFB	PLBOFB
8×16	44	96	**32**	72
8×24	60	144	**48**	108

Table 2: Comparison of the number of merged rounding operations.

	[6]	[7]	PLPUFB	PLBOFB
8×16	132 → 62	112 → **20**	192 → 24	192 → 24
8×24	180 → 82	176 → **28**	288 → 36	288 → 36

$$\tilde{\mathbf{X}}_k = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{W}_k^2 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{W}_k^{-1} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{W}_k \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{W}_k^{-1} & \mathbf{I} \end{bmatrix}. \quad (7)$$

However, (7) is not the complete lifting structure due to $\mathbf{W}_k^2$. Let impose a condition $\mathbf{W}_k^{-1} = \mathbf{W}_k$ for the completion of lifting factorization of PRFB. In case of PUFB, $\mathbf{W}_k$ must be a symmetric orthogonal matrix where $\mathbf{W}_k^{-1} = \mathbf{W}_k^T = \mathbf{W}_k$ as follows [10]:

$$\mathbf{W}_k = \begin{bmatrix} \mathbf{U}_{k,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{k,1} \end{bmatrix} \begin{bmatrix} \mathbf{C}_k & \mathbf{S}_k \\ \mathbf{S}_k & -\mathbf{C}_k \end{bmatrix} \begin{bmatrix} \mathbf{U}_{k,0}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{k,1}^T \end{bmatrix} \quad (8)$$

where $\mathbf{U}_{k,0}$ and $\mathbf{U}_{k,1}$ are $M/2 \times M/2$ arbitrary orthogonal matrices, $[\mathbf{C}_k]_{ii} = \cos\alpha_{ki}$ and $[\mathbf{S}_k]_{ii} = \sin\alpha_{ki} (0 \leq i < M/2)$, respectively. In case of BOFB, (8) is extended as follows:

$$\mathbf{W}_k = \begin{bmatrix} \mathbf{V}_{k,0} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{k,1} \end{bmatrix} \begin{bmatrix} \mathbf{C}_k & \mathbf{S}_k \\ \mathbf{S}_k & -\mathbf{C}_k \end{bmatrix} \begin{bmatrix} \mathbf{V}_{k,0}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_{k,1}^{-1} \end{bmatrix} \quad (9)$$

where $\mathbf{V}_{k,0}$ and $\mathbf{V}_{k,1}$ are $M/2 \times M/2$ arbitrary nonsingular matrix, $[\mathbf{C}_k]_{ii} = \cos\alpha_{ki}$ and $[\mathbf{S}_k]_{ii} = \sin\alpha_{ki} (0 \leq i < M/2)$, respectively.

As a result, $\mathbf{X}_k$ in (2) can be factorized into the complete block lifting structure such as

$$\mathbf{X}_k = \begin{bmatrix} \mathbf{0} & -\mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{W}_k & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{W}_k \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{W}_k & \mathbf{I} \end{bmatrix}. \quad (10)$$

The block parallel system $\mathbf{X}_k$ and its lifting factorization in (10) are shown in Fig. 7(a) and (b).

PRFB using (10) is called parallel lifting-based PRFB (PLPRFB). Especially, PLPRFB based on (8) and (9) are called parallel lifting-based PUFB (PLPUFB) and parallel lifting-based BOFB (PLBOFB). Table 1 shows the comparison of the number of free parameters (PLPUFB: $KM^2/4$, PLBOFB: $KM(M+1)/2$).

In (10), it is clear that the each building block $\mathbf{X}_k$ is constructed by using three block lifting matrices with same lifting coefficient matrix without an inverse operation. This is a practical structure.

3.3 Merging Rounding Operations

Many rounding operations can be merged because the lifting coefficient of PLPRFB can be expressed as a matrix (block). This is an efficient for lossless image coding [7]. Table 2 shows the number of rounding operations when the numbers are in one system. Although the numbers in [7] are the least, PLPRFB can reduce it to the almost same number.

4. RESULTS

4.1 Filter Bank Design

In this paper, we focus on image coding applications. The cost function to design the FBs is a weighted linear combination of coding gain C_{CG}, stopband attenuation C_{STOP} and DC leakage C_{DC} [8].

$$C = -w_1 C_{CG} + w_2 C_{STOP} + w_3 C_{DC}$$

where w_1, w_2 and w_3 are weighting factors. We designed only 8×24 PLPUFB and PLBOFB because its filter size have better performance than the other size. Their frequency responses are depicted in Fig. 8.

4.2 Application to Lossless-to-Lossy Image Coding

In this subsection, our PLPRFBs are applied to lossless image coding by using the rounding operations in the each lifting structure. We adopted the periodic extension at the image boundaries and EZW-IP as a wavelet based coder [11]. The lossless image coding results are compared by

$$\text{Lossless bit rate[bpp]} = \frac{\text{Total number of bits[bit]}}{\text{Total number of pixels[pixel]}}$$

which indicate how the FBs efficiently reduce the spatial redundancy of the input signals.

If lossy compressed data is required, it can be achieved by interrupting the lossless bit stream obtained. The lossy image coding results are compared by

$$\text{PSNR[dB]} = 10 \log_{10} \frac{255^2}{\text{MSE}}$$

where MSE is the mean squared error. Table 3 and Fig. 9 show the comparison of lossless bits rate and PSNRs between our PLPRFBs and the conventional methods and lossy compressed images of 'Barbara' when bit rate is 0.25[bpp]. 5/3-tap DWT [3], 8×24 PUFB [6] and 8×24 BOFB [7] are selected as the conventional method for lossless image coding. Similarly, 9/7-tap discrete wavelet transform (DWT) [3], 8×24 PUFB [6] and 8×24 BOFB [7] are selected as the conventional method for lossy image coding. Our PLPRFBs present same or better performance than the conventional FBs in both lossless and lossy image coding.

5. CONCLUSION

In this paper, we proposed a more practical lifting factorization of M-channel perfect reconstruction filter banks (PRFBs) using the block parallel system. This PRFBs are called parallel lifting-based PRFBs (PLPRFBs). The every building blocks can be factorized into the three block lifting matrix with same lifting coefficient matrix without an inverse operation. Although the condition for lifting factorization is imposed, our PLPRFBs are comparable or even better performance in comparison with the conventional FBs in lossless-to-lossy image coding.

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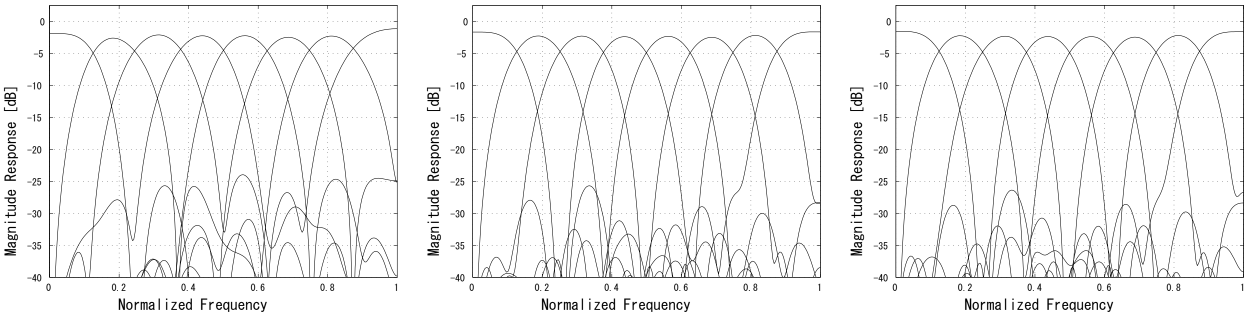

(a) (b) (c)

Figure 8: Frequency responses of PLPRFBs: (a)8×24 PLPUFB, (b)8×24 PLBOFB(analysis), (c)8×24 PLBOFB(synthesis).

Table 3: Comparison of lossless-to-lossy image coding (Lossless bit rate[bpp] and PSNR[dB]).

FILTER	Lossless bit rate [bpp]	PSNR[dB] 1.00 [bpp]	0.50 [bpp]	0.25 [bpp]
'Barbara'				
5/3 and 9/7-tap DWT	4.87	34.88	30.49	27.25
8×24 PUFB [6]	4.82	36.07	31.98	28.38
8×24 BOFB [7]	4.77	36.18	**32.03**	**28.40**
8×24 PLPUFB	4.77	36.14	31.92	28.33
8×24 PLBOFB	**4.76**	**36.21**	31.99	28.36
'Elaine'				
5/3 and 9/7-tap DWT	5.11	34.61	32.94	**31.51**
8×24 PUFB [6]	5.06	35.24	33.29	31.34
8×24 BOFB [7]	5.04	35.32	33.24	31.26
8×24 PLPUFB	5.05	35.31	33.28	31.33
8×24 PLBOFB	**5.03**	**35.38**	**33.33**	31.33
'Finger'				
5/3 and 9/7-tap DWT	5.84	29.07	25.99	23.51
8×24 PUFB [6]	5.68	30.21	26.59	23.79
8×24 BOFB [7]	**5.65**	30.25	**26.62**	23.77
8×24 PLPUFB	5.66	**30.26**	26.58	23.71
8×24 PLBOFB	5.66	30.21	**26.62**	**23.91**
Average of 10 test images				
5/3 and 9/7-tap DWT	5.31	32.95	30.06	27.62
8×24 PUFB [6]	5.32	33.46	30.33	27.68
8×24 BOFB [7]	**5.29**	33.53	30.31	27.65
8×24 PLPUFB	**5.29**	**33.56**	30.32	27.62
8×24 PLBOFB	**5.29**	**33.56**	**30.37**	**27.70**

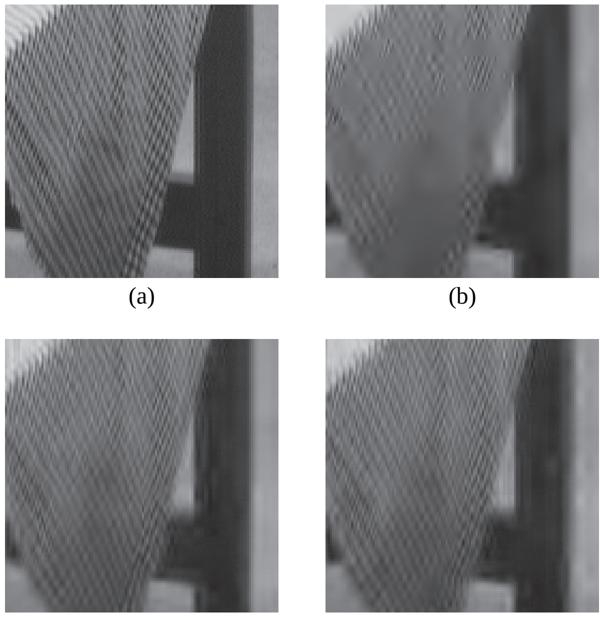

Figure 9: Enlarged image of 'Barbara' (Bit rate: 0.25[bpp]): (a)Original image, (b)9/7-tap DWT, (c)8×24 PLPUFB, (d)8×24 PLBOFB.

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