Logical Representation of Musical Information

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Musical Informatics

- Investigating higher level fusion of intellect and sensibility of human via music
- Understanding how to play artistic music performance by computers
- Importance of logical representation of musical information
Musical Information

Expression Rules and Fingering Rules based on Musical Structures and Structural Functions of Music
Musical Structures

The 1st 5 measures of Chopin’s Polonaise Militaire Op. 40, No. 1

- **Group:** short sequence of notes
- **Motif:** 2 measures
- **Phrase:** 2 motifs
- **Sentence:** 2 phrases
Structural Functions

The 1st motif of Chopin’s Polonaise Militaire Op. 40, No. 1

- Inceptive: the starting point.
- Anacrusic: getting tenser to the initiative.
- Tiara: a climax area of the tension.
- Initiative: an ideal climax note in the structure.
- Desinence: diminishing the tension.
- Conclusive: the end point.
The 1st sentence of Chopin’s Polonaise Militaire Op. 40, No. 1, Performed by J. Olejniczak.
Agogic Rules:

- The inceptive is played **slowly**. How slow?  
- The performance of the anacrusis **accelerates**. How much is the rate of acceleration?  
- The initiative is **emphasized**. What means “emphasize”?  
- etc.

**Musical description in natural language is ambiguous.**
Fingering Rules

Fingering:

- a basis for expression of performance based on musical structures, esp. small groups (e.g. beat, semimotif, etc.)
- logical representation (rule) is useful for decision of fingers.
The 1st motif of Burgmüller’s Innocence

Rule:

- **St₁**: for every adjacent pair of notes in a group, a reachable pair of fingers must be used if they are performed *legato*.

- **St₃**: two sequences of fingers for two groups must be the same if two scores and two performance plans are both similar.
Musical Structures

\[ S[F_S] : \text{score with fingering} \]

\[ S : \text{score}, \; F_S : \text{fingering for } S \]

\[ S[F_S]= (\sigma_1, \sigma_2, \ldots, \sigma_m), \; \sigma_i : \text{structure} \]

\[ \sigma_i=(\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{il}), \; \sigma_{il} : \text{lower structure}, \; \text{or} \]

\[ \sigma_i=(\nu_{i1}, \nu_{i2}, \ldots, \nu_{in}), \; \nu_{ij} : \text{note} \]
$v_{ij} = <\text{num}_{ij}, \text{ptc}_{ij}, \text{vol}_{ij}, \text{lng}_{ij}, \text{itv}_{ij}, \text{prt}_{ij}, \text{msr}_{ij}, \text{nvl}_{ij}, \text{fgn}_{ij}>$

- $\text{num}_{ij}$: index number
- $\text{ptc}_{ij}$, $\text{vol}_{ij}$: pitch and volume
- $\text{lng}_{ij}$, $\text{itv}_{ij}$: length and interval
- $\text{prt}_{ij}$, $\text{msr}_{ij}$: part and measure
- $\text{nvl}_{ij}$: note value
- $\text{fgn}_{ij}$: finger number
Notation

\[ \nu_{ij}^{\text{num}} = \nu_{ij}^{1} = \text{num}_{ij}, \quad \nu_{ij}^{\text{ptc}} = \nu_{ij}^{2} = \text{ptc}_{ij}, \quad \text{etc.} \]

\[ \mathcal{S}[F_S](k) : k\text{-th note in the score } \mathcal{S}[F_S]. \]

\[ \sigma = \text{incp}(\sigma) \cdot \text{anac}(\sigma) \cdot \text{tiar}(\sigma) \cdot \text{init}(\sigma) \cdot \text{desn}(\sigma) \cdot \text{cncl}(\sigma) \]

\[ \text{mid}(\sigma) = \text{tiar}(\sigma) \cdot \text{init}(\sigma) \cdot \text{desn}(\sigma) \]
Example

\[ S[F_S] = (Mtf_1, Mtf_2) \]

\[ Mtf_1 = (n_{1,1}, n_{1,2}, \ldots, n_{1,10}) \]

\[ n_{1,1} = <1, A_4, 80, 500, 0, 1, 1, 1/8, 5> \]

\[ n_{1,2} = <2, \text{rest}, 0, 250, 500, 1, 1, 1/16, 5> \]

\[ n_{1,3} = <3, E_4, 76, 250, 250, 1, 1, 1/16, 4> \]

...
Agogic Rules

The performance of the inceptive starts slowly:
\[ \text{ave} \circ \text{tpbs} \circ \text{mid}(\sigma) < \text{ave} \circ \text{tpbs} \circ \text{incp}(\sigma) \]
The preformance accelerates in the anacrusis, while it decelerates in the desinance:

\[
Dcr(\text{aprx} \circ \text{agc} \circ \text{anac}(\sigma)) \land \text{Inc}(\text{aprx} \circ \text{agc} \circ \text{desn}(\sigma))
\]
The tiara is played longer than the last note of the anacrusis:

$$tpb \circ nth(anac(\sigma), |anac(\sigma)|) < ave \circ tpbs \circ tiar(\sigma)$$
The initiative is played longer than the "neighbor" notes other than the tiara

\[
\text{tpb} \circ \text{nth}(\text{anac}(\sigma), |\text{anac}(\sigma)|) < \text{tpb} \circ \text{init}(\sigma)
\]
\[
\wedge \text{tpb} \circ \text{nth}(\text{desn}(\sigma), 1) < \text{tpb} \circ \text{init}(\sigma)
\]
The performance ends slowly in the conclusive:
\[ tpb \circ \text{nth}(\text{anac}(\sigma), |\text{anac}(\sigma)|) < tpb \circ \text{init}(\sigma) \]
Fingering Information

Distance between 2 keys $x$ and $y$:

$$kd(x, y) = \max\{x, y\} - 1$$

$$kd(x, y) = \sum_{i=\min\{x, y\}}^{\max\{x, y\} - 1} kd^*(i)$$

Signed key distance:

$$skd(x, y) = \begin{cases} 
kd(x, y) \\
-kd(x, y)
\end{cases}$$

where

$$kd^*(x) = \begin{cases} 
2 & \text{if } x \mod 12 \in \{4, 12\} \\
1 & \text{otherwise}
\end{cases}$$
Signed Key Distance
Signed Key Distance

\[ \text{skd}(C_4, E_4) = 4 \]
Signed Key Distance

\[ \text{skd}(C_4, E_4) = 4 \]
\[ \text{skd}(C_4, A_3) = -4 \]
Spans between two fingers

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<thead>
<tr>
<th></th>
<th>Maximum</th>
<th>Minimum</th>
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<tbody>
<tr>
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<td>$\text{maxpr}$</td>
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<tr>
<td>Relaxed</td>
<td>$\text{maxrel}$</td>
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$maxpr(1, 5) = 17$
### Spans between two fingers

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- $minpr(1, 5) = -5$
Spans between two fingers

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\[
max_{rel}(1, 5) = 15
\]
(Predicate) Two notes $v_1$ and $v_2$ are adjacent in the score, but some rests may occur between them:

$$
\text{Adj}(v_1, v_2) \equiv v_1^{ptc} \neq \text{rest} \land v_2^{ptc} \neq \text{rest} \\
\land v_1^{\text{num}} < v_2^{\text{num}} \\
\land \forall i (v_1^{\text{num}} < i < v_2^{\text{num}} \supset S(i)^{ptc} = \text{rest})
$$
(Predicate) Two notes $v_1$ and $v_2$ can be played simultaneously for two fingers with a practical span, i.e. the fingers are reachable:

$$\text{Reach}(v_1, v_2) =$$

$$\minpr(v_{1 \ fgn}, v_{2 \ fgn}) \leq \skd(v_{1 \ fgn}, v_{2 \ fgn})$$

$$\leq \maxpr(v_{1 \ fgn}, v_{2 \ fgn})$$

$$\land v_1^{ptc} \neq \text{rest} \land v_2^{ptc} \neq \text{rest}.$$
(Predicate) Two notes \( v_1 \) and \( v_2 \) is played simultaneously for two fingers with opposite order, i.e. the fingers are crossing:

\[
\text{Cross}(v_1, v_2) \equiv \\
(\nu_1^{ptc} \neq \text{rest} \land \nu_2^{ptc} \neq \text{rest}) \\
\land \text{Cross}^*(\langle \nu_1^{fgn}, \nu_2^{fgn} \rangle, \langle \nu_1^{ptc}, \nu_2^{ptc} \rangle)
\]

where

\[
\text{Cross}^*(\langle f_1, f_2 \rangle, \langle x_1, x_2 \rangle) \equiv \\
(f_1 < f_2 \land x_1 > x_2) \lor (f_1 > f_2 \land x_1 < x_2)
\]
Fingering Rules

Rule \( St_1 \): If a group \( \sigma \) is played legato, then reachable fingers must be used:

\[
St_1(\sigma) \equiv \forall i (1 \leq i \leq |\sigma| - 1 \supset \nexists \) \ntag{1544}\n\]

\[
St_1^*(\sigma(i), \sigma(i + 1)) \tag{1544}
\]

where

\[
St_1^*(v_1, v_2) \equiv \text{Adj}(v_1, v_2) \land \text{Lgt}(v_1, v_2) \supset \text{Reach}(v_1, v_2)
\]

Performance plan: they are played legato.
Rule $St_{1G}$: If adjacent notes of adjacent group is played legato, then reachable fingers must be used:

$$St_{1G}(\sigma_1, \sigma_2) \equiv St_1^*(\sigma_1(|\sigma_1|), \sigma_2(1))$$
Rule $St_2$: If a group $\sigma$ is played smoothly, then fingers must not cross:

\[
St_2(\sigma) \equiv Phrs(\sigma) \supset \\
\forall i(1 \leq i \leq |\sigma| - 1 \land \text{Adj}(\sigma(i), \sigma(i + 1)) \supset \\
\neg \text{cross}(\sigma(i), \sigma(i + 1))
\]

Performance plan: it is played smoothly, emphasizing its phrasing.
Rule $St_3$: For two groups $\sigma_1$ and $\sigma_2$, if both the scores and the performance plans are similar, then these fingers must be the same:

$$St_3(\sigma_1, \sigma_2) \equiv |\sigma_1| = |\sigma_2| \land Sim(\sigma_1, \sigma_2) \land Smxp(\sigma_1, \sigma_2) \supset \forall i (1 \leq i \leq |\sigma_1| \supset \sigma^f_{\text{gn}}(i) = \sigma^f_{\text{gn}}(i))$$

Scores are similar.  Performance plans are similar.
From these 4 rules, we have candidates of fingering including the correct answer (4, 3, 2, 1, 4, 3, 2, 1, 4, 3, 2, 1, 4, 3, 2).

If the correct answer want be obtained, then the notions of *penalty point, difficulty, weight*, etc. are needed.
Conclusions

- Some logical representations are given.
  - The first trial for musical expression to describe in logical language.
  - Quantification is unfinished.
    (eg. “how much is the rate of the acceleration?”)
  - An automatic ensemble system has already been represented logically, implemented, verified and analyzed.
Future Work

Application of the research of the musical informatics to North African music.