-  $|let x = t in u|_e = Pushenv$ ,  $|t|_e$ , Extend,  $|u|_{e, x}$ , Popenv

For example, the compilation of

generates the sequence of instructions Pushenv, Mkclos [Search 0, Test([Ldi 1], [Pushenv, Ldi 1, Push, Search 0, Sub, Push, Search 1, Apply, Popenv, Push, Search 0, Mult])], Extend, Pushenv, Ldi 6, Push, Search 0, Apply, Popenv, Popenv and the result of its execution is the number 720.

The correctness of the compilation, and of the semantics of the abstract machine, can be stated as follows: if V is a numeric value, then  $\vdash t \hookrightarrow V$  if and only if  $|t| \Rightarrow V$ .

Exercise 4.2 Write an abstract machine and a compiler for PCF.



The state of the abstract machine at the beginning of the 14th execution step for the program Pushenv, Ldi 1, Extend, Ldi 6, Push, Ldi 5, Push, Ldi 4, Push, Ldi 3, Push, Ldi 2, Push, Search 0, Add, Add, Add, Add, Add, Add, Popenv.

**Exercise 4.3** We extend PCF with the tree operators described in Exercise 3.15. Write a compiler and an abstract machine for this extension of PCF.

**Exercise 4.4** (A bootstrapping compiler) Many kinds of data structures can be represented using the trees described in Exercise 3.15. To start with, we can represent a natural number n as a tree L n. The character c can be represented by the tree L n where n is a code, for instance the ASCII code of the character c. If  $t_1, t_2, \ldots, t_n$  are trees, the list  $t_1, t_2, \ldots, t_n$  can be represented by the tree  $N(t_1, N(t_2, \ldots, N(t_n, L 0) \ldots))$ . Finally, values of a type defined by constructors that are themselves representable could be defined by enumerating the constructors and representing the value  $C(V_1, V_2, \ldots, V_n)$  by the list L p,  $t_1, t_2, \ldots, t_n$  represent the values  $V_1, V_2, \ldots, V_n$ .