DESIGN OF BLOCK LIFTING-BASED DISCRETE COSINE TRANSFORM TYPE-II AND IV

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ABSTRACT

This paper presents a novel lifting factorization of discrete cosine transform type-II and IV (DCT-II and IV). Although some conventional integer DCT-IIs (IntDCT-IIs) with block size 8 have been proposed, they are not generalized as arbitrary block size M. Using block lifting factorization which has an efficient structure for lossless-to-lossy image coding, we present IntDCT-IIs and IVs with arbitrary block size M that is called block lifting-based DCT-IIs and IVs (BLDCT-IIs and IVs). Finally, the validity of our method is proved by showing the results of lossless-to-lossy image coding in the most general case of the block size 8 and the extended size 16.

***Index Terms*—** discrete cosine transform (DCT), block lifting factorization, lossless-to-lossy image coding

1. INTRODUCTION

The discrete cosine transform (DCT) [1] is used to image, video and audio coding (compression) standards such as JPEG [2] and MPEG [3], because it has excellent property and many fast algorithms. There are several types in DCT. DCT type-II (DCT-II) and DCT type-III (DCT-III) which is the inverse transform of DCT-II are adapted to image and video coding. And DCT type-IV (DCT-IV) is also important for some fast algorithms of DCT-II or modified DCT (MDCT) [4] in audio coding. In JPEG, DCT-II is applied to lossy image coding, and a linear prediction is applied to lossless image coding [5]. We have to prepare both lossy and lossless compressed data because DCT-II and linear prediction do not have correlation each other.

Recently, higher quality data is demanded by the spread of broadbands and the development of multimedia contents. Some integer DCT-IIs (IntDCT-IIs) have been proposed for lossless-to-lossy image coding [6–8]. However the conventional methods are described about only block size 8.

This paper presents a general block lifting factorization of M-channel DCT-II and IV. We call them M-channel block lifting-based DCT-II and IV (BLDCT-II and IV). Finally, the validity is shown by the comparison of our methods and conventional methods in lossless-to-lossy image coding.

Notations: $\mathbf{I}$, $\mathbf{J}$, $\mathbf{M}^T$ and $\mathbf{M}^{[N]}$ are an identity matrix, a reversal matrix, a transpose of matrix $\mathbf{M}$ and a $N \times N$ square matrix.

2. REVIEW

2.1. Discrete Cosine Transform (DCT)

The m column and n row element of M-channel DCT-II matrix $\mathbf{C}_{II}^{[M]}$, DCT-III matrix $\mathbf{C}_{III}^{[M]}$ and DCT-IV matrix $\mathbf{C}_{IV}^{[M]}$ are defined as

$$\left[\mathbf{C}_{II}^{[M]}\right]_{m,n} = \sqrt{\frac{2}{M}} c_m \cos\left(\frac{m(n+1/2)\pi}{M}\right)$$
$$\left[\mathbf{C}_{III}^{[M]}\right]_{m,n} = \sqrt{\frac{2}{M}} c_n \cos\left(\frac{(m+1/2)n\pi}{M}\right)$$
$$\left[\mathbf{C}_{IV}^{[M]}\right]_{m,n} = \sqrt{\frac{2}{M}} \cos\left(\frac{(m+1/2)(n+1/2)\pi}{M}\right)$$

where $0 \leq m, n \leq M-1$ and $c_m = 1/\sqrt{2}$ $(m=0)$ or 1 $(m \neq 0)$.

DCT can be factorized to some matrices for a fast algorithm. Chen $et.\ al.$, present a fast algorithm of DCT-II [1] as follows:

$$\mathbf{C}_{II}^{[M]} = \mathbf{PUW} = \mathbf{P}\begin{bmatrix}\mathbf{U}_0 & \mathbf{U}_0 \\ \mathbf{U}_1 & -\mathbf{U}_1\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}\end{bmatrix}$$
$$\text{where}\begin{cases}\mathbf{U}_0 = \frac{1}{\sqrt{2}}\mathbf{C}_{II}^{[\frac{M}{2}]} \\ \mathbf{U}_1 = \frac{1}{\sqrt{2}}\mathbf{C}_{IV}^{[\frac{M}{2}]} \\ \mathbf{P} \ : \ \text{a permutation matrix}\end{cases} \tag{1}$$

Using $\mathbf{C}_{III}^{[M]} = \mathbf{C}_{II}^{[M]T}$, we can easily obtain a matrix factorized DCT-III.

Next, note that the matrix of DCT-IV can be presented as

$$\mathbf{C}_{IV}^{[M]} = \begin{bmatrix}\mathbf{V}_0 & \mathbf{V}_1 \\ \mathbf{V}_1^T & \mathbf{V}_2\end{bmatrix}$$
$$\text{where}\begin{cases}\mathbf{V}_0 = \mathbf{V}_0^T \\ \mathbf{V}_2 = \mathbf{V}_2^T = -\mathbf{V}_1^{-1}\mathbf{V}_0\mathbf{V}_1\end{cases} \tag{2}$$

because $\mathbf{C}_{IV}^{[M]} = \mathbf{C}_{IV}^{[M]T}$ and

$$\begin{aligned}\mathbf{C}_{IV}^{[M]}\mathbf{C}_{IV}^{[M]} &= \begin{bmatrix}\mathbf{V}_0 & \mathbf{V}_1 \\ \mathbf{V}_1^T & \mathbf{V}_2\end{bmatrix}\begin{bmatrix}\mathbf{V}_0 & \mathbf{V}_1 \\ \mathbf{V}_1^T & \mathbf{V}_2\end{bmatrix} \\ &= \begin{bmatrix}\mathbf{V}_0^2 + \mathbf{V}_1\mathbf{V}_1^T & \mathbf{V}_0\mathbf{V}_1 + \mathbf{V}_1\mathbf{V}_2 \\ \mathbf{V}_1^T\mathbf{V}_0 + \mathbf{V}_2\mathbf{V}_1^T & \mathbf{V}_1^T\mathbf{V}_1 + \mathbf{V}_2^2\end{bmatrix} \\ &= \mathbf{I}.\end{aligned}$$

2.2. Lifting Structure

2.2.1. Basic Lifting Structure

Lifting structure [9] with rounding operators which quantize input signals can achieve lossless-to-lossy image coding. Fig.1(a) shows basic lifting structure. Analysis input signals x_i and x_j, analysis output and synthesis input signals y_i and y_j, synthesis output signals z_i

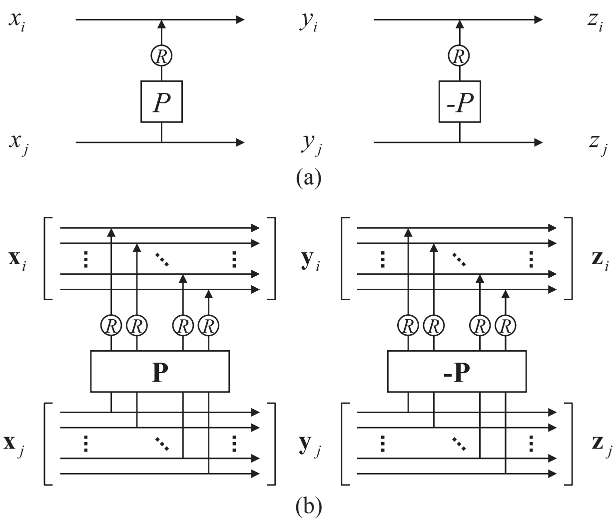


Fig. 1. Lifting structure: (a)basic lifting structure, (b)block lifting structure.

and z_j, lifting coefficient P and rounding operator R are represented as

$$\begin{aligned} y_i &= x_i + R[Px_j] \\ y_j &= x_j \\ z_i &= y_i - R[Py_j] = x_i \\ z_j &= y_j = x_j \end{aligned}$$

where $R[.]$ is rounding operation. In this case, the lifting matrix and its inverse matrix are as follows:

$$\begin{bmatrix} 1 & P \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & P \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -P \\ 0 & 1 \end{bmatrix}.$$

2.2.2. Block Lifting Structure

Block lifting structure [10] is an efficient structure for lossless-to-lossy image coding. The structure achieves higher compression ratio because it can merge many rounding operators. Fig.1(b) shows block lifting structure. Analysis input signal vectors $\mathbf{x}_i$ and $\mathbf{x}_j$, analysis output and synthesis input signal vectors $\mathbf{y}_i$ and $\mathbf{y}_j$, synthesis output signal vectors $\mathbf{z}_i$ and $\mathbf{z}_j$, $N \times N$ lifting coefficient matrix $\mathbf{P}$ and rounding operator R are represented as

$$\begin{aligned} \mathbf{y}_i &= \mathbf{x}_i + R[\mathbf{P}\mathbf{x}_j] \\ \mathbf{y}_j &= \mathbf{x}_j \\ \mathbf{z}_i &= \mathbf{y}_i - R[\mathbf{P}\mathbf{y}_j] = \mathbf{x}_i \\ \mathbf{z}_j &= \mathbf{y}_j = \mathbf{x}_j \end{aligned}$$

where $R[.]$ is rounding operation. In this case, the block lifting matrix and its inverse matrix are as follows:

$$\begin{bmatrix} \mathbf{I} & \mathbf{P} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \begin{bmatrix} \mathbf{I} & \mathbf{P} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{I} & -\mathbf{P} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}.$$

3. BLOCK LIFTING-BASED DISCRETE COSINE TRANSFORM TYPE-II AND IV (BLDCT-II AND IV)

The block lifting structure is an efficient structure for lossless-to-lossy image coding [10]. In this paper, LUL (lower-upper-lower triangular matrix) and ULU (upper-lower-upper triangular matrix) decomposition based block lifting factorization of DCT-II and IV are proposed.

3.1. Block Lifting Factorization based on LUL Decomposition

A novel block lifting factorization of DCT-II based on LUL decomposition is explained. First, the block lifting matrix are multiplied from the right sides of $\mathbf{U}$ in (1) as

$$\begin{bmatrix} \mathbf{U}_0 & \mathbf{U}_0 \\ \mathbf{U}_1 & -\mathbf{U}_1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{S}_0 & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{U}_0 \\ \mathbf{S}_1 & -\mathbf{U}_1 \end{bmatrix}$$
$$\text{where} \begin{cases} \mathbf{S}_0 &= \mathbf{I} - \mathbf{U}_0^{-1} \\ \mathbf{S}_1 &= 2\mathbf{U}_1 - \mathbf{U}_1\mathbf{U}_0^{-1} \end{cases}. \quad (3)$$

And the other block lifting matrix is multiplied from the right side of (3) as

$$\begin{bmatrix} \mathbf{I} & \mathbf{U}_0 \\ \mathbf{S}_1 & -\mathbf{U}_1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & -\mathbf{S}_2 \\ \mathbf{0} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{S}_1 & \mathbf{S}_3 \end{bmatrix}$$
$$\text{where} \begin{cases} \mathbf{S}_2 &= \mathbf{U}_0 \\ \mathbf{S}_3 &= -2\mathbf{U}_1\mathbf{U}_0 \end{cases}. \quad (4)$$

Then, the other block lifting matrix is multiplied from the right side of (4) as

$$\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{S}_1 & \mathbf{S}_3 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{S}_4 & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_3 \end{bmatrix}$$
$$\text{where} \quad \mathbf{S}_4 = \frac{1}{2}\mathbf{U}_0^{-2} - \mathbf{U}_0^{-1}. \quad (5)$$

Consequently, $\mathbf{U}$ can be factorized into the lifting structures using (3)-(5) as

$$\begin{bmatrix} \mathbf{U}_0 & \mathbf{U}_0 \\ \mathbf{U}_1 & -\mathbf{U}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_3 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{S}_4 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{S}_2 \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{S}_0 & \mathbf{I} \end{bmatrix}.$$

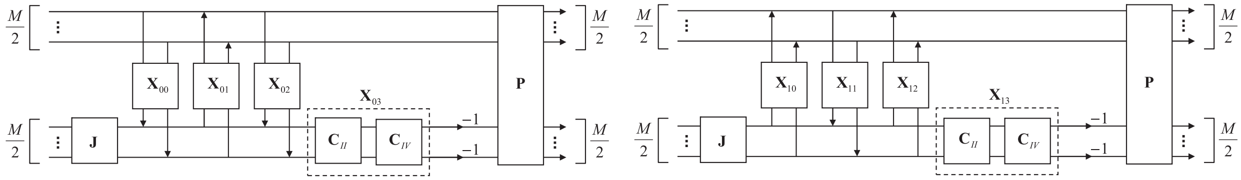
Using $\mathbf{U}_0 = \frac{1}{\sqrt{2}}\mathbf{C}_{II}$, $\mathbf{U}_0^{-1} = \sqrt{2}\mathbf{C}_{III}$ and $\mathbf{U}_1 = \frac{1}{\sqrt{2}}\mathbf{C}_{IV}$, (1) can be factorized into the lifting structures as follows:

$$\mathbf{C}_{II}^{[M]} = \mathbf{P} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{X}_{03} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{X}_{02} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{X}_{01} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{X}_{00} & \mathbf{I} \end{bmatrix} \mathbf{W}$$
$$\text{where} \begin{cases} \mathbf{X}_{00} &= \mathbf{I} - \sqrt{2}\mathbf{C}_{III}^{[\frac{M}{2}]} \\ \mathbf{X}_{01} &= \frac{1}{\sqrt{2}}\mathbf{C}_{II}^{[\frac{M}{2}]} \\ \mathbf{X}_{02} &= \mathbf{C}_{III}^{[\frac{M}{2}]^2} - \sqrt{2}\mathbf{C}_{III}^{[\frac{M}{2}]} \\ \mathbf{X}_{03} &= \mathbf{C}_{IV}^{[\frac{M}{2}]}\mathbf{C}_{II}^{[\frac{M}{2}]} \end{cases}. \quad (6)$$

Fig.2(a) shows M-channel BLDCT-II in (6).

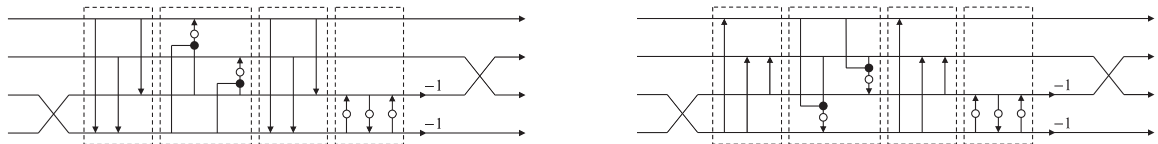
Next, a novel block lifting factorization of DCT-IV is explained. The block lifting matrix are multiplied from the right sides of (2) as

$$\begin{bmatrix} \mathbf{V}_0 & \mathbf{V}_1 \\ \mathbf{V}_1^T & \mathbf{V}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{Y}_{00} & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{V}_1 \\ -\mathbf{Y}_{00} & \mathbf{T}_0 \end{bmatrix}$$
$$\text{where} \begin{cases} \mathbf{Y}_{00} &= \mathbf{V}_1^{-1}(\mathbf{V}_0 - \mathbf{I}) \\ \mathbf{T}_0 &= -\mathbf{V}_1^{-1}\mathbf{V}_0\mathbf{V}_1 \end{cases}. \quad (7)$$



(a) (b)

Fig. 2. M-channel BLDCT-II: (a)based on LUL decomposition, (b)based on ULU decomposition.



$\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{X}_{00}^{[2]} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{X}_{01}^{[2]}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{X}_{02}^{[2]} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_{03}^{[2]}\end{bmatrix}$ $\begin{bmatrix}\mathbf{I} & \mathbf{X}_{10}^{[2]}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{X}_{11}^{[2]} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{X}_{12}^{[2]}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{X}_{13}^{[2]}\end{bmatrix}$

(a) (b)

Fig. 3. 4-channel BLDCT-II: (a)based on LUL decomposition, (b)based on ULU decomposition.

And the other block lifting matrix is multiplied from the right side of (7) as

$$\begin{bmatrix}\mathbf{I} & \mathbf{V}_1\\ -\mathbf{Y}_{00} & \mathbf{T}_0\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathbf{Y}_{01}\\ \mathbf{0} & \mathbf{I}\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{Y}_{00} & \mathbf{I}\end{bmatrix}$$
$$\text{where}\quad \mathbf{Y}_{01} = \mathbf{V}_1. \tag{8}$$

Consequently, (2) can be factorized into the lifting structures using (7) and (8) as follows:

$$\mathbf{C}_{IV}^{[M]} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{Y}_{00} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{Y}_{01}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{Y}_{00} & \mathbf{I}\end{bmatrix}. \tag{9}$$

3.2. Block Lifting Factorization based on ULU Decomposition

Previous block lifting factorizations are based on LUL decomposition. We also propose another novel block lifting factorization of DCT-II and IV based on ULU decomposition.

Computing three matrices without $\mathbf{P}$, $diag\{\mathbf{I}, -\mathbf{X}_{03}\}$ and $\mathbf{W}$ in (6), we can obtain the following matrix.

$$\begin{aligned}\mathbf{X} &\triangleq \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{X}_{02} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{X}_{01}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{X}_{00} & \mathbf{I}\end{bmatrix}\\ &= \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{C}_{II}^{[\frac{M}{2}]} & \mathbf{C}_{II}^{[\frac{M}{2}]}\\ -\mathbf{C}_{III}^{[\frac{M}{2}]} & \mathbf{C}_{III}^{[\frac{M}{2}]}\end{bmatrix}\end{aligned} \tag{10}$$

The block lifting matrices are multiplied from the left side of $\mathbf{X}$ as

$$\begin{bmatrix}\mathbf{I} & -\mathbf{X}_{10}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ -\mathbf{X}_{11} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathbf{X}_{12}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{X} = \mathbf{I}$$
$$\text{where}\begin{cases}\mathbf{X}_{10} &= \sqrt{2}\mathbf{C}_{II}^{[\frac{M}{2}]} - \mathbf{I}\\ \mathbf{X}_{11} &= -\frac{1}{\sqrt{2}}\mathbf{C}_{III}^{[\frac{M}{2}]}\\ \mathbf{X}_{12} &= \sqrt{2}\mathbf{C}_{II}^{[\frac{M}{2}]} - \mathbf{C}_{II}^{[\frac{M}{2}]^2}\end{cases}.$$

Hence, we can obtain M-channel BLDCT-II based on ULU decomposition as follows:

$$\mathbf{C}_{II}^{[M]} = \mathbf{P}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{X}_{13}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{X}_{12}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{X}_{11} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{X}_{10}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{W}$$
$$\text{where}\quad \mathbf{X}_{13} = \mathbf{X}_{03} = \mathbf{C}_{IV}^{[\frac{M}{2}]}\mathbf{C}_{II}^{[\frac{M}{2}]}. \tag{11}$$

Fig.2(b) shows M-channel BLDCT-II in (11).

Similarly, we can obtain M-channel BLDCT-IV based on ULU decomposition as follows:

$$\mathbf{C}_{IV}^{[M]} = \begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{Y}_{10}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{Y}_{11} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{Y}_{10}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}$$
$$\text{where}\begin{cases}\mathbf{Y}_{10} &= -\mathbf{V}_1^T\\ \mathbf{Y}_{11} &= (\mathbf{I} - \mathbf{V}_0)\mathbf{V}_1^{-T}\end{cases}. \tag{12}$$

3.3. Completion of Lifting Factorization

Although (6) and (11) are not complete lifting structure due to $\mathbf{X}_{03}$ and $\mathbf{X}_{13}$ in (6) and (11), we can achieve the completeness by iterating lifting factorization of $\mathbf{C}_{II}^{[M]}$ and $\mathbf{C}_{IV}^{[M]}$ with shrunk size shown as Fig.2. Therefore, $\mathbf{C}_{II}^{[4]}$ has special lifting structures as

$$\begin{aligned}\mathbf{X}_{00}^{[2]} &= \mathbf{X}_{02}^{[2]} = \begin{bmatrix}0 & -1\\ -1 & 2\end{bmatrix},\ \mathbf{X}_{01}^{[2]} = \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\\ \mathbf{X}_{10}^{[2]} &= \mathbf{X}_{12}^{[2]} = \begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix},\ \mathbf{X}_{11}^{[2]} = \frac{1}{2}\begin{bmatrix}-1 & -1\\ -1 & 1\end{bmatrix}\\ \mathbf{X}_{03}^{[2]} &= \mathbf{X}_{13}^{[2]} = \begin{bmatrix}1 & \alpha\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \beta & 1\end{bmatrix}\begin{bmatrix}1 & \alpha\\ 0 & 1\end{bmatrix}\end{aligned}$$
$$\text{where}\quad \alpha = \frac{\cos(-\frac{\pi}{8}) - 1}{\sin(-\frac{\pi}{8})},\ \beta = \sin(-\frac{\pi}{8}).$$

Fig.3(a) and (b) show 4-channel BLDCT-IIs in (6) and (11). The white circles are rounding operators and the black ones are adders. It is clear that 4-channel BLDCT-IIs have an efficient structure because they have only five rounding operators, respectively.

Table 1. Comparison of lossless image coding (Entropy [bpp]).

	$M = 8$				$M = 16$	
Image	Komatsu's	Tran's	BLDCT-II		BLDCT-II	
(512×512)	RDCT [6]	BinDCT [8]	LUL	ULU	LUL	ULU
Baboon	6.275	6.896	6.273	6.273	6.257	**6.239**
Barbara	5.022	5.511	5.015	5.010	5.082	**5.002**
Boat	5.217	5.732	5.216	**5.211**	5.275	5.231
Elaine	5.248	5.913	5.249	**5.244**	5.289	5.250
Finger1	6.071	6.384	6.072	6.067	5.917	**5.870**
Finger2	5.869	6.145	5.866	5.859	5.702	**5.624**
Goldhill	5.189	5.711	5.183	**5.179**	5.248	5.195
Grass	6.179	6.679	6.178	6.177	6.148	**6.129**
Lena	4.686	5.217	**4.676**	**4.676**	4.830	4.754
Pepper	4.976	5.516	4.975	**4.973**	5.077	5.032
Ave.	5.473	5.970	5.470	5.467	5.483	**5.433**

Table 2. Comparison of lossy image coding (PSNR [dB]).

		$M = 8$					$M = 16$	
Image	Bit rate	Komatsu's	Chen's	Tran's	BLDCT-II		BLDCT-II	
(512×512)	[bpp]	RDCT [6]	IntDCT [7]	BinDCT [8]	LUL	ULU	LUL	ULU
Baboon	0.25	22.562	22.559	20.713	22.566	22.567	**22.711**	22.709
	0.50	24.799	24.768	21.851	24.794	24.803	**25.033**	25.019
	1.00	28.237	28.159	24.542	28.241	28.245	**28.394**	28.366
Barbara	0.25	26.928	26.903	23.702	26.935	26.936	27.900	**27.904**
	0.50	30.653	30.548	27.141	30.657	30.660	**31.625**	31.600
	1.00	35.882	35.827	31.183	35.893	35.895	**36.180**	35.979
Goldhill	0.25	29.355	29.360	27.352	29.354	29.364	**29.607**	29.606
	0.50	31.920	31.938	29.274	31.927	31.926	**32.112**	32.064
	1.00	35.195	35.307	31.306	**35.210**	**35.210**	34.898	34.744
Lena	0.25	31.834	31.797	29.028	31.832	31.836	**32.522**	32.508
	0.50	35.452	35.414	32.254	35.459	35.458	**35.658**	35.501
	1.00	38.702	**38.951**	35.515	38.774	38.765	36.815	36.980
Pepper	0.25	31.380	31.378	28.487	31.389	31.392	**31.924**	31.923
	0.50	34.377	34.371	31.525	34.363	**34.373**	34.192	34.127
	1.00	36.678	36.842	34.159	36.700	**36.703**	35.835	35.660

4. RESULTS

In this paper, we design four 8 and 16-channel BLDCT-IIs based on LUL and ULU decomposition. They are compared to the conventional methods, and the validity of them are shown in lossless-to-lossy image coding.

4.1. Application to lossless image coding

Our BLDCT-IIs are applied to lossless-to-lossy image coding. The set partitioning in hierarchical trees (SPIHT) progressive image transmission algorithm [11] was used to encode the transformed images. The comparison of

$$\text{Entropy [bpp]} = \frac{\text{Total number of bits [bit]}}{\text{Total number of pixels [pixel]}}$$

in lossless image coding are shown in Table 1. We chose Komatsu's reversible DCT (RDCT) [6] and Tran's binary DCT (BinDCT) [8] as the conventional methods. Chen's IntDCT [7] was not chosen because it has so bad results for lossless image coding. In Table 1, it is obvious that our BLDCT-IIs present better performance than the conventional IntDCT-IIs in lossless image coding. Each of BLDCT-II shows the different performance depending on an image.

4.2. Application to lossy image coding

If lossy compressed data is required, it can be achieved by interrupting the obtained lossless bit stream. The comparison of

$$\text{PSNR [dB]} = 10 \log_{10} \left(\frac{255^2}{\text{MSE}} \right)$$

where MSE is the mean squared error in lossy image coding are shown in Table 2. We chose Komatsu's RDCT [6], Chen's IntDCT [7] and Tran's BinDCT [8] as the conventional methods. In Table 2, our BLDCT-IIs present better performance than the conventional IntDCT-IIs in lossy image coding. In low bit rate, the texture remained by using 16-channel BLDCT-II is more crear than 8-channel IntDCT-IIs because high frequency components can be analyzed by bigger block size. Fig.4 shows the enlarged images of 'BARBARA' which are the original image and its lossy compressed images by the conventional IntDCT-IIs and the proposed BLDCT-IIs when bit rate is 0.25 [bpp].

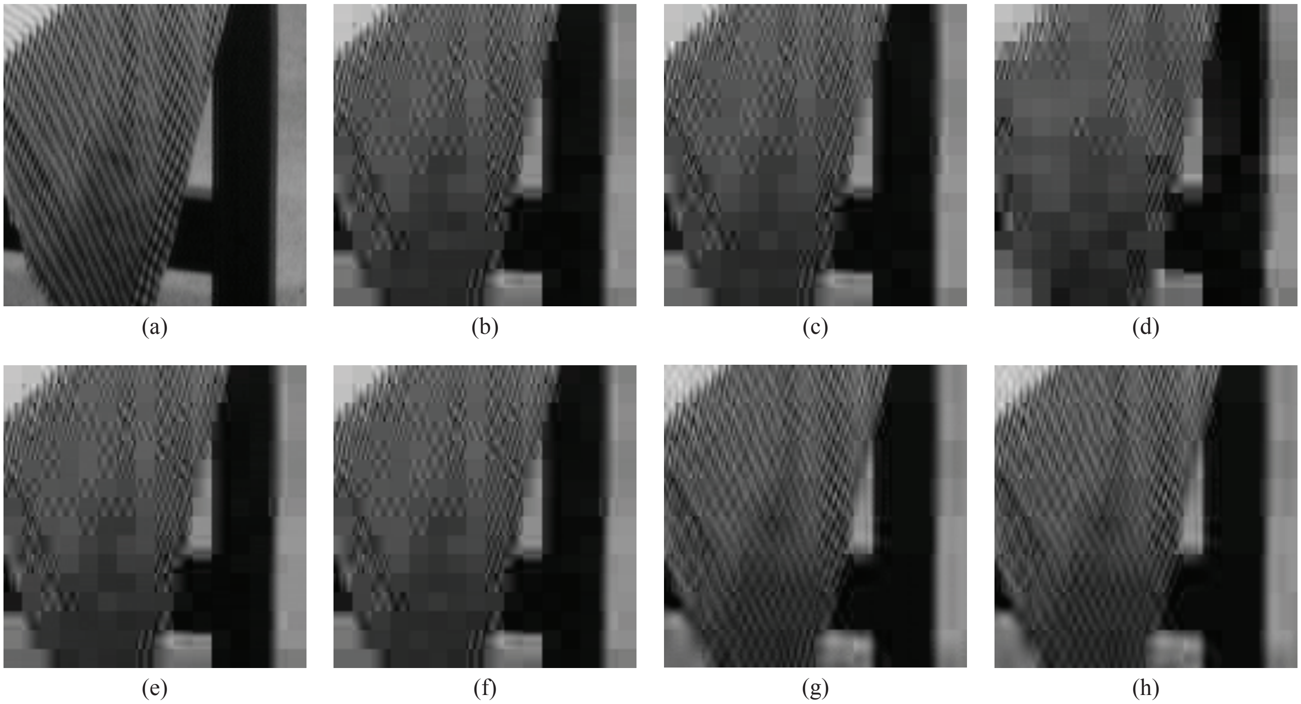


Fig. 4. Enlarged image of 'Barbara' (bit rate: 0.25 [bpp]): (a)original image, (b)compressed image by 8-channel Komatsu's RDCT [6], (c)8-channel Chen's IntDCT [7], (d)8-channel Tran's BinDCT [8], (e)8-channel BLDCT-II (LUL), (f)8-channel BLDCT-II (ULU), (g)16-channel BLDCT-II (LUL), and (h)16-channel BLDCT-II (ULU) .

5. CONCLUSION

In this paper, we proposed a novel IntDCT-II and IV based on block lifting structure. Our IntDCT-II and IV, called M-channel BLDCT-II and IV, have arbitrary block size M that is not only 8. In addition, our BLDCT-II included BLDCT-IV are more suitable for lossless-to-lossy image coding, because it shows better performance than the conventional IntDCT-IIs. Our BLDCT-II and IV can be easily extended into ones with different size which is more suitable.

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7. REFERENCES

[1] K. R. Rao and P. Yip, *Discrete Cosine Transform Algorithms*, Academic Press, 1990.

[2] W. Pennebaker and J. Mitchell, *JPEG, Still Image Data Compression Standard*, NY: Van Nostrand, 1993.

[3] T. Sikora, "MPEG digital video-coding standards," *IEEE Signal Process. Mag.*, vol. 14, no. 5, pp. 82–100, Sept. 1997.

[4] J. P. Princen and A. B. Bradley, "Analysis/synthesis filter bank design based on time domain aliasing cancellation," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 34, no. 5, pp. 1153–1161, Oct. 1986.

[5] M. J. Weinberger, G. Seroussi, and G. Sapir, "The LOCO-I lossless image compression algorithm: principles and standardization into JPEG-LS," *IEEE Trans. Image Process.*, vol. 9, no. 8, pp. 1309–1324, Aug. 2000.

[6] K. Komatsu and K. Sezaki, "Reversible discrete cosine transform," in *Proc. of ICASSP'98*, Seattle, WA, May. 1998.

[7] Y. J. Chen, S. Oraintara, and T. Q. Nguyen, "Integer discrete cosine transform (IntDCT)," in *Proc. of 2nd ICICS'99*, Singapore, Dec. 1999, Invited paper.

[8] T. D. Tran, "The binDCT: fast multiplierless approximation of the DCT," *IEEE Signal Process. Lett.*, vol. 7, pp. 145–149, 2000.

[9] W. Sweldens, "The lifting scheme: a custom-design construction of biorthogonal wavelets," *Appl. Comput. Harmon. Anal.*, vol. 3, no. 2, pp. 186–200, 1996.

[10] S. Iwamura, Y. Tanaka, and M. Ikehara, "An efficient lifting structure of biorthogonal filter banks for lossless image coding," in *Proc. of ICIP'07*, San Antonio, TX, Sept. 2007.

[11] A. Said and W. A. Pearlman, "A new, fast, and efficient image codec based on set partitioning in hierarchical trees," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 6, no. 3, pp. 243–250, 1996.