DESIGN OF BLOCK LIFTING-BASED DISCRETE COSINE TRANSFORM TYPE-II AND IV

Taizo Suzuki and Masaaki Ikehara

Department of Electronics and Electrical Engineering, Keio University Yokohama, Kanagawa 223-8522 Japan
Email: {suzuki, ikehara}@tkhm.elec.keio.ac.jp

ABSTRACT

This paper presents a novel lifting factorization of discrete cosine transform type-II and IV (DCT-II and IV). Although some conventional integer DCT-IIs (IntDCT-IIs) with block size 8 have been proposed, they are not generalized as arbitrary block size M. Using block lifting factorization which has an efficient structure for lossless-to-lossy image coding, we present IntDCT-IIs and IVs with arbitrary block size M that is called block lifting-based DCT-IIs and IVs (BLDCT-IIs and IVs). Finally, the validity of our method is proved by showing the results of lossless-to-lossy image coding in the most general case of the block size 8 and the extended size 16.

Index Terms— discrete cosine transform (DCT), block lifting factorization, lossless-to-lossy image coding

1. INTRODUCTION

The discrete cosine transform (DCT) [1] is used to image, video and audio coding (compression) standards such as JPEG [2] and MPEG [3], because it has excellent property and many fast algorithms. There are several types in DCT. DCT type-II (DCT-II) and DCT type-III (DCT-III) which is the inverse transform of DCT-II are adapted to image and video coding. And DCT type-IV (DCT-IV) is also important for some fast algorithms of DCT-II or modified DCT (MDCT) [4] in audio coding. In JPEG, DCT-II is applied to lossy image coding, and a linear prediction is applied to lossless image coding [5]. We have to prepare both lossy and lossless compressed data because DCT-II and linear prediction do not have correlation each other.

Recently, higher quality data is demanded by the spread of broadband and the development of multimedia contents. Some integer DCT-IIs (IntDCT-IIs) have been proposed for lossless-to-lossy image coding [6–8]. However the conventional methods are described about only block size 8.

This paper presents a general block lifting factorization of M-channel DCT-II and IV. We call them M-channel block lifting-based DCT-II and IV (BLDCT-II and IV). Finally, the validity is shown by the comparison of our methods and conventional methods in lossless-to-lossy image coding.

Notations: I, J, M T and M N are an identity matrix, a reversal matrix, a transpose of matrix M and a N × N square matrix.

2. REVIEW

2.1. Discrete Cosine Transform (DCT)

The m column and n row element of M-channel DCT-II matrix C M [I] I, DCT-III matrix C M [IV] III and DCT-IV matrix C M [IV] IV are defined as

\[
\begin{bmatrix}
C_{M[I]}^{[I]}[m,n] = \frac{2}{M} c_m \cos \left( \frac{m(n+1/2)\pi}{M} \right) \\
C_{M[III]}^{[II]}[m,n] = \frac{2}{M} c_n \cos \left( \frac{(m+1/2)n\pi}{M} \right) \\
C_{M[IV]}^{[IV]}[m,n] = \frac{2}{M} \cos \left( \frac{(m+1/2)(n+1/2)\pi}{M} \right)
\end{bmatrix}
\]

where 0 ≤ m, n ≤ M − 1 and c m = 1/√2 (m = 0) or 1 (m ≠ 0). DCT can be factorized to some matrices for a fast algorithm. Chen et. al., present a fast algorithm of DCT-II [1] as follows:

\[
C_{M[I]}^{[I]} = P U W = P \begin{bmatrix} U_0 & U_0 \\ U_1 & -U_1 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & J \end{bmatrix}
\]

where

\[
\begin{align*}
U_0 & = \frac{1}{\sqrt{2}} C_{M[I]}^{[I]} \\
U_1 & = \frac{1}{\sqrt{2}} C_{M[IV]}^{[IV]}
\end{align*}
\]

P : a permutation matrix

Using \(C_{M[I]}^{[I]} = C_{M[I]}^{[I]^T}\), we can easily obtain a matrix factorized DCT-III.

Next, note that the matrix of DCT-IV can be presented as

\[
C_{M[IV]}^{[IV]} = \begin{bmatrix} V_0 & V_1 \\ V_1^T & V_2 \end{bmatrix}
\]

where

\[
\begin{align*}
V_0 & = V_0^T \\
V_2 & = -V_1^{-1} V_0 V_1
\end{align*}
\]

because \(C_{M[IV]}^{[IV]} = C_{M[IV]}^{[IV]^T}\) and

\[
C_{M[IV]}^{[IV]} C_{M[IV]}^{[IV]} = \begin{bmatrix} V_0 & V_1 \\ V_1^T & V_2 \end{bmatrix} \begin{bmatrix} V_0 & V_1 \\ V_1^T & V_2 \end{bmatrix} = \begin{bmatrix} V_0^2 + V_1 V_1^T & V_0 V_1 + V_1 V_2 \\ V_1 V_0 + V_2 V_1^T & V_1 V_1 + V_2^2 \end{bmatrix} = I.
\]

2.2. Lifting Structure

2.2.1. Basic Lifting Structure

Lifting structure [9] with rounding operators which quantize input signals can achieve lossless-to-lossy image coding. Fig.1(a) shows basic lifting structure. Analysis input signals \(x_i\) and \(x_j\), analysis output and synthesis input signals \(y_i\) and \(y_j\), synthesis output signals \(z_i\)
3. BLOCK LIFTING-BASED DISCRETE COSINE TRANSFORM TYPE-II AND IV (BLDCT-II AND IV)

The block lifting structure is an efficient structure for lossless-to-lossy image coding [10]. In this paper, LUL (lower-upper-lower triangular matrix) and ULU (upper-lower-upper triangular matrix) decomposition based block lifting factorization of DCT-II and IV are proposed.

3.1. Block Lifting Factorization based on LUL Decomposition

A novel block lifting factorization of DCT-II based on LUL decomposition is explained. First, the block lifting matrix are multiplied from the right sides of U in (1) as

\[
\begin{bmatrix}
U_0 & U_0 \\
U_1 & -U_1
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
S_0 & I
\end{bmatrix}
= \begin{bmatrix}
I & U_0 \\
S_1 & -U_1
\end{bmatrix}
\]

where

\[
S_0 = I - U_0^{-1} \quad S_1 = 2U_1 - U_1U_0^{-1}
\]

(3)

And the other block lifting matrix is multiplied from the right side of (3) as

\[
\begin{bmatrix}
I & U_0 \\
S_1 & -U_1
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
0 & S_1
\end{bmatrix}
= \begin{bmatrix}
I & 0 \\
0 & S_3
\end{bmatrix}
\]

where

\[
S_3 = \frac{1}{2}U_0^{-2} - U_0^{-1}
\]

(4)

Consequently, U can be factorized into the lifting structures using (3)-(5) as

\[
\begin{bmatrix}
U_0 & U_0 \\
U_1 & -U_1
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
I & 0
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
I & 0
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
0 & 1
\end{bmatrix}
= \begin{bmatrix}
I & 0 \\
S_0 & I
\end{bmatrix}
\]

Using \(U_0 = \frac{1}{\sqrt{2}}C_{II}, U_0^{-1} = \frac{1}{\sqrt{2}}C_{II}^T\) and \(U_1 = \frac{1}{\sqrt{2}}C_{IV}, (1)\) can be factorized into the lifting structures as follows:

\[
C_{II}^{[M]} = P \begin{bmatrix}
I & 0 \\
0 & -X_{03}
\end{bmatrix} \begin{bmatrix}
I & 0 \\
X_{02} & I
\end{bmatrix} \begin{bmatrix}
I & X_{01} \\
X_{00} & I
\end{bmatrix} W
\]

where

\[
\begin{bmatrix}
X_{00} = I - \sqrt{2}C_{II}^{[M]} \\
X_{01} = \sqrt{2}C_{II}^{[M]} \\
X_{02} = C_{II}^{[M]} - \sqrt{2}C_{II}^{[M]} \\
X_{03} = C_{II}^{[M]} + \sqrt{2}C_{II}^{[M]}
\end{bmatrix}
\]

(6)

Fig.2(a) shows M-channel BLDCT-II in (6).

Next, a novel block lifting factorization of DCT-IV is explained. The block lifting matrix are multiplied from the right sides of (2) as

\[
\begin{bmatrix}
V_0 & V_1 \\
Y_{00} & T_0
\end{bmatrix}
\begin{bmatrix}
I & 0 \\
-\sqrt{2} & 0
\end{bmatrix}
= \begin{bmatrix}
I & V_1 \\
-\sqrt{2} & T_0
\end{bmatrix}
\]

where

\[
Y_{00} = V_1^{-1}(V_0 - I) \quad T_0 = -V_1^{-1}V_0V_{10}
\]

(7)
And the other block lifting matrix is multiplied from the right side of (7) as
\[
\begin{bmatrix}
1 & V_1 \\
-Y_{00} & T_0
\end{bmatrix}
\begin{bmatrix}
1 & -Y_{01} \\
0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
Y_{01} = V_1. 
(8)
Consequently, (2) can be factorized into the lifting structures using (7) and (8) as follows:
\[
C_{II}^{[M]} = \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
Y_{00} & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
Y_{01} = V_1. 
(9)

3.2. Block Lifting Factorization based on ULU Decomposition

Previous block lifting factorizations are based on LUL decomposition. We also propose another novel block lifting factorization of DCT-II and IV based on ULU decomposition.

Computing three matrices without P, diag{1, -X_{03}} and W in (6), we can obtain the following matrix.
\[
\begin{bmatrix}
1 & V_1 \\
-Y_{00} & T_0
\end{bmatrix}
\begin{bmatrix}
1 & -X_{01} \\
0 & 1
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
X_{00} & 1
\end{bmatrix}
Y_{01} = V_1. 
(10)

The block lifting matrices are multiplied from the left side of X as
\[
\begin{bmatrix}
1 & -X_{10} \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
X_{12} & 1
\end{bmatrix}
X = \begin{bmatrix}
1 & 0 \\
X_{11} & 1
\end{bmatrix}
\]
where
\[
\begin{align*}
X_{10} &= \sqrt{2}C_{II}^{[4]} - I \\
X_{11} &= -\frac{1}{\sqrt{2}}C_{II}^{[4]} \\
X_{12} &= \sqrt{2}C_{II}^{[4]} - C_{II}^{[4]} - \frac{1}{2}
\end{align*}
\]

Hence, we can obtain M-channel BLDCT-II based on ULU decomposition as follows:
\[
C_{II}^{[M]} = P \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
X_{12} & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
Y_{10} = V_1. 
(11)

Fig. 2(b) shows M-channel BLDCT-II in (11).

Similarly, we can obtain M-channel BLDCT-IV based on ULU decomposition as follows:
\[
C_{IV}^{[M]} = \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
Y_{10} & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
Y_{11} & 1
\end{bmatrix}
Y_{10} = V_1. 
(12)

3.3. Completion of Lifting Factorization

Although (6) and (11) are not complete lifting structure due to X_{03} and X_{13} in (6) and (11), we can achieve the completeness by iterating lifting factorization of C_{II}^{[4]} and C_{IV}^{[4]} with shrunken size shown as Fig.2. Therefore, C_{II}^{[4]} has special lifting structures as
\[
\begin{align*}
X_{00}^{[2]} &= \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} , \\
X_{10}^{[2]} &= \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} , \\
X_{10}^{[2]} &= \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix} , \\
X_{10}^{[2]} &= \begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\end{align*}
\]
where \(\alpha = \cos(-\frac{\pi}{8}) - 1\) and \(\beta = \sin(-\frac{\pi}{8})\).

Fig. 3(a) and (b) show 4-channel BLDCT-IIs in (6) and (11). The white circles are rounding operators and the black ones are adders. It is clear that 4-channel BLDCT-IIs have an efficient structure because they have only five rounding operators, respectively.
4. RESULTS

In this paper, we design four 8 and 16-channel BLDCT-IIs based on LUL and ULU decomposition. They are compared to the conventional methods, and the validity of them are shown in lossless-to-lossy image coding.

4.1. Application to lossless image coding

Our BLDCT-IIs are applied to lossless-to-lossy image coding. The set partitioning in hierarchical trees (SPIHT) progressive image transmission algorithm [11] was used to encode the transformed images. The comparison of

\[
\text{Entropy [bpp]} = \frac{\text{Total number of bits [bit]}}{\text{Total number of pixels [pixel]}}
\]

in lossless image coding are shown in Table 1. We chose Komatsu’s reversible DCT (RDCT) [6] and Tran’s binary DCT (BinDCT) [8] as the conventional methods. Chen’s IntDCT [7] was not chosen because it has so bad results for lossless image coding. In Table 1, it is obvious that our BLDCT-IIs present better performance than the conventional IntDCT-IIs in lossless image coding. Each of BLDCT-II shows the different performance depending on an image.

4.2. Application to lossy image coding

If lossy compressed data is required, it can be achieved by interrupting the obtained lossless bit stream. The comparison of

\[
\text{PSNR [dB]} = 10 \log_{10} \left( \frac{255^2}{\text{MSE}} \right)
\]

where MSE is the mean squared error in lossy image coding are shown in Table 2. We chose Komatsu’s RDCT [6], Chen’s IntDCT [7] and Tran’s BinDCT [8] as the conventional methods. In Table 2, our BLDCT-IIs present better performance than the conventional IntDCT-IIs in lossy image coding. In low bit rate, the texture remained by using 16-channel BLDCT-II is more crear than 8-channel IntDCT-IIs because high frequency components can be analyzed by bigger block size. Fig.4 shows the enlarged images of ‘BARBARA’ which are the original image and its lossy compressed images by the conventional IntDCT-IIs and the proposed BLDCT-IIs when bit rate is 0.25 [bpp].
5. CONCLUSION

In this paper, we proposed a novel IntDCT-II and IV based on block lifting structure. Our IntDCT-II and IV, called $M$-channel BLDCT-II and IV, have arbitrary block size $M$ that is not only 8. In addition, our BLDCT-II included BLDCT-IV are more suitable for lossless-to-lossy image coding, because it shows better performance than the conventional IntDCT-Is. Our BLDCT-II and IV can be easily extended into ones with different size which is more suitable.

6. ACKNOWLEDGMENT

This work is supported in part by a Grant-in-Aid for the Global Center of Excellence for High-Level Global Cooperation for Leading-Edge Platform on Access Spaces from the Ministry of Education, Culture, Sport, Science, and Technology in Japan.

7. REFERENCES