## 〈Analysis〉

Q1　Select the answer for $\lim_{x\to 0}\frac{x^3}{x-\sin x}$.

① $1$　② $2$　③ $3$　④ $6$　⑤ $0$　⑥ $-6$　⑦ $-3$　⑧ $-2$　⑨ $-1$

Q2　Select the answer for $\int_0^{\infty}\int_0^{\infty}\frac{dxdy}{(1+x^2)(1+y^2)}$.

① $1$　② $\frac{1}{3}$　③ $\frac{1}{4}$　④ $0$　⑤ $\frac{\pi}{4}$　⑥ $\frac{\pi^2}{4}$　⑦ $\pi$　⑧ $2\pi$　⑨ $\pi^2$

〈Linear Algebra〉

Let matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Fill (a) to (e) below using the given options.

Q1 The eigenvalues of $A$ are $\lambda_1 = (a)$ and $\lambda_2 = (b)$, where $\lambda_1 > \lambda_2$.

Q2 The matrix $A$ can be diagonalized as $X^{\mathrm{T}}AX = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$ by the orthogonal matrix $X = \frac{1}{\sqrt{(c)}}\begin{pmatrix} 1 & 1 \\ (d) & (e) \end{pmatrix}$.

Options (You can use the same option multiple times.)

① $1$ ② $2$ ③ $3$ ④ $4$ ⑤ $5$ ⑥ $-1$ ⑦ $-2$ ⑧ $-3$ ⑨ $-4$ ⑩ $-5$

## ⟨Discrete Mathematics and Logic⟩

Let $\mathcal{N}_5 = \{0, 1, 2, 3, 4\}$. For a positive integer $n$ , let a relation $E_n = \{\langle x, y\rangle \in \mathcal{N}_5 \times \mathcal{N}_5 \mid x \bmod n = y \bmod n\}$, where $x \bmod n$ is the remainder after division of $x$ by $n$. Answer the following questions.

Q1 $E_4 = \{\langle 0,0\rangle, \langle 1,1\rangle, \langle 2,2\rangle, \langle 3,3\rangle, \langle 4,4\rangle, \boxed{\phantom{xxxxxxxx}}\}$

Choose the appropriate number from the following options to fill in the blank box.

**Options** ① $\langle 0,1\rangle, \langle 1,0\rangle$ ② $\langle 1,3\rangle, \langle 3,1\rangle$ ③ $\langle 2,4\rangle, \langle 4,2\rangle$ ④ $\langle 0,4\rangle, \langle 4,0\rangle$

Q2 Choose the appropriate numbers from the following options corresponding to (a), (b), (c).

When $R$ is an equivalence relation, $R$ is reflexive, $\boxed{\text{(a)}}$, and transitive. $R$ is transitive, if $\langle x, y\rangle, \boxed{\text{(b)}} \in R$ implies $\boxed{\text{(c)}} \in R$ .

**Options** ① symmetric ② antisymmetric ③ ⟨*x*, *z*⟩ ④ ⟨*y*, *x*⟩ ⑤ ⟨*y*, *z*⟩ ⑥ ⟨*z*, *x*⟩

# 〈Fundamentals of Programming〉

The following function written in Python sorts an integer list `a[0]` to `a[n-1]` of length `n` in ascending order.

```
def selection_sort(n, a):
    for i in range(0, n-1):
        for t in range(0, n):
            print(" %d " % a[t], end='')
        print()
        m = a[i]; k = i;
        for j in range(i+1,n):
            if a[j] < m:
                m = a[j]; k = j;
        a[k] = a[i]; a[i] = m;
```

Q1

Given an integer list `a` of length 5, the below shows the output of `selection_sort(5, a)` , where some parts are hidden by ■ .
Answer each appropriate value for (a) - (d).