Localization using uniaxial laser rangefinder and IMU for MAV

Kohei Ito and Justin Han and Akihisa Ohya

Abstract— **In this paper we describe a localization method based on an inertial measurement unit (IMU) and a uniaxial laser rangefinder suitable for use on a micro aerial vehicle (MAV). Because it is not possible to obtain the surrounding conditions all at once using the uniaxial rangefinder, it is necessary to determine the position in the environment in terms of the MAV's own movements. We demonstrated our method experimentally on a Quadrotor multicopter flying in a wide area enclosed by pillars.**

I. INTRODUCTION

In recent years, research on micro aerial vehicles (MAV) has become more popular. Exploration in GPS-denied environments such as indoor areas is one such research. In order to fly indoors, it is necessary to pass through doorways and hallways. Taking the width of these passageways under consideration, a MAV with a width of 30 cm or less would be the most suitable for free flight. Figure 1 shows the estimated relationship between the size and the flyable weight of the MAV. Using the required wattage for each propeller, we calculated the weight that would allow each propeller to provide approximately 10 minutes of flight with an assumed battery. According to this data, as the 30 cm class MAV will weigh about 1300 g, it would not be possible to load it with a high performance sensor. For the MAV to become a high-performance search machine, it is necessary to miniaturize both the mounted sensors and control devices.

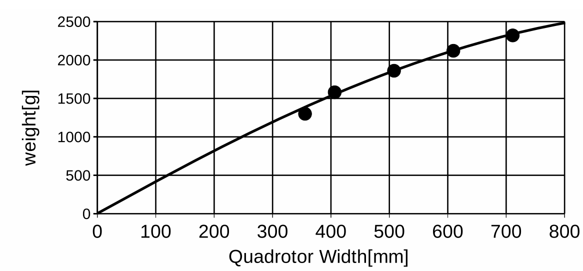

Fig. 1. Maximum Load per Quadrotor size

Today, rangefinders are essential for localization and mapping techniques such as Simultaneous Localization and Mapping (SLAM). In terms of the miniaturization of the rangefinder, the main sources of the weight are the optical system and the scanning mechanism. While the optical system cannot be removed, if the rotation mechanism is removed, it can still be used as a rangefinder. In considering weight reduction, for the time being, uniaxial laser rangefinders, that are unable to rotate and can only scan in one direction, seem to be promising. The meaning of uniaxial is that an optical axis direction is fixed for the main body of sensor. The authors investigated various uniaxial laser sensors with respect to the performance of Hokuyo's Top URG sensor. The main sensors considered are compared in TABLE I. I

Attempting to select a sensor based on measurement distance and measurement period failed to uncover a suitable sensor. Therefore, after considering diverting the signal processing circuit from the existing URG to construct a sensor capable of high speed measurements at a range of up to 30 m, we developed a prototype to test it (Fig.2). This time, for the purpose of confirming this idea, we only developed a prototype with the rotation function stopped. However, if the rotation mechanism were to be taken out, it is expected that a laser range finder that is around 100 g lighter can be implemented.

The URG sensor originally reads a reference plate a set distance away whilst scanning to calibrate its reading but cannot do so if it cannot scan. Due to this, the output will flux in response to the internal temperature. As such, we considered using another object at a known distance away to calibrate the measured value. However, the uniaxial sensor, due to not being able to change its bearing, cannot measure two distances. However, by using the multi-echo measurement function in the URG, we succeeded in measuring the distance to both the reference plate and the object in the same axial direction. As the location of the reference plate is already known, it became possible to calibrate the reading.

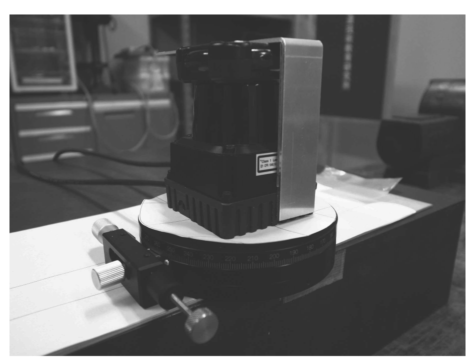

Fig. 2. Unaxial Laser Rangefinder

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TABLE I
RANGE FINDER

Manufacturer	Name	Weight	Range	Resolution	Update rate	Interface
Hokuyo	UTM-30LX	210g	30m	50mm	40Hz	USB
Nikon	COOLSHOT	165g	500m	0.5m	Unknown	No interface
NEWCONOPTIK	LRF Mod 2	180g	2500m	1m	2Hz	RS232C
Sharp	GP2Y0A21YK	20g	0.8m	1mm	20Hz	Analog
Parallax	Laser Range Finder	50g	2.4m	1mm	1Hz	Serial
SICK	DT60	202g	5.3m	1.5mm	66Hz	Analog

Because it is not possible to obtain the surrounding conditions all at once with the uniaxial rangefinder, it is necessary to determine the position of the objects in the environment in terms of the MAV's own movements. In this experiment, using an Inertial Measurement Unit (IMU) loaded onto the MAV, by combining the attitude information and the distance information from the uniaxial rangefinder, the MAV was able to grasp its own location. The IMU gives the angular velocity and the acceleration while the MAV obtains its location and attitude from the integration of the angular velocity and acceleration. Also, while it is possible to use the information from the laser rangefinder to discover its own location, because the attitude information estimated from the IMU contains integration error, it is necessary to first minimize the errors first.

Although we wanted to solve the problem of self-localization and mapping simultaneously, for this process, we are handling the self-localization based upon a previously obtained map. The method used for this is the widely used Markov localization algorithm. However, even in the case of the Markov localization method, in order to apply a high likelihood for the various locations from the uniaxial rangefinder, instead of using one round of the estimations for localization, storing several of the attitude estimation updates is necessary to validate the localization.

For us, the most important part that must be considered for obtaining the environmental information is the movement of the MAV. Because the information obtained by the uniaxial rangefinder on its surroundings is small, it is necessary to gather the environmental information by converting the attitude and location of the MAV in order to obtain the information. For that purpose, we developed a new method, tested it, and are explaining it in this paper.

II. RELATED WORK

There is much research into the practicality of MAVs in GPS-denied environments[1][2]. For the small MAVs, because the payload is small, the performance of the loaded sensor will also be low. In addition, because directly obtaining the odometry becomes impossible, it becomes necessary to measure the attitude from the integration of the information from the IMU. As the location measurement is a second-order integration of the acceleration, the drift is also large. Much research into self-localization and mapping under such conditions exists[3]. In many studies, the MAV is equipped with a MEMS gyro, an accelerometer, and a two-dimensional laser scanner. After performing scan matching using the Extended Kalman Filter (EKF) and the two-dimensional laser, by using the SLAM algorithm[4], the MAVs could perform self-localization and mapping. Also, in terms of SLAM, there is research into EKF as well as a recent study with particle filters. Furthermore, research into calculating the location and attitude of the MAV using a stereo camera or a monocular camera instead of the two-dimensional laser scanner exists as well[5][6]. On the other hand, although there are a large number of SLAM related studies using a sensor with two-dimensional scanning capability, research into self-localization and mapping using a uniaxial rangefinder is exceedingly unique.

III. MAV SYSTEM

A. *Quadrotor*

Shown Fig.3, the MAV used for this research is a Quadrotor with four propellers, each with a diameter of 14 in. The body is one meter long and one meter wide at a height of about 0.3 m. Two pairs of propellers spin in opposite directions to eliminate the torque reaction from the other. The propellers spin using high torque brushless motors. A PWM signal input Electric Speed Controller (ESC) controls the motors. The ESC is connected to an APM 2.6 Arduino flight controller board. Including the battery, the body weighs around 3.0 kg. As the thrust from the propellers is roughly 4.8 kgf, it is able to handle a payload of over 1 kg.

B. *Arduino microcontroller*

A gyroscope, an accelerometer, a barometer, a magnetic compass, and an ultrasonic altimeter are connected to the APM 2.6, which controls the stabilization of the Quadrotor.

C. *BeagleBoard-xM*

The APM 2.6 connects to a Linux installed miniature BeagleBoard-xM single-board computer via USB serial. The BeagleBoard-xM carries out obstacle avoidance, navigation, communication with ground equipment, and data collection.

D. *Unaxial Laser Rangefinder*

We remodeled the UTM-30LX-EW and produced the uniaxial laser rangefinder. Because the MAVs are small and cannot be loaded with large baggage, the ability of the sensors that it can be equipped with, in comparison with those that can be loaded onto ground robots, is lacking in many ways. Furthermore, when considering the future of MAVs, in order to penetrate deeply into narrow indoor corridors, a necessity for even smaller MAVs can be expected. To that extent, in favor of developing a lighter laser rangefinder, we diverted the processing circuit and the optics system of the scanning type laser rangefinder. This sensor is capable of a high speed sampling rate (3 ms) and a large measurement range (30 m). This is a feature not found in other rangefinders. It is connected to the Beagleboard-xM via USB.

E. Inertial Measurement Unit

The Quadrotor is equipped with an Inertial Measurement Unit for self-localization, which can obtain angles, angular velocity, and acceleration. It connects to the Beagleboard-xM through USB.

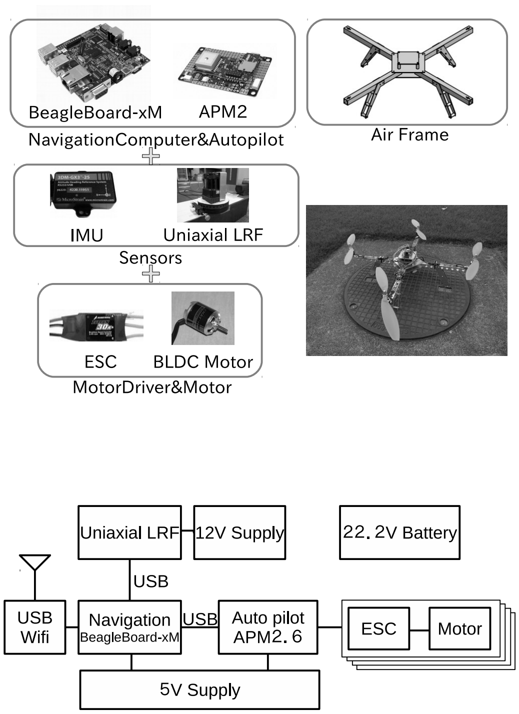

Fig. 3. Quadrotor system

IV. CONTROL

The Quadrotor is controlled by four inputs: pitch angle, roll angle, yaw angle, and thrust. Shown in Fig.4, to control the roll angle, by introducing a difference in the rotational speed between the left and right propellers, generating a rotational moment that causes rotation around the roll axis. The same principle applies to the other angles. Because the Quadrotor is an unstable system, a stabilization control must be included. For the control, the attitude of the Quadrotor is fed back and used in a PD control for stabilization. Using the ultrasonic sensor, the Quadrotor can detect the altitude and fix the height at a constant value. With this, it becomes possible to observe the plane at which it hovers.

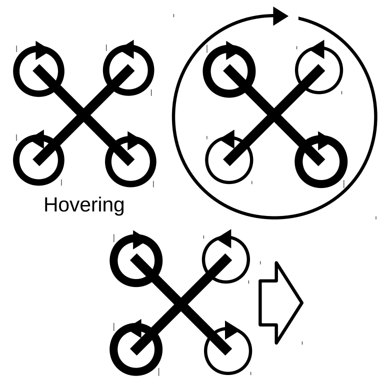

Fig. 4. Quadrotor control

V. LOCALIZATION

A. MONTE CARLO LOCALIZATION

The Quadrotor uses Monte Carlo Localization (MCL) as its localization scheme. MCL is a direct application of the Bayesian filter. The attitude of the robot cannot be assumed to be a unimodal distribution such as a normal distribution as it is a nonparametric filter. MCL approximates the distribution of the attitude of the Quadrotor using a large amount of particles. At every step, every particle undergoes a transition based on the movement model and each calculated, transitioned particle undergoes a selection process called resampling where particles with a high likelihood are duplicated and particles with a low likelihood are discarded. Algorithm 1 below shows the algorithm Thrun et al.[8] used for this process.

Algorithm 1 MCL(χ_{t-1}, u_t, z_t, m)

$\bar{\chi}_t = \chi_t = \emptyset$
for $m = 1$ to M **do**
  $x_t^{[m]} =$ sample_motion_model ($u_t, x_{t-1}^{[m]}$)
  $w_t^{[m]} =$ measurement_model ($z_t, x_{t-1}^{[m]}$)
  $\bar{\chi}_t = \bar{\chi}_t + < x_t^{[m]}, w_t^{[m]} >$
end for
for $m = 1$ to M **do**
  draw i with probablity $w_t^{[i]}$
  add $x_t^{[i]}$ to χ_t
end for
return χ_t

Here, χ_t is a set of M number of attitudes $x_t^{[m]}$ and is believed to be an approximate representation of a collection of particles. The sample_motion_model is the sampling function for the robot's movements and is calculated from the control u_t and the previous attitudes x_{t-1}.

B. QUADROTOR MOTION MODEL

In order to use MCL, we must create a motion model for the Quadrotor. The motion model for the Quadrotor using the IMU is shown below. The Quadrotor cannot obtain odometry nor is it equipped with a sensor to directly receive the speed. Due to using the IMU, the robot's own location can be found with the strapdown inertia calculation. Figure 5 also shows the coordinate system of the Quadrotor.

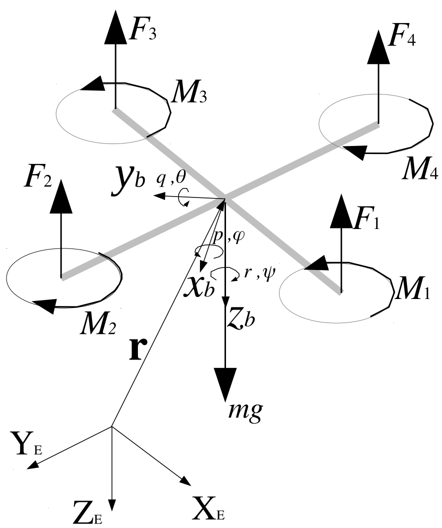

Fig. 5. Coordinate system

The relationship between Euler angles is given by:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{1}$$

where p is the angular velocity around the x-axis, q is the angular velocity around the y-axis, and r is the angular velocity around the z-axis in the coordinate system fixed to the Quadrotor.

The Euler angles can be obtained by integrating Eq.1. However, in this case, when $\theta = 90deg$, a discontinuity arises, so the Euler angles cannot be used. Therefore, for the attitude angle calculation, we use the quaternions. The four parameters for the quaternions is related to the axial angular velocities p, q, r by the relationships below.

$$\dot{e_0} = -\frac{1}{2}(e_1 p + e_2 q + e_3 r) \tag{2}$$

$$\dot{e_1} = \frac{1}{2}(e_0 p + e_2 q - e_3 r) \tag{3}$$

$$\dot{e_2} = \frac{1}{2}(e_0 p + e_3 q - e_1 r) \tag{4}$$

$$\dot{e_3} = \frac{1}{2}(e_0 p + e_1 q - e_2 r) \tag{5}$$

Theses parameters also always satisfy the following relationship.

$$e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1 \tag{6}$$

With the above equations, we can continuously calculate the parameters e_0, e_1, e_2, e_3. The initial value of each parameter can be expressed in terms of the Euler angles.

$$e_0 = \cos\frac{\psi}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2} \tag{7}$$

$$e_1 = \cos\frac{\psi}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} - \sin\frac{\psi}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} \tag{8}$$

$$e_2 = \cos\frac{\psi}{2}\sin\frac{\theta}{2}\cos\frac{\phi}{2} + \sin\frac{\psi}{2}\cos\frac{\theta}{2}\sin\frac{\phi}{2} \tag{9}$$

$$e_3 = -\cos\frac{\psi}{2}\sin\frac{\theta}{2}\sin\frac{\phi}{2} + \sin\frac{\psi}{2}\cos\frac{\theta}{2}\cos\frac{\phi}{2} \tag{10}$$

Calculating the four parameters simultaneously, the Euler angles can be solved using the following equations.

$$\theta = \sin^{-1}[-2(e_1 e_3 - e_0 e_2)] \tag{11}$$

$$\phi = \cos^{-1}\left[\frac{e_0^2 - e_1^2 - e_2^2 + e_3^2}{\sqrt{1 - 4(e_1 e_3 - e_0 e_2)^2}}\right] sign[2(e_2 e_3 + e_0 e_1)] \tag{12}$$

$$\psi = \cos^{-1}\left[\frac{e_0^2 + e_1^2 - e_2^2 - e_3^2}{\sqrt{1 - 4(e_1 e_3 - e_0 e_2)^2}}\right] sign[2(e_1 e_2 + e_0 e_3)] \tag{13}$$

From these equations, the attitude angle of the Quadrotor can be obtained. The accelerations $\dot{u}, \dot{v}, \dot{w}$ can be calculated in terms of the angular acceleration a_x, a_y, a_z, the angular velocity p, q, r, the velocity u, v, w, and the gravitational acceleration g values obtained from the IMU.

$$\dot{u} = a_x + vr - wq + g\sin\theta \tag{14}$$

$$\dot{v} = a_y - ur + wp - g\cos\theta\sin\phi \tag{15}$$

$$\dot{w} = a_z - uq - vp - g\cos\theta\cos\phi \tag{16}$$

The Earth rotates around the its axis at $\Omega(15deg/hour)$.

$$\boldsymbol{\Omega} = \begin{bmatrix} \Omega\cos\lambda \\ 0 \\ -\Omega sin\lambda \end{bmatrix} \tag{17}$$

Here, λ is latitude.

When the Quadrotor flies at a set speed, in regards to the Earth, there is a new rotational movement represented by the following equation:

$$\boldsymbol{\omega'} = \begin{bmatrix} \dot{\mu}\cos\lambda \\ -\dot{\lambda} \\ -\dot{\mu} sin\lambda \end{bmatrix} \tag{18}$$

where μ is the longitude. The angular velocity measured by the IMU includes Ω and ω' so the actual angular velocity is represented by the following equation.

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_m - \boldsymbol{DCM}[\boldsymbol{\Omega} + \boldsymbol{\omega'}] \tag{19}$$

The $\boldsymbol{DCM}$ in this equation refers to the direction cosine matrix or the transformation matrix shown below used to convert to the Quadrotor's body coordinates from the inertial coordinates.

$$\boldsymbol{DCM} = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\theta s\phi c\psi - s\psi c\phi & s\psi s\theta s\phi + c\psi c\phi & s\phi c\theta \\ s\theta c\phi c\psi + s\psi s\phi & s\phi s\theta c\phi - c\psi s\theta & c\phi c\theta \end{bmatrix} \tag{20}$$

Integrating $\dot{u}, \dot{v}, \dot{w}$ and then transforming the result with $\boldsymbol{DCM}^T$, we obtain V_N, the northward velocity, V_E, the eastward velocity, and V_D, the downward velocity in terms of the navigation frame or the local earth frame,

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} V_N \\ V_E \\ V_D \end{bmatrix} = \boldsymbol{DCM}^T \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{21}$$

Integrating V_N,V_E,V_D, we obtain the location in relation to the Earth's surface. Looking at the relationship between the latitude λ, the longitude μ, and the height H.

$$\dot{\lambda} = \frac{V_N}{R_e} \tag{22}$$

$$\dot{\mu} = \frac{V_E}{R_e \cos \lambda} \tag{23}$$

$$\dot{H} = -V_D \tag{24}$$

Here,R_e is the Earth's radius.

With this, we can calculate the Quadrotor's coordinates on the Earth. Regarding the above equations, by replacing the integration with numerical integration and adding a random perturbation to the normal distribution, we can use them as the sample_motion_model. However, because the calculations from here on are large, the actual calculations will be done by a separate computer.

C. MEASUREMENT MODEL

For the measurement model of the uniaxial laser rangefinder sensor, in accordance with Thrun et al.[8], we considered small instrumentation noise, unexpected object errors, object detection failure, and unknown random noise.

Appointing z_t as the measured value with the small measurement noise and the true distance as z_t^*, the measurement probability is:

$$p_{hit}(x_t|x_t, m) = \begin{cases} \eta \mathcal{N}(x_t; z_t^*, \sigma_{hit}^2) & \text{if } 0 \leq z_t \leq z_{max} \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

Here, η is the normalization parameter for setting the integration of the probability to one.

For unexpected objects, the value will undoubtedly be detected as smaller than z_t^*. This can be modeled by an exponential distribution and is represented by the following equation.

$$p_{short}(z_t^k|x_t, m) = \begin{cases} \eta \lambda e^{-\lambda_{short} z_t} & \text{if } 0 \leq z_t \leq z_t^* \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

In this case η is also the normalization parameter for setting the integration of the probability to one again.

The measurement failure refers to, in terms of the laser rangefinder, a situation where an object absorbs the light. If the sensor cannot detect a distance, it will return the maximum distance. The probability for the maximum distance is represented by:

$$p_{max}(z_t|x_t, m) = \begin{cases} 1 & \text{if } z = z_{max} \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

In terms of a nonsense measurement, there are times where the sensor will return an incomprehensible output. As it can cross over the entire measurement range, it can be represented by a uniform distribution.

$$p_{rand}(z_t|x_t, m) = \begin{cases} \frac{1}{z_{max}} & \text{if } 0 \leq z_t \leq z_t^* \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

Algorithm 2 shows the algorithm for finding the likelihood of the measurements with the above errors included.

Algorithm 2 beam_range_finder_model(z_t, x_t, m)

1: compute z_t^* for the measurement z_t using ray casting
2: $p = z_{hit} p_{hit}(z_t|x_t, m) + z_{short} p_{short}(z_t|x_t, m)$
3: $+ z_{max} p_{max}(z_t|x_t, m) + z_{rand} p_{rand}(z_t|x_t, m)$
4: **return** p

VI. SCANNING BY MANEUVER

When using a uniaxial laser rangefinder for self-localization and mapping, with a single measurement, only the distance to an object in the direction the sensor is facing is known. Therefore, unlike the scanning type laser rangefinder, other than by matching measurement data, things like attitude cannot be estimated. To that extent, by building in an attitude sensor and estimating the sensor direction from the attitude sensor's information, we can obtain the surrounding conditions. Accordingly, we, as shown in Fig.6, will have the Quadrotor hover in place along the search route at fixed time intervals and rotate around the z-axis.

Before the rotation movement is carried out, because we do not know anything besides the distance from the bodies in front of its travel path, although we can estimate the distribution of every particle, in terms of the rotational movement, the likelihood that particles will be in its actual area increases. The distribution can then be throttled..

VII. EXPERIMENTS

The Quadrotor carried out the self-localization experiment by measuring its environment while pivoting itself in the air. The experiment environment is a roofed plaza in front of the school building enclosed by pillars. As the area is smaller than the uniaxial laser rangefinder's measurement range, it is possible to obtain the entirety of the area's circumstances with one revolution. The test machine, at 10 m from a pillar, hovered and rotated to confirm its surroundings.

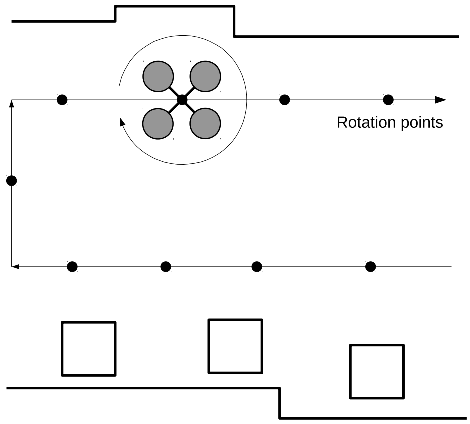

Fig. 6. Scanning by Maneuver

During the experiment, the Beagleboard-xM, loaded onto the Quadrotor, did not perform the self-localization. Instead, it recorded the output data from the uniaxial laser and the IMU and afterwards, we performed the localization on a computer using the recorded data. As the purpose of this experiment was to verify the validity of the algorithm, we did not perform an examination into the efficiency or the speed of the algorithm. Also, because the amount of calculations require was large, simultaneous self-localization would have been difficult.

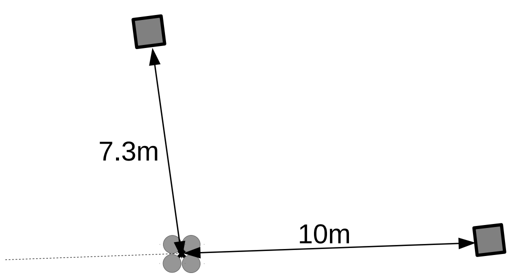

Fig. 7. Experiments Field

As shown Fig.8, in terms of the experimental results, we understood it was roughly possible to self-localize.

VIII. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

We loaded a uniaxial laser rangefinder without a scanning function and an IMU onto a Quadrotor and tested self-localization with it. As it does not have a scanning function, in order to self-locate, the Quadrotor itself has to move to improve the accuracy of its self-localization. In this experiment, in order to confirm its surroundings, every 10 m, the Quadrotor hovers and pivots to confirms its surroundings. The result is that we confirmed a similar self-localization efficiency in comparison to the dead reckoning from using the acceleration and angular velocity obtained from the IMU.

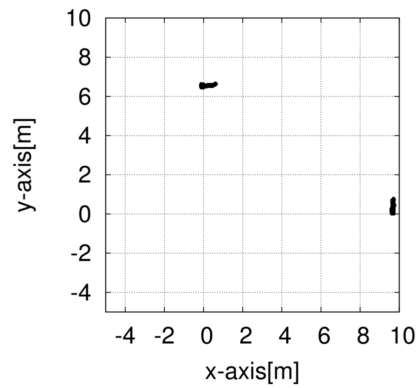

Fig. 8. Mapped data obtained localization

B. Future Works

In the current experiment, when the Quadrotor was rotating, we observed it drawing an arc and deviating sideways from its position. We believe that not taking the actual balance of the body into account is the cause. Because the disturbance from not giving much attention to location control with the machine's control scheme is small,, if the location slips when at rest, steering could correct the slip. However, because the Quadrotor changes its bearing during rotations, manual steering is difficult. For this phenomenon, when the machine is pivoting to confirm its surroundings, the bearing as well as its location will have an effect on the precision of the self-localization. Future work will focus on building location control and rotation stabilization.

In the current experiment, we implemented manual steering but we want to aim at autonomous flight where the MAV itself will decide the best actions.

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