# Nonlinear k-subspaces based appearances clustering of objects under varying illumination conditions

Xi LI and Kazuhiro Fukui
Graduate School of Systems and Information Engineering
University of Tsukuba, JAPAN
{xili@viplab.is,kfukui@cs}.tsukuba.ac.jp

## Abstract

*Unsupervised clustering of image sets of 3D objects has been an active research field within vision community. It is a challenging task since the appearance variation of the same object under different illumination condition is often larger than the appearance variation of different object under the same illumination condition. Some previous methods perform the appearance clustering using k-subspaces algorithm by assuming that the set of images of a Lambertian object approximately reside in a low dimensional linear subspace. This paper further extends the original k-subspaces clustering algorithm to the nonlinear case. The sum of the squares of distance to corresponding feature points of each nonlinear subspace cluster centers is minimized using Expectation-Maximization like iteration procedure. Those distances can be novelly defined via inner product by kernel trick. Experiments on different datasets show that the proposed kernel-based nonlinear k-subspaces clustering algorithm achieves much higher clustering rate than its linear counterpart.*

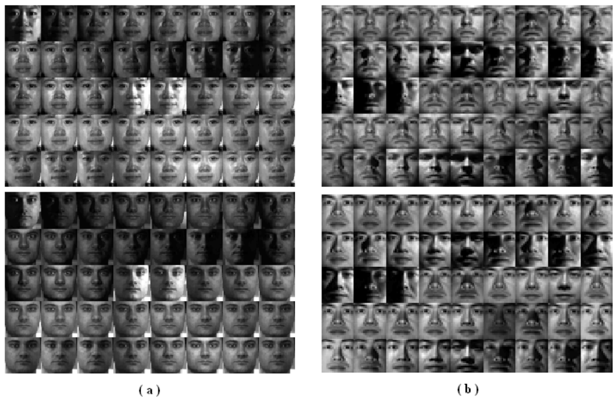



**Figure 1. Sample images of two persons under varying illumination conditions in the a) PIE database[8] b) YaleB database[9]. The appearance variation of the same object under different illumination condition is larger than the appearance variation of different object under the same illumination condition which renders the clustering according to the underlying identity a difficult task.**

## 1 Introduction

Unsupervised clustering of image sets of 3D objects under varying viewing conditions has been an active research field within computer vision community[1, 2, 3, 4, 5, 6, 7]. Typically, there are several viewing aspects that could affect the 2D image appearances during the projection procedure: the relative orientation between the viewing camera and the target object, the illumination conditions under which the images are acquired, and the reflective properties of the surface. As in [7], this paper studies the problem of unsupervised clustering of images sets of objects with Lambertian surface taken under varying illumination conditions while the target objects are in fixed poses. The objective is to partition the given image sets into disjoint subsets corresponding to underlying identities. It is a challenging task since the appearance variation of the same object under different illumination condition is often larger than the appearance variation of different object under the same illumination condition. Figure 1 shows an example of such case using CMU PIE face dataset[8] and YaleB face dataset[9]. It can be clearly seen that a direct standard clustering using Euclidean distance metric such as k-means algorithm will yield poor result.

Previously, several algorithms have been proposed for the problem of appearances clustering of objects under varying illumination conditions[1, 2, 3, 4, 5, 6, 7]. The most related to our work is[7]. In their work, J. Ho et al.

presented an appearance-based methods for clustering image sets of 3-D objects, acquired under varying illumination conditions, into disjoint subsets corresponding to each subject.They iteratively performed appearances clustering using K-subspaces algorithm by assuming that images for the same object approximately reside in a low dimension linear subspace[2, 5, 6]. The K-subspaces clustering algorithm can fully exploit the linear geometric structure hidden among the image sets. They proposed two method for the initialization: One is based on the concept of illumination cones and the other is based on spectral clustering, where the affinity matrix is computed by image gradient comparisons.

Recent studies show that non-linear subspace approximation via kernel trick is superior compared to their linear counterpart because the real life high dimensional data, such as the vectorized image data, is often inherently nonlinear rather than simple normally distribution[10]. One of the most representative innovation has been the kernel principal component analysis(KPCA), which makes use of the kernel trick to non-linearize PCA and extract nonlinear subspaces. This kind of kernel based algorithms can model complex real life data structures more faithfully and have achieved much success within machine learning and pattern recognition communities[10]. Motivated by those successes, this paper proposes a novel algorithm that performs nonlinear subspace clustering in the mapped high dimensional feature space. Firstly, the input patterns are mapped into a high-dimensional feature space via a nonlinear mapping function. Then the nonlinear subspaces are extracted in the feature space and distances between the mapped feature points and extracted nonlinear subspaces are defined via inner products by kernel trick. The objective function, which is the sum of the squares of distance to corresponding feature points of each nonlinear subspace cluster centers, is minimized using Expectation-Maximization like iteration procedure. Experiments on two different face datasets show that the proposed nonlinear Kernel K-subspaces Clustering(Kernel-KsC) algorithm converges quickly and achieves much higher clustering rate than that of the original Linear K-subspaces Clustering (Linear-KsC) algorithm.

The rest of this paper is organized as follows: Firstly, we describe the Linear-KsC algorithm for unsupervised appearances clustering of objects under varying illumination conditions in Section 2. Section 3 describes the proposed Kernel-KsC algorithm in detail. Experimental results of the clustering performance comparison between the Linear-KsC algorithm and the proposed Kernel-KsC algorithm using CMU PIE face dataset and Yale face dataset are presented in Section 4. Section 5 draws the conclusion.

## 2 Unsupervised appearances clustering

J. Ho et al.[7] showed that both the illumination cones based method and the gradient metric based method give reasonable results for the initialization of the iteration procedure of the linear K-subspaces clustering. They claimed that the computation of gradient metric was reliable in low-resolution images and could give promising clustering results. Here we adopt the similar framework as in [7]. That is to say, firstly we also use the gradient metric based method for the initialization and then the initial rough clustering results are further refined using subspace clustering. We put the emphasis on showing the superiority of the proposed kernel K-subspaces clustering algorithm over its linear counterpart. For the sake of completeness, we describe the main idea of gradient metric based clustering initialization and the iterative procedure of the original linear K-subspaces clustering algorithm briefly in the next subsections.

### 2.1 Spectral clustering based on gradient affinity

Suppose there are $N$ input images $\{I_1, ..., I_N\}$ where each image has $s$ pixels. The idea of gradient affinity is simple to directly compare between image gradient pairs. The differences in the magnitude of the image gradient and the relative orientation over the whole image plane are summed. Once we get the affinity matrix, standard spectral based algorithms[11] can be used to perform unsupervised clustering. For more details, refer to literatures[7]. Also, Some variants of the spectral clustering algorithm have been developed for the problem of automatic determination of the number of cluster centers[12]. This paper focuses on the superior performance of the proposed nonlinear K-subspaces clustering algorithm over its linear counterpart and we simply assume that the number of the cluster centers in known in advance.

Previous studies on illumination invariants show that the set of monochrome images of a convex object with a Lambertian reflectance forms a convex polyhedral cone when illuminated by an arbitrary number of point light sources at infinity[4, 5]. This implies that the collection of appearances of objects can be approximated by some low dimensional linear subspaces. So the initial rough clustering results using spectral method based on gradient affinity can be further refined via subspace clustering algorithm.

### 2.2 Linear K-subspaces Clustering

Linear K-subspaces Clustering(Linear-KsC) algorithm is an extension of the traditional K-means clustering algorithm. While the K-means clustering algorithm tries to find

$K$ cluster centers using Euclidean distance metric between point pairs, the objective of K-subspaces clustering algorithm is to find $K$ linear subspace base clusters using distance metric between points and subspace bases. The K-subspaces clustering algorithm shares the similar idea with the k-means algorithm and the flowcharts of both iteration procedure are almost the same. Firstly, each point is assigned to the nearest subspace cluster base. The distance is computed as the length of the difference vector between the original point and its reconstruction using the corresponding subspace base center ( In the next section, we will show that the computation of the distance can be written in the form of inner product, which renders the extension to the nonlinear case possible ) . Then the subspace bases are updated by principal component analysis. Usually the iteration procedure converges quickly in just several number of loops. It should be noted that the original 2D image matrix representation is firstly transformed into 1D vector form.

Specifically, the linear K-subspaces clustering algorithm can be described as follows:

**Algorithm 1: Linear K-subspaces Clustering(Linear-KsC)**

1. *Initialization*: Suppose there are $N$ input images $\{I_1, ..., I_N\}$ where each image has $s$ pixels. Starting with a collection $\{S_1, ..., S_K\}$ of $K$ subspaces of dimension $d$, where $S_i \in R^s$. The corresponding orthnormal bases for each subspace is denoted as $U_i$ with size $s \times d$;

2 *Points assignment*: Denotes $\rho(x_i) \in \{1, ..., K\}$ as the cluster label for point $x_i$. Then each point is assigned a new label as follows:

$$\rho(x_i) = argmin_k \|(I_{s\times s} - U_k U_k^T)x_i\| \tag{1}$$

where $k \in 1, ..., K$;

3 *Subspace update*: Update each subspace bases $U_i, i \in \{1, ..., K\}$ using the new label information. $U_i$ can be formed by retaining the eigenvectors corresponding to the top $d$ eigenvalues of the scatter matrix constructed using those sample points with label $i$. This can be easily computed via principal component analysis[10];

4 *Repeat step 2 and 3 until convergence*: The iteration procedure will stop if the label information does not change in two successive iteration steps.

# 3 Kernel K-subspaces Clustering(Kernel-KsC)

Although the method in the previous section can give the clustering result reasonable to some extent. It still has the limitation that the refining procedure using Linear-KsC is based on the assumption that the appearances of a target object can be approximated well using linear subspace. Recent studies show that often the distributions of the high dimensional image data are inherently nonlinear. Many successful algorithm for extracting those complex nonlinear structures in real life data have been proposed and one of the most representative ones is the Kernel Principal Component Analysis(KPCA)[10]. KPCA has achieved great success in the areas of pattern recognition and image processing, such as face recognition and image de-noising . We will show that combining the K-type clustering framework with the kernel based nonlinear feature extraction would yield much better result for the problem of appearances clustering of objects under varying illumination conditions.

In the next of this section, we first review the nonlinear subspace extraction using KPCA for completeness. Then we define the distance between point and nonlinear subspace in the transformed feature space. Intuitively, the distance can be defined as the "length" of the difference vector between points and nonlinear subspace in the transformed feature space. But a direct computation of the distance is infeasible due to the high, or even infinite, dimensional space. Fortunately, those difference vectors can be written in the form of linear combination of transformed high dimensional feature points, which makes it possible to compute the distance via "kernel trick" without explicitly implement the inner product in the high dimensional feature space. Next we will describe the proposed nonlinear Kernel K-subspaces Clustering(Kernel-KsC) algorithm in detail. Promising experimental results will be presented in section 4.

## 3.1 Nonlinear subspace extraction via KPCA

Recent studies in pattern recognition community show that often the target distributions, such as those of multi-view patterns of a 3D object or image sets of a single objects under varying illumination conditions, is highly nonlinear. The simple linear subspace representation is not suitable for representing highly nonlinear structures. Several non-linear dimension reduction methods have been proposed. One representative is the kernel principal component algorithm which is an unsupervised non-linear feature extractor[10]. Kernel principal component analysis allows estimation of non-linear subspace for the data distribution such as face images.

First, the input pattern $x_i \in R^s, i \in \{1, ..., m\}$ is transformed from $s$ dimensional input space $I$ onto an higher dimensional feature space $F$ via a nonlinear mapping $\phi : R^s \rightarrow R^{s_\phi}, x \rightarrow \phi(x)$. To perform the standard PCA on the mapped patterns, we need to calculate the inner product $(\phi(x_i) \bullet \phi(x_j))$ between the function values. Direct calculation of those inner products is difficult since the dimension of the feature space $F$ could be very high, possibly infinite. Kernel learning theory shows that if the nonlinear map $\phi$ is defined through a kernel function $k(x, y)$ which satisfies Mercers conditions, the inner products $(\phi(x_i) \bullet \phi(x_j))$ can be calculated from the inner products $k(x \bullet y)$. A common choice is to use the Gaussian kernel function: $k(x, y) = exp(-\frac{\|x_i - x_j\|^2}{\sigma^2})$ where $\sigma$ is the scale parameter. The $N$ orthonormal basis vectors $e_i, i = \{1, ..., N\}$, which span the nonlinear subspace, can be represented by the linear combination of all the $m$ transformed patterns in the feature space $\phi(x_j), j = \{1, ..., m\}$, i.e. $e_i = \sum_{j=1}^{m} a_{ij}\phi(x_j)$ where the coefficient $a_{ij}$ is the $j$-th component of the eigenvector $a_i$ corresponding to the $i$-th largest eigenvalue $\lambda_i$ of the $m \times m$ matrix $K$ defined by $Ka = \lambda a$ where $k_{ij} = (\phi(x_i) \bullet \phi(x_j)) = k(x_i, x_j)$. Each $a_i$ is normalized to satisfy $\lambda_i(a_i, a_i) = 1$ The projection of the mapped $\phi(x)$ onto the $i$-th orthonormal basis vector $e_i$ of the nonlinear subspace base can be computed by the following equation via the kernel trick: $(\phi(x), e_i) = \sum_{j=1}^{m} a_{ij}k(x, x_j)$

### 3.2 Kernel K-subspaces Clustering

The purposed Kernel K-subspaces Clustering(Kernel-KsC) algorithm assigns each input pattern to its nearest nonlinear subspace base. Denote $D(x, S)$ as the difference between an input pattern in the transformed space $\phi(x)$ and its reconstruction using a nonlinear subspace $S$ with the corresponding orthonormal basis vector defined in Section 3.1. Here we only keep the first $d$ basis vectors the nonlinear subspace $S$ with higher eigenvalues. Then

$$D(x, S) = \phi(x) - \Theta_S(\Xi_S(\phi(x))) \tag{2}$$

where $\Xi_S(\phi(x)) \in R^d$ is the projection of $\phi(x)$ onto the nonlinear subspace $S$ and $\Theta_S(\Xi_S(\phi(x)))$ is its reconstruction. Denote the $d$ dimensional nonlinear subspace $S$ as

$$\begin{aligned} S &= [e_1, ..., e_d] \\ &= [\sum_{j=1}^{m} a_{1j}\phi(x_j), ..., \sum_{j=1}^{m} a_{dj}\phi(x_j)] \end{aligned} \tag{3}$$

then

$$\begin{aligned} &\Theta_S(\Xi_S(\phi(x))) \\ &= [\sum_{t=1}^{m} a_{1t}\phi(x_t), ..., \sum_{t=1}^{m} a_{dt}\phi(x_t)] \times \\ &[\sum_{s=1}^{m} a_{1s}\phi(x_s), ..., \sum_{s=1}^{m} a_{ds}\phi(x_s)]^T\phi(x) \\ &= [\sum_{t=1}^{m} a_{1t}\phi(x_t), ..., \sum_{t=1}^{m} a_{dt}\phi(x_t)] \times \\ &[\sum_{s=1}^{m} a_{1s}\phi(x_s) \bullet \phi(x), ..., \sum_{s=1}^{m} a_{ds}\phi(x_s) \bullet \phi(x)]^T \\ &= [\sum_{t=1}^{m} a_{1t}\phi(x_t), ..., \sum_{t=1}^{m} a_{dt}\phi(x_t)] \times \\ &[\sum_{s=1}^{m} a_{1s}k(x_s, x), ..., \sum_{s=1}^{m} a_{ds}k(x_s, x)]^T \end{aligned} \tag{4}$$

Here the inner product in the transformed space , which is difficult to compute due to the inherently high or infinite dimension of the transformed space, is implemented via the kernel trick. After some algebraic derivations, we obtain:

$$\begin{aligned} &\Theta_S(\Xi_S(\phi(x))) \\ &= \sum_{t=1}^{m}\{\sum_{r=1}^{d} a_{rt} \sum_{s=1}^{m} a_{rs}k(x_s, x)\}\phi(x_t) \\ &= \sum_{t=1}^{m} B_t\phi(x_t) \end{aligned} \tag{5}$$

where

$$B_t = \sum_{r=1}^{d} a_{rt} \sum_{s=1}^{m} a_{rs}k(x_s, x), t = 1, ..., m \tag{6}$$

So from the definition of equation2

$$D(x, S) = \phi(x) - \sum_{t=1}^{m} B_t\phi(x_t) \tag{7}$$

For the sake of clearness, we represent $x$ as $x_0$ and let $B_0$ equals to the value of $-1$, then

$$D(x, S) = -\sum_{t=0}^{m} B_t\phi(x_t) \tag{8}$$

That is to say, the difference vector $D(x, S)$ can be written in the form of linear combination of nonlinear transformed input patterns. The square of the length of the difference vector $D(x, S)$ can be computed using the inner product as follows:

$$
\begin{aligned}
\|D(x,S)\|^2 &= \sum_{i=0}^{m}\sum_{j=0}^{m} B_i B_j \phi(x_i) \bullet \phi(x_j) \\
&= \sum_{i=0}^{m}\sum_{j=0}^{m} B_i B_j k(x_i, x_j)
\end{aligned}
\tag{9}
$$

Based on the definition of the distance between input patterns in the transformed space and nonlinear subspace bases, we can define the objective function to be minimized as follows:

$$
\sum_{i=1}^{K} \sum_{\rho(x_j)\in i} \|D(x_j, S_i)\|^2 \tag{10}
$$

The iteration procedure of the proposed Kernel K-subspaces Clustering (Kernel-KsC) method can be described as follows:

**Algorithm 2: Kernel K-subspaces Clustering(Kernel-KsC)**

1. *Initialization*: Starting with an initial labeling of the input image pattern $\{x_1, ..., x_n\}$ into $K$ clusters. Compute the nonlinear subspaces $S_i, i = \{1, ..., K\}$ of the corresponding data with label $i$ via kernel principal component analysis defined in Section 3.1;

2 *Points assignment*: Denotes $\rho(x_i)$ as the cluster label for point $x_i$. Then each point is assigned a new label as follows:

$$
\rho(x_i) = argmin_k \|D(x_i, S_k)\|^2 \tag{11}
$$

where $k \in \{1, ..., K\}$. The $\|D(x_i, S_k)\|^2$ can be computed using equation 9;

3 *Non-linear subspaces update*: Update each nonlinear subspace bases $S_i, i \in \{1, ..., K\}$ using the new label information via kernel principal component analysis;

4 *Repeat step 2 and 3 until convergence*. The iteration procedure will stop if the change of the value of the objection function is small enough, or equivalently if the label information does not change anymore in two successive iteration steps.

The iterative procedure of the proposed Kernel-KsC algorithm implement appearances clustering by assigning each input pattern(the vector form of the original 2D image matrix) according to the underlying nonlinear subspace distribution structure, which is a more accurate description of the real life data than its simple linear counterpart. Thus a higher correct clustering rate can be achieved, which will be demonstrated in the next section. The correct clustering rate can be defined as:

$$
\sum_{i=1}^{K} \tau_i / n \tag{12}
$$

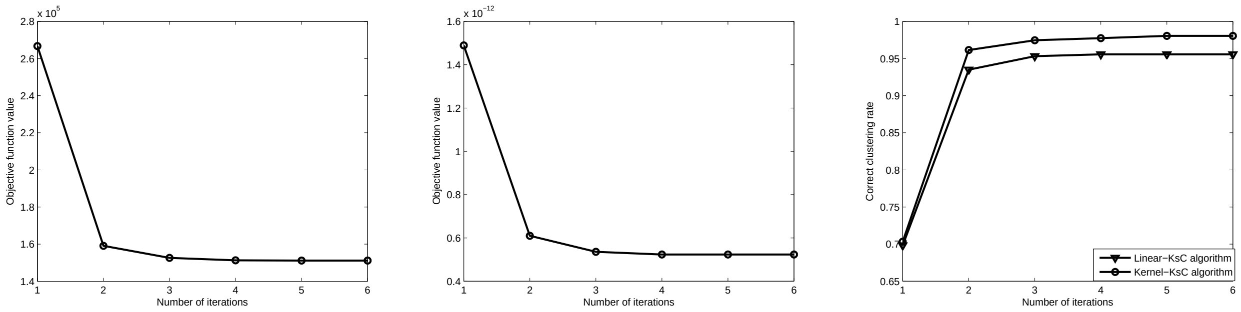
where $n$ denotes the total number of images and $\tau_i$ denotes the maximum number of images with the same true identity clustered into the class $i$.

## 4 Experimental results

We used the CMU PIE database[8] and the Yale Face Database[9] as the test sets for performing unsupervised appearance clustering under varying illumination conditions. We compared the clustering performance of the proposed nonlinear kernel K-subspaces method with that of its linear counterpart. The iteration procedures of both algorithms were initialized using the gradient affinity based spectral clustering method proposed in [7]. For both of the data sets, the proposed nonlinear K-subspaces clustering method achieves satisfactory clustering results and outperforms its linear counterpart. The detail of the experiments will be given below. We use the Gaussian kernel function[10] in all the following experiments.

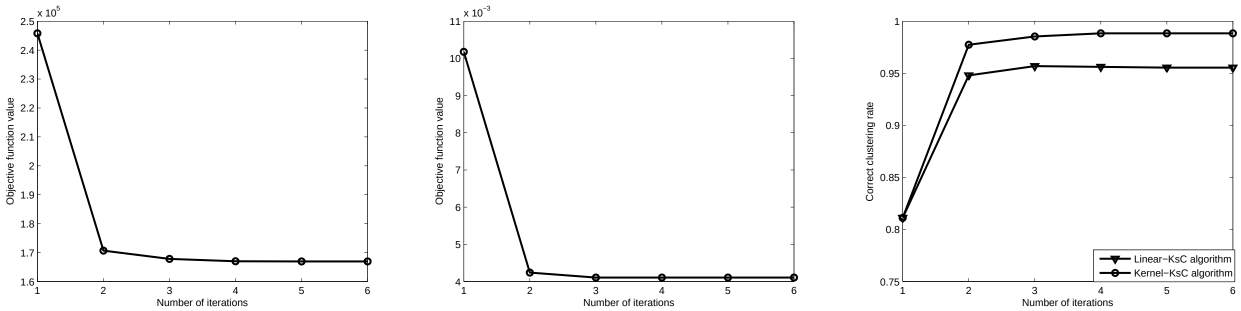
For the CMU PIE database, we used a subset of 40 frontal or near-frontal images of 68 individuals which were taken under different illumination conditions. Figure 1(a) shows the sample image sets for two specific subjects. First, the resolution of the original images are resized from original $32 \times 32$ pixels to $16 \times 16$ pixels and the range of image intensities are normalized to $\{0, 1\}$. Then the gradient fields are computed and the spectral clustering was implemented using the similarity measurement matrix . After initialization, the proposed nonlinear Kernel K-subspaces Clustering (Kernel-KsC) and Linear K-subspaces Clustering (Linear-KsC) algorithms are implemented respectively. Although there are several studies show that the number of cluster centers can be selected automatically by analyzing the distributions of the corresponding eigenvalues such as in [12], in this paper we assume the number of clusters, i.e. $K$, is known in advance since the emphasis of this work is to show the superiority of the proposed Kernel-KsC algorithm, which is a nonlinear extension to the original Linear-KsC algorithm by exploiting the inherent nonlinear distribution property in the input patterns, over its linear counterpart. Figure 2(a) and (b) show the objective function value during iteration procedure for the proposed nonlinear K-subspaces clustering algorithm and the original K-subspaces clustering algorithm, respectively. Figure 2(c) shows the correct clustering rate comparison of the two methods during iteration. Although the appearance variation of the same person is fairly large due to the varying illumination conditions,the proposed nonlinear K-subspaces clustering method achieved satisfactory clustering results and outperforms its linear counterpart greatly.

The Yale B database used in our experiment consists of 450 images with 45 frontal images of each person captured under varying light directions. Figure 1(b) shows sample




(a) (b) (c)

**Figure 2. Experimental results for PIE database:(a) and (b)are the objective function values as a function of the number of iterations for Linear-KsC and Kernel-KsC,respectively. (c) shows correct clustering rate comparison of the two methods during iteration.**




(a) (b) (c)

**Figure 3. Experimental results for YaleB database:(a) and (b)are the objective function values as a function of the number of iterations for Linear-KsC and Kernel-KsC,respectively. (c) shows correct clustering rate comparison of the two methods during iteration.**

images of two persons from these subsets. Each image is resize to resolution of $16 \times 14$ pixels and initialized using the same method as for the PIE database. Figure 3 show the experiment results.

Both of the above experiments clearly show that nonlinear K-subspaces and linear K-subspaces method converge quickly in just several iteration steps. And the proposed nonlinear kernel-based K-subspaces clustering algorithm achieves much lower error rate than the linear K-subspaces clustering algorithm.

## 5 Conclusions and future work

This paper studies the problem of appearance clustering under varying illumination conditions and a novel nonlinear kernel K-subspaces clustering algorithm is presented. The proposed Kernel-KsC algorithm further extends the original K-subspaces clustering algorithm to the nonlinear case since inherently the distribution of the real life image data has a complex nonlinear structure rather than simple linear case. Firstly, the input space is mapped into a high-dimensional feature space using nonlinear mapping function. Then the nonlinear subspaces are extracted in the feature space and distances between the mapped feature points and those nonlinear subspaces are computed via inner products by kernel trick. Experiments on several real life data sets show that the proposed nonlinear kernel K-subspaces clustering algorithms converges quickly and achieves higher clustering rate that that of the original linear K-subspaces clustering algorithm.

Besides successful applications in computer vision community, recently subspace clustering algorithm also achieves successes in other areas such as data mining and bio-informatics, where the data may also have inherent nonlinear properties. We believe that the proposed kernel-based nonlinear subspace clustering algorithm can outperform its linear counterpart for those problems. The application of the proposed nonlinear K-subspaces clustering in other fields might be future research directions.

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