Propositional Dynamic Logic for Higher-Order Functional Programs

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Higher-Order Functional Programs

- A higher-order function takes a function as its argument and/or returns a function as its return value

$$\mathbf{twice} : (\mathbf{int} \rightarrow \mathbf{int}) \rightarrow \mathbf{int} \rightarrow \mathbf{int}$$
$$\mathbf{twice}\ f\ x = f\ (f\ x)$$

- Support in Programming Languages:
  - OCaml, Haskell, Rust
  - Jave(+1.8), Scala, Ruby, Python

Contribute significantly to modularity

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takes a function as its argument

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returns a function as its result

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Example: Higher-Order Program

Higher Order Program

$$\text{let } r = * \text{ in}$$
$$\text{let inc } x = x + 1 \text{ in}$$
$$\text{let twice } f\ x = f\ (f\ x) \text{ in}$$
$$\text{twice inc } r$$

Reduction Sequence for r=0

$$\underline{\text{twice inc}}\ 0$$

Example: Higher-Order Program

Non-deterministic integer

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Higher-Order function **twice**

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Higher-Order function twice

Reduction Sequence for r=0

$$\underline{\text{twice inc}}\ 0$$
$$\rightarrow \underline{(\lambda x.\ \text{inc}\ (\text{inc}\ x))\ 0}$$

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Higher-Order function **twice**

Reduction Sequence for r=0

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$$\rightarrow\ (\lambda\mathbf{x.\ inc\ (inc\ x))\ 0}$$
$$\rightarrow\ \mathbf{inc\ (inc\ 0)}$$

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Reduction Sequence for r=0

$\mathbf{twice\ inc\ 0}$
$\rightarrow\ (\lambda x.\ \mathbf{inc}\ (\mathbf{inc}\ x))\ 0$
$\rightarrow\ \mathbf{inc}\ (\mathbf{inc}\ 0)$
$\rightarrow^*\ \mathbf{inc}\ 1 \rightarrow^*\ 2$

Temporal Verification of Higher-Order Programs

- Specification languages used in existing verification methods
  - (ω-)regular word languages (that subsume LTL) [Disney+ '11; Hofmann+ '14; Murase+ '16]
  - Modal μ-calculus (that subsumes CTL) [Fujima+ '13; Lester+ '11; Suzuki+ '17]
  - (Extended) dependent refinement types
    - Temporal properties [Koskinen+ '14; Nanjo+ '18]
    - Temporal and branching properties [Unno+ '18]

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☹ **they cannot sufficiently express temporal specifications that involve higher-order functions**

Example: Property that cannot be expressed by the previous specification languages

$$
\begin{aligned}
& \mathbf{twice\ inc\ 0} \\
\rightarrow\ & (\boldsymbol{\lambda}\mathbf{x.\,inc\ (inc\ x)})\ \ \mathbf{0} \\
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If the function returned by a partial application of twice to some function is called with some integer n, then the function argument passed to twice is eventually called with n

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\to\ & \textbf{inc (inc 0)} \\
\to^*\ & \textbf{inc 1} \to^* \textbf{2}
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If **the function returned** by a partial application of twice to some function is called with some integer n, then the function argument passed to twice is eventually called with n

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Previous languages cannot refer to the control flow of higher-order programs involving a function that is passed to or returned by a higher-order function

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Our Contributions

- **Higher-Order Trace (HOT)** that captures the control flow of higher-order programs
- **Higher-Order Trace Propositional Dynamic Logic (HOT-PDL)**:
  - Propositional Dynamic Logic (PDL) over HOTs for specifying temporal properties of higher-order programs
- Decidability of HOT-PDL model checking of higher-order programs via a reduction to higher-order model checking

  (See the paper for details)
- Applications (See the paper for details)
  - Modeling and extending dependent refinement types
  - Modeling and extending stack-based access control properties

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Higher-Order Trace (HOT)

- A sequence of **call and return events** equipped with **two kinds of pointers**:
  - **Call and return events**
    - call(f,x): a call event of the function f with the argument x.
    - ret(f, x): a return event of the function f with the return value x.

Inspired by the notion of justification pointers from the game semantics of PCF

  - **Two kinds of pointers labeled with:**
    - CR : capture the correspondence between call and return events
    - CC, RC: capture higher-order control flow involving a function that is passed to or returned by a higher-order function

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    - CR : capture the correspondence between call and return events
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Example: Event sequence of $\mathbf{twice\ inc\ 0}$

$$
\begin{aligned}
&\mathbf{twice\ inc\ 0} \\
\to\ &(\lambda \mathbf{x}.\ \mathbf{inc\ (inc\ x)})\ \ \mathbf{0} \\
\to\ &\mathbf{inc\ (inc\ 0)} \\
\to^{*}\ &\mathbf{inc\ 1} \to^{*} \mathbf{2}
\end{aligned}
$$

Example: Event sequence of twice inc 0

$$
\begin{aligned}
& \underline{\text{twice inc }} 0 \\
\to\ & (\lambda x.\ \text{inc } (\text{inc } x))\ \ 0 \\
\to\ & \text{inc } (\text{inc } 0) \\
\to^*\ & \text{inc } 1 \to^* 2
\end{aligned}
$$

call(twice, ●)

Example: Event sequence of $\mathbf{twice\ inc\ 0}$

$$\underline{\mathbf{twice\ inc}}\ \mathbf{0}$$
$$\rightarrow\ (\lambda x.\,\mathbf{inc}\ (\mathbf{inc}\ x))\ \ \mathbf{0}$$
$$\rightarrow\ \mathbf{inc}\ (\mathbf{inc}\ \mathbf{0})$$
$$\rightarrow^*\ \mathbf{inc}\ \mathbf{1} \rightarrow^*\ \mathbf{2}$$

The function passed to a higher-order function

call(twice, ●)

Example: Event sequence of twice inc 0

$$\begin{aligned}
&\underline{\mathbf{twice\ inc}}\ \mathbf{0}\\
\to\ &(\lambda x.\ \mathbf{inc}\ (\mathbf{inc}\ x))\ \mathbf{0}\\
\to\ &\mathbf{inc}\ (\mathbf{inc}\ 0)\\
\to^*\ &\mathbf{inc}\ 1 \to^*\ 2
\end{aligned}$$

call(twice, ●) **ret(twice, ●)**

Example: Event sequence of twice inc 0

twice inc 0

$\rightarrow$ (λx. inc (inc x)) 0

$\rightarrow$ inc (inc 0)

$\rightarrow^*$ inc 1 $\rightarrow^*$ 2

call(twice, ●) ret(twice, ●)

The function returned by a higher-order function

Example: Event sequence of $\mathbf{twice\ inc\ 0}$

$$
\begin{aligned}
&\mathbf{twice\ inc\ 0} \\
\to\ & \underline{\mathbf{(\lambda x.\, inc\ (inc\ x))\ \ 0}} \\
\to\ & \mathbf{inc\ (inc\ 0)} \\
\to^*\ & \mathbf{inc\ 1} \to^* \mathbf{2}
\end{aligned}
$$

$\mathrm{call(twice, \bullet)\ ret(twice, \bullet)}$ $\mathbf{call(\bullet, 0)}$

Example: Event sequence of $\mathbf{twice\ inc\ 0}$

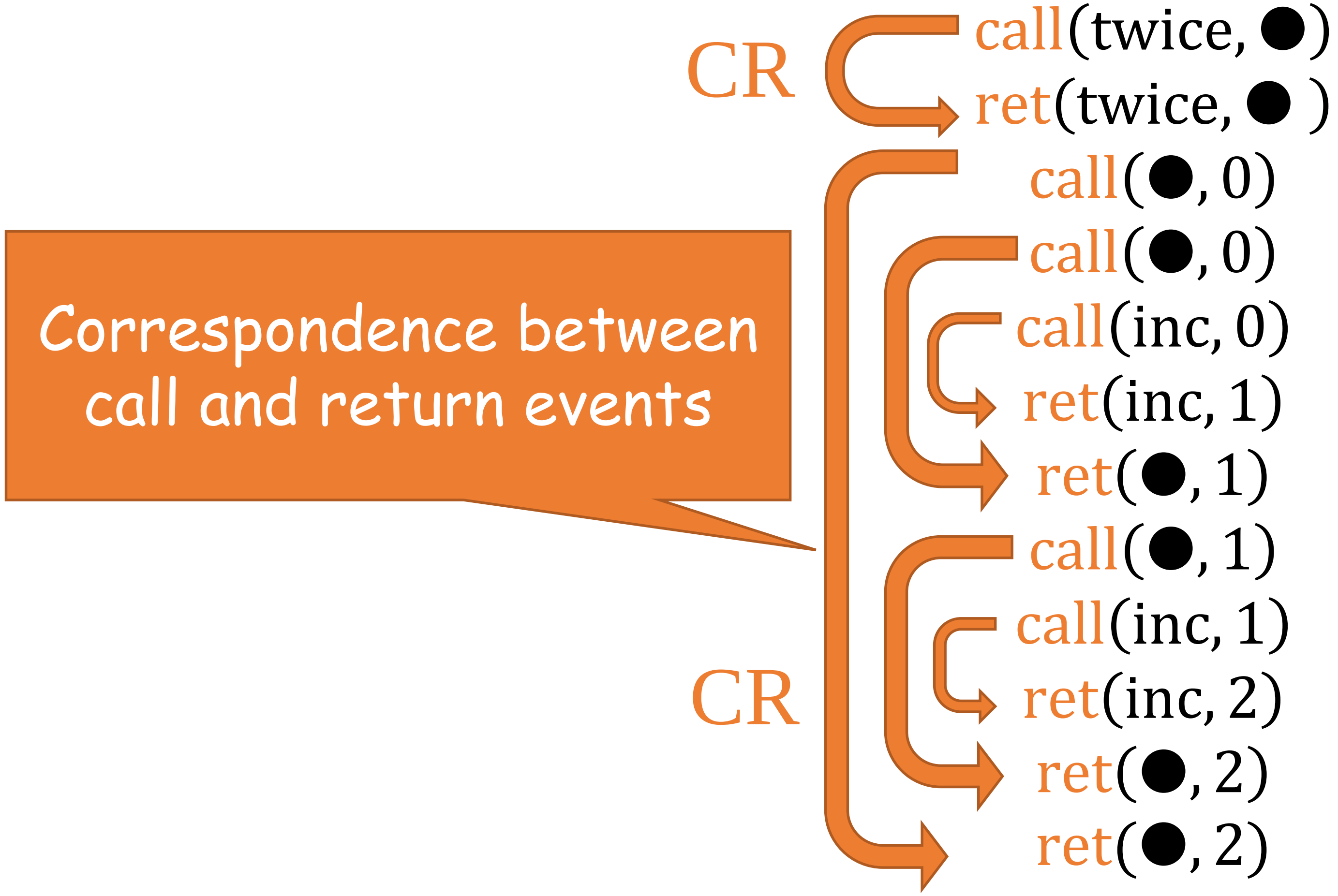
$$\begin{aligned} & \mathbf{twice\ inc\ 0} \\ \to\ & (\lambda \mathbf{x}.\ \mathbf{inc\ (inc\ x)})\ \ \mathbf{0} \\ \to\ & \mathbf{inc\ (inc\ 0)} \\ \to^{*}\ & \mathbf{inc\ 1} \to^{*} \mathbf{2} \end{aligned}$$

call(twice, ●) ret(twice, ●) call(●, 0) call(●, 0) call(inc, 0) ret(inc, 1) ret(●, 1) call(●, 1) call(inc, 1) ret(inc, 2) ret(●, 2) ret(●, 2)

Example: HOT that models the reduction sequence of $\mathbf{twice\ inc\ 0}$

call(twice, ●)
ret(twice, ●)
call(●, 0)
call(●, 0)
call(inc, 0)
ret(inc, 1)
ret(●, 1)
call(●, 1)
call(inc, 1)
ret(inc, 2)
ret(●, 2)
ret(●, 2)

Example: HOT that models the reduction sequence of twice inc 0



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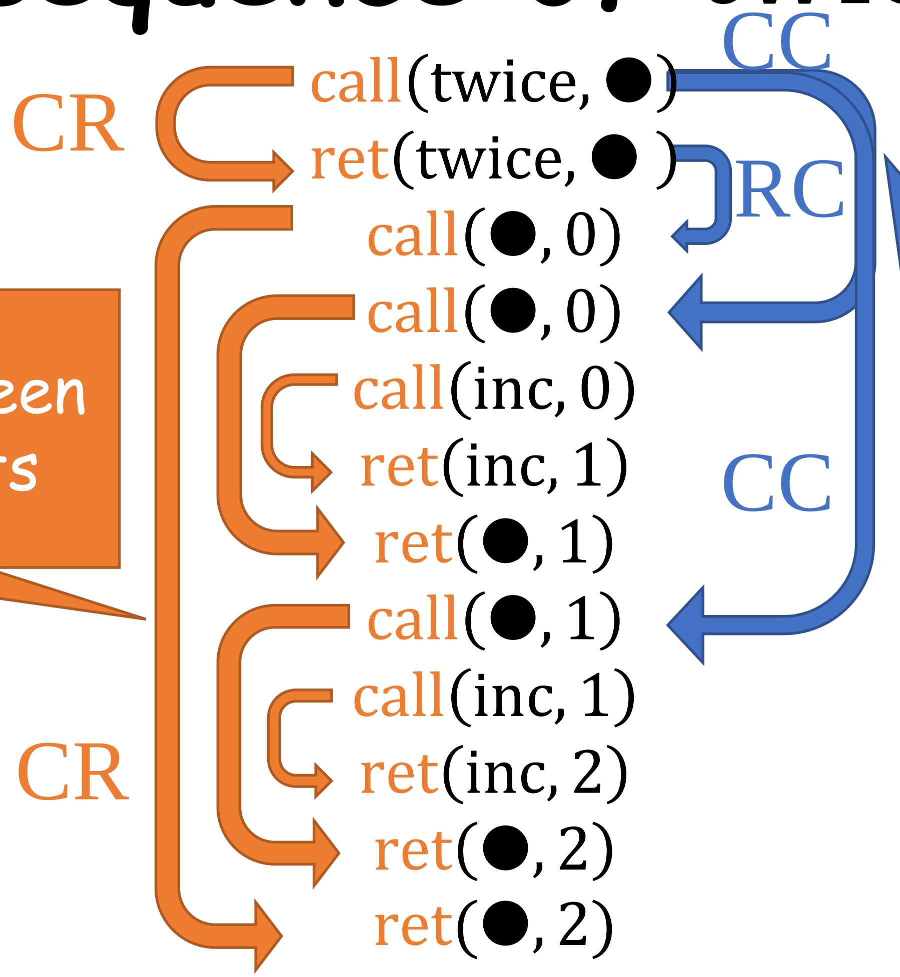

Correspondence between call and return events

Higher-order control flow
CC : call of the function passed to a higher-order function
RC : call of the function returned by a higher-order function

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Higher-Order Propositional Dynamic Logic (HOT-PDL)

- To traverse the pointers, HOT-PDL extends PDL with new path expressions ($\rightarrow_{ret}, \rightarrow_{call}$)
- Each node of a HOT is assigned a truth value indicating whether the given formula is satisfied
- Syntax:
  - (Formulas)

$$\boldsymbol{\phi} ::= \mathbf{call}(\boldsymbol{t_1}, \boldsymbol{t_2}) | \mathbf{ret}(\boldsymbol{t_1}, \boldsymbol{t_2}) | \ldots | [\boldsymbol{\pi}]\boldsymbol{\phi} | \langle\boldsymbol{\pi}\rangle\boldsymbol{\phi}$$

  - (Path expression)

$$\boldsymbol{\pi} ::= \rightarrow | \rightarrow_{ret} | \rightarrow_{call} | \boldsymbol{\pi_1} \cdot \boldsymbol{\pi_2} | \boldsymbol{\pi_1} + \boldsymbol{\pi_2} | \boldsymbol{\pi}^* | \{\phi\}?$$

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Any of top-level functions f, bounded integers $\boldsymbol{n} \in \mathbb{Z}_b$, or anonymous functions ●

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A call event of a function t_1 with an argument t_2

$$\phi ::= \mathbf{call}(t_1, t_2) | \mathbf{ret}(t_1, t_2) | \ldots | [\pi]\phi | \langle\pi\rangle\phi$$

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A return event of a function t_1 with a return value t_2

$$\phi ::= \mathbf{call}(t_1, t_2) \mid \mathbf{ret}(t_1, t_2) \mid \ldots \mid [\pi]\phi \mid \langle\pi\rangle\phi$$

  - (Path expression)

$$\pi ::= \rightarrow \mid \rightarrow_{ret} \mid \rightarrow_{call} \mid \pi_1 \cdot \pi_2 \mid \pi_1 + \pi_2 \mid \pi^* \mid \{\phi\}?$$

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Box: ϕ holds at every node reached by the path represented by π

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Diamond: there exists a node that can be reached by π and ϕ holds

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$$\pi ::= \rightarrow | \rightarrow_{ret} | \rightarrow_{call} | \pi_1 \cdot \pi_2 | \pi_1 + \pi_2 | \pi^* | \{\phi\}?$$

Concatenation ($\pi_1 \cdot \pi_2$) — alternation ($\pi_1 + \pi_2$) — Kleene star (π^*)

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tests if ϕ holds at the current node

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move to the next event in the sequence

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traverse an edge labeled with CR

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- Syntax:
  - (Formulas)

$$\boldsymbol{\phi} ::= \mathbf{call}(t_1, t_2) | \mathbf{ret}(t_1, t_2) | \ldots | [\pi]\phi | \langle\pi\rangle\phi$$

  - (Path expression)

$$\pi ::= \rightarrow | \rightarrow_{ret} | \rightarrow_{call} | \pi_1 \cdot \pi_2 | \pi_1 + \pi_2 | \pi^* | \{\phi\}?$$

traverse an edge labeled with CC or RC

Example : The property that cannot be expressed by the previous specification languages

If the function returned by a partial application of twice to some function is called with some integer n, then the function argument passed to twice is eventually called with n

$$[\rightarrow^*] \bigwedge x \in \mathbb{Z}_b \left(\begin{array}{r} (\mathbf{call}(\mathbf{twice},\cdot) \wedge \langle \rightarrow_{\mathbf{ret}} \cdot \rightarrow_{\mathbf{call}} \rangle \mathbf{call}(\cdot, x)) \\ \Rightarrow \langle \rightarrow_{\mathbf{call}} \rangle \mathbf{call}(\cdot, x) \end{array} \right)$$

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$\mathbb{Z}_b$: bounded integers

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$[\rightarrow *]\phi = \mathrm{G}\phi$ (in LTL)

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Globally, for all bounded integer x,

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If the function twice is called

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and the function returned by twice is called with an argument x,

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then the function argument passed to twice is eventually called with the argument x

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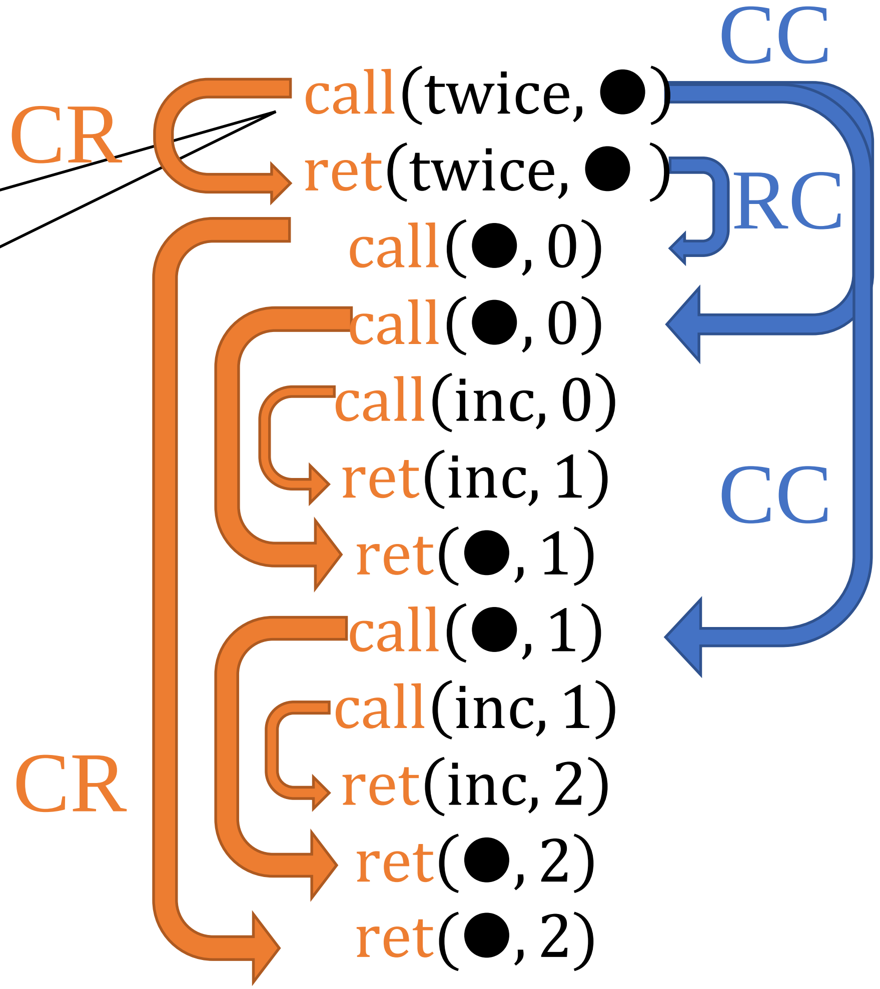
$$[\to^*] \bigwedge_{x \in \mathbb{Z}_b} \left(\left(\mathbf{call}(\mathbf{twice}, \cdot) \wedge \langle \to_{\mathbf{ret}} \cdot \to_{\mathbf{call}} \rangle \mathbf{call}(\cdot, \boldsymbol{x}) \right) \Rightarrow \langle \to_{\mathbf{call}} \rangle \mathbf{call}(\cdot, \boldsymbol{x}) \right)$$

The formula holds at the node labeled with the event call(twice, ●)



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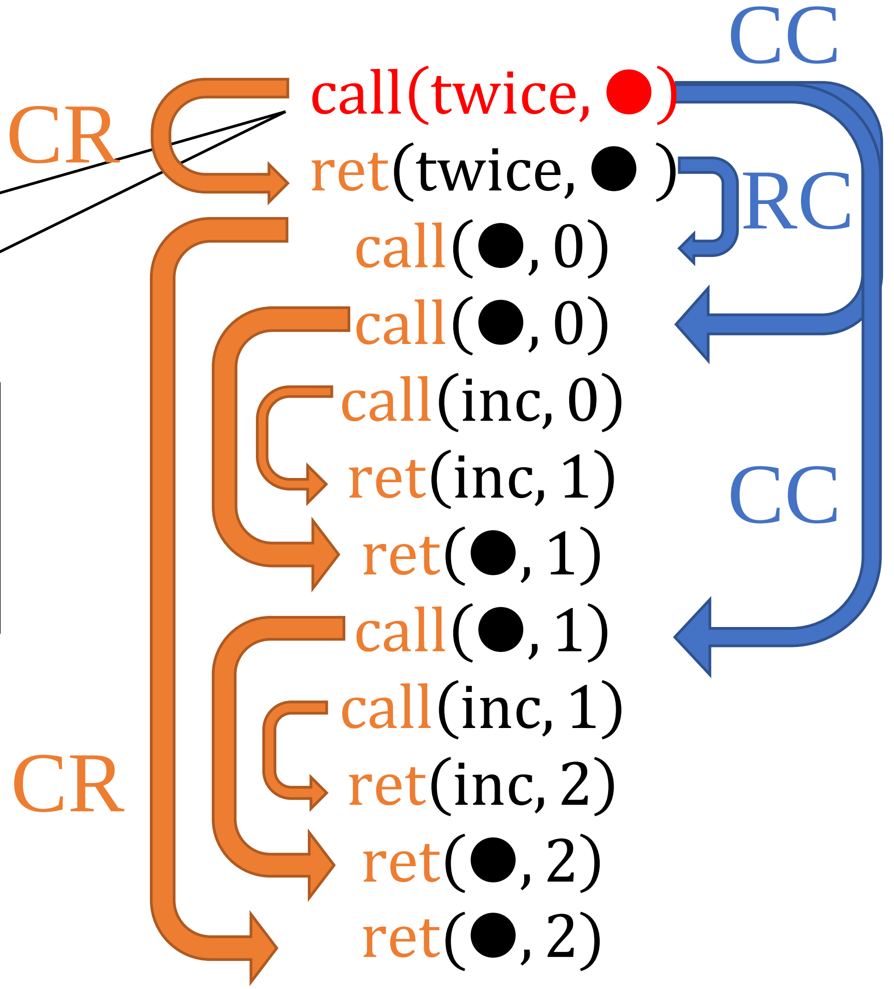

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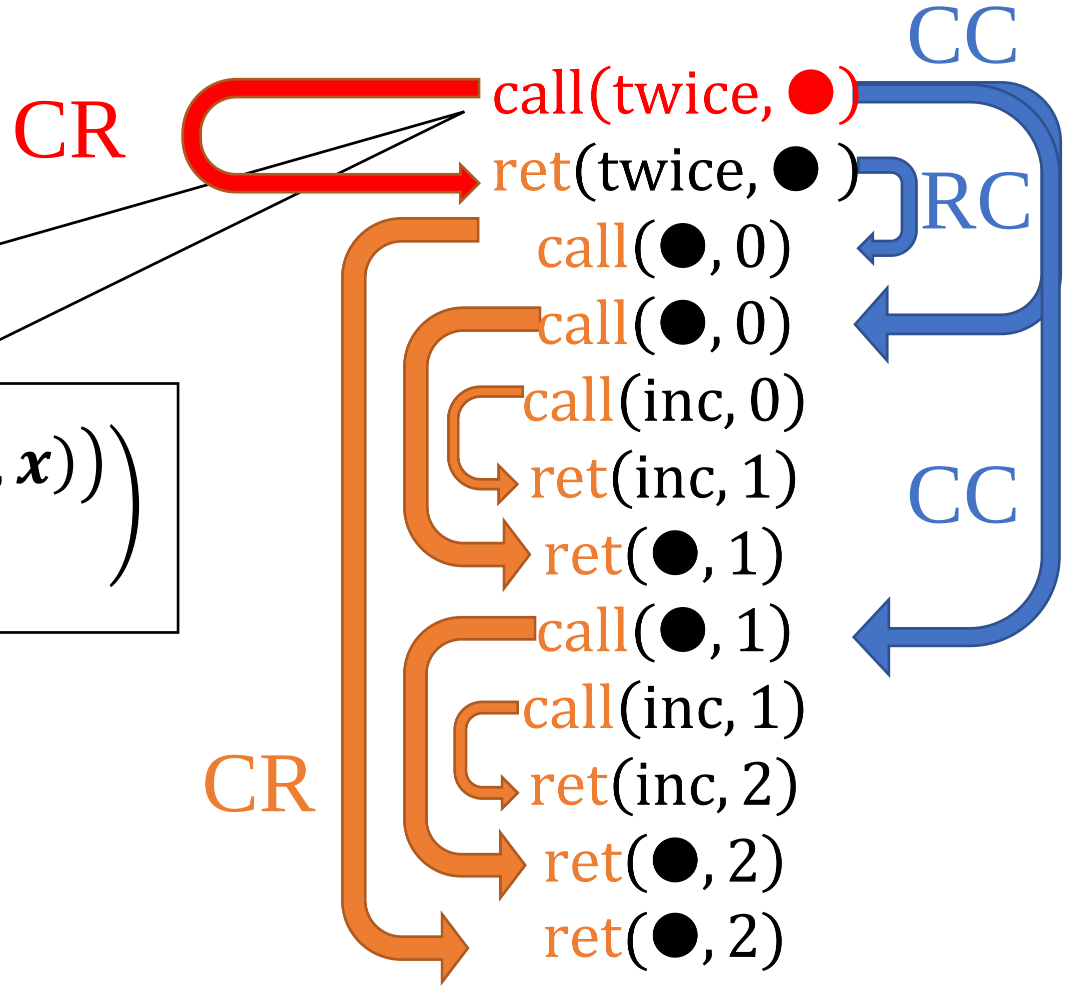

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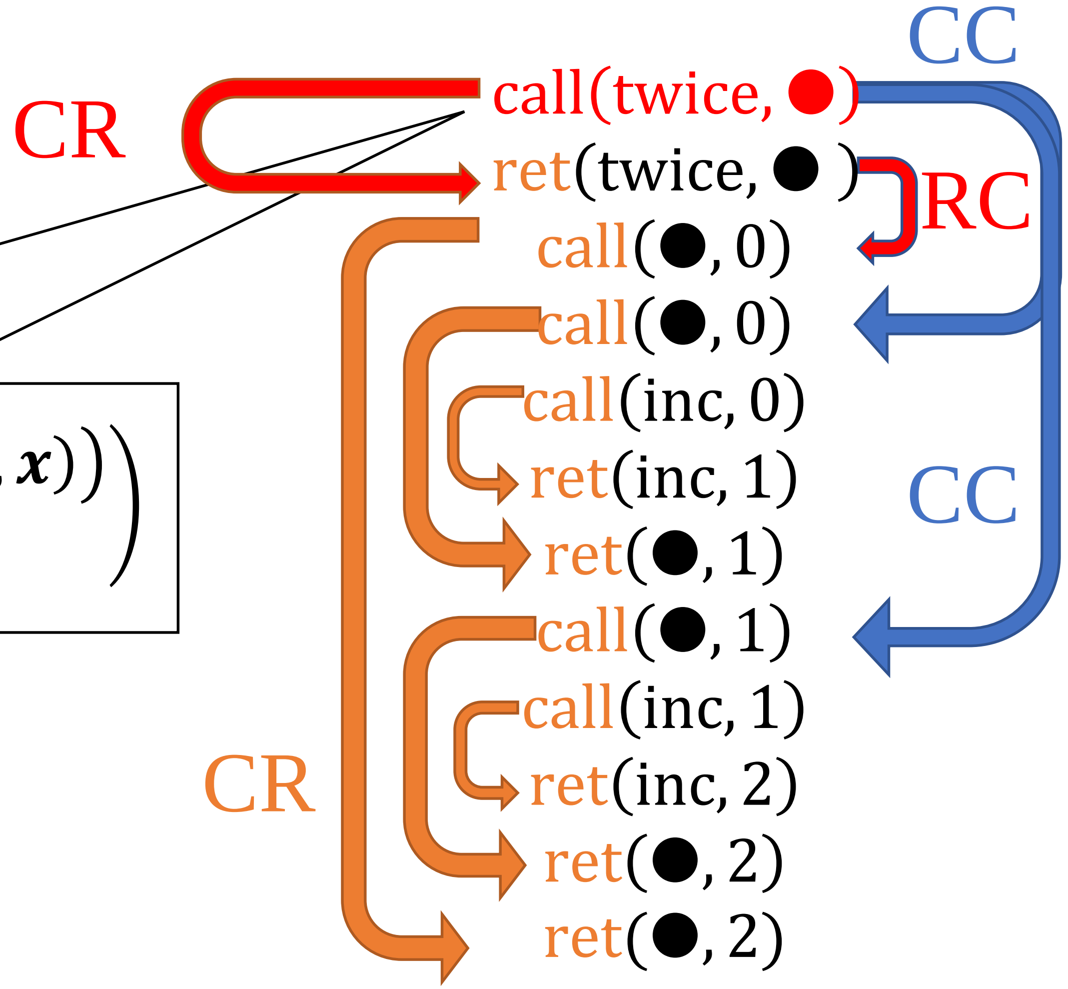

Example : The property that cannot be expressed by the previous specification languages

$x = 0$

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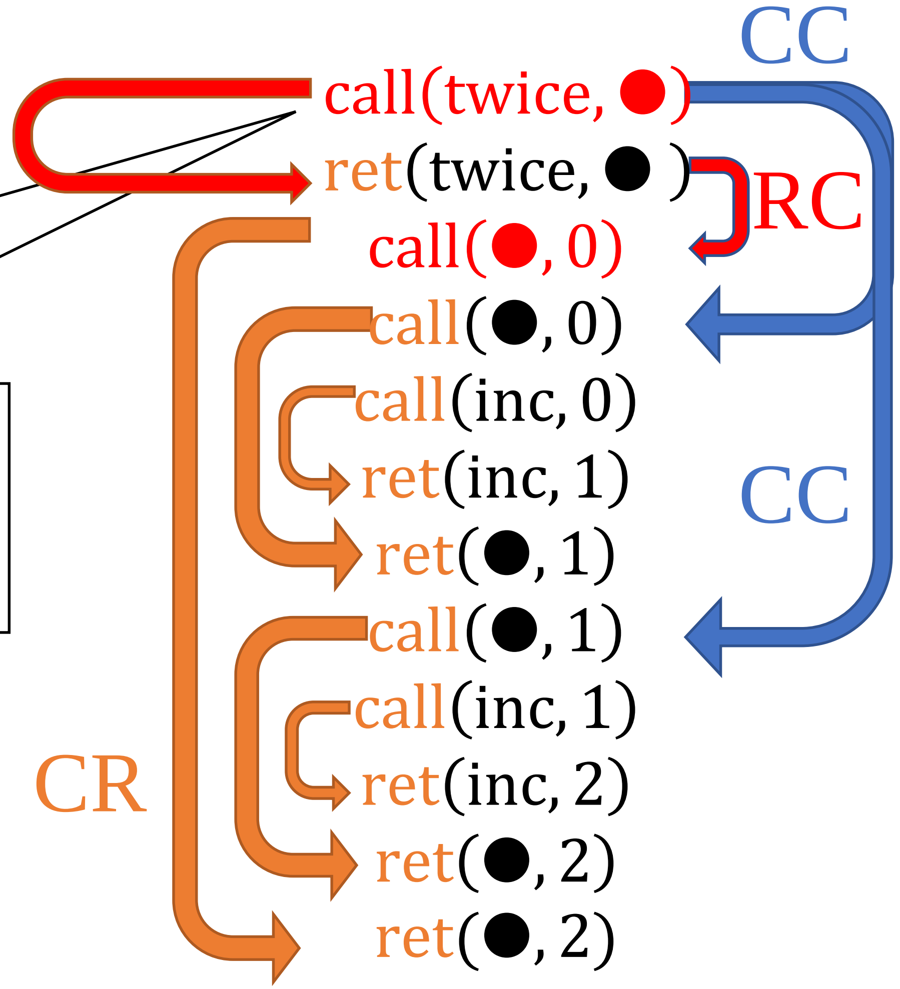

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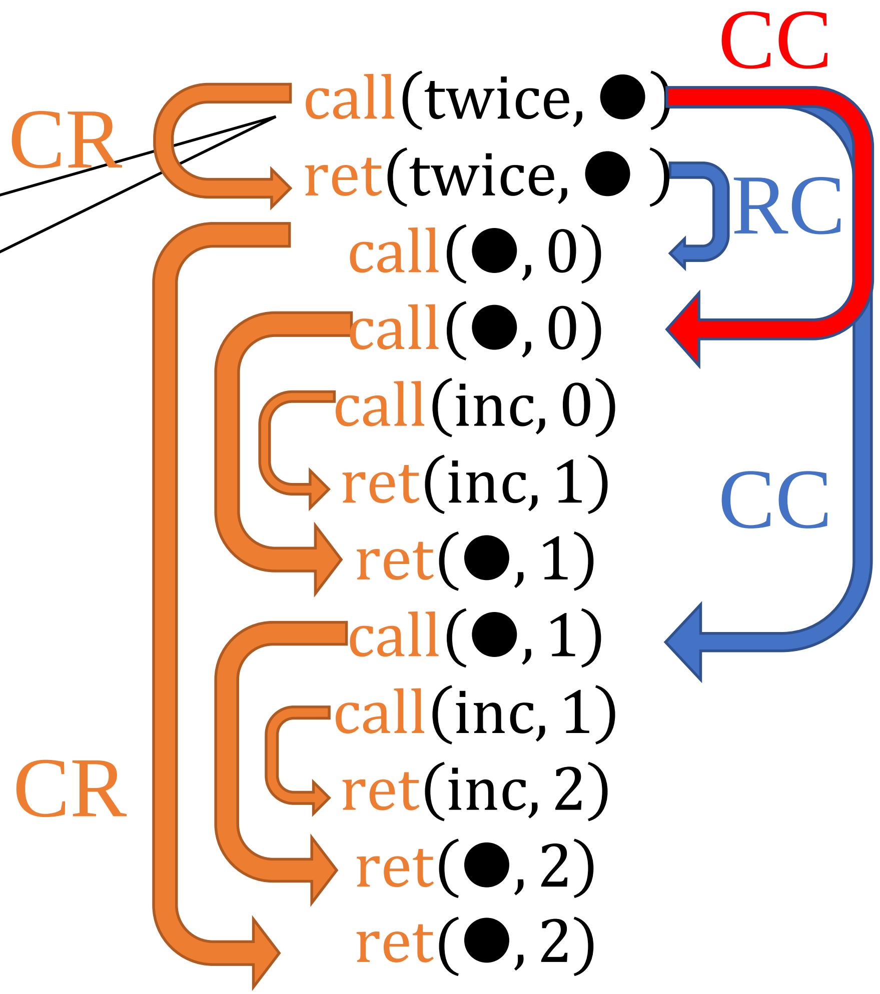

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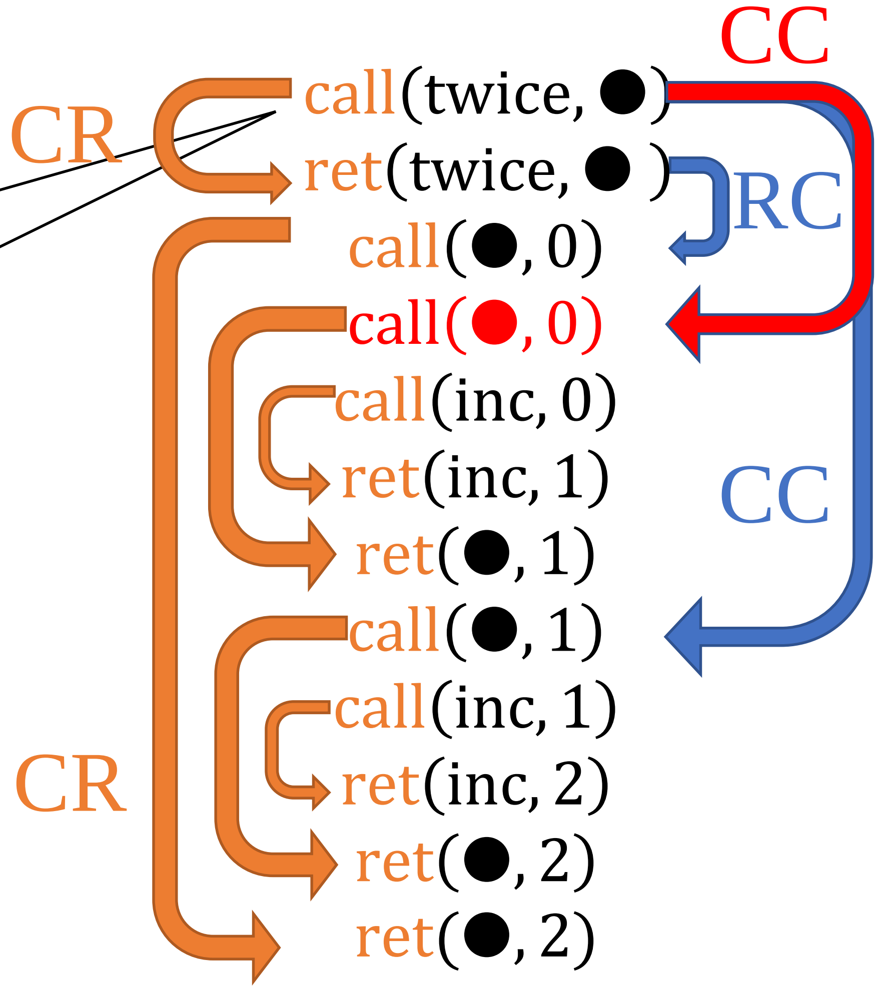

Conclusion

- HOT captures the control flow of higher-order programs
- HOT-PDL is an extension of PDL defined over HOTs
  - Enables a precise specification of temporal trace properties for higher-order programs
  - Provides a foundation for specification in various application domains
    - stack-based access control properties
    - dependent refinement types
- HOT-PDL model checking of higher-order programs is shown decidable via a reduction to higher-order model checking
- **Future work:** extend HOTs with new kinds of events and pointers for capturing call-by-name and/or effectful computations by incorporating more ideas from game semantics