4.3.5 Small-Step Operational Semantics

The machine state, the contents of its registers, is a tuple consisting of a value (the accumulator), a list where each element is either a value or a list of values (the stack), a list of values (the environment), and a sequence of instructions (the code).

A small execution step consists of getting an instruction from the code register and executing it. The small-step semantics of the machine can be easily defined:

- (a,s,e,((**Mkclos** i),c)) \longrightarrow ((i,e),s,e,c) $-(a,s,e,(\mathbf{Push},c)) \rightarrow (a,(a,s),e,c)$ $-(a,s,e,(Extend,c)) \longrightarrow (a,s,(e,a),c)$ $-(a,s,e,((Search n),c)) \rightarrow (V,s,e,c)$ if V is the nth value in e (starting from the end) $-(a, s, e, (Pushenv, c)) \rightarrow (a, (e, s), e, c)$ $-(a, (e', s), e, (Popenv, c)) \longrightarrow (a, s, e', c)$ $-(\langle i,e' \rangle, (W,s),e, (Apply,c)) \longrightarrow (\langle i,e' \rangle, s, (e', \langle i,e' \rangle, W),$ i c) $-(a,s,e,((Ldi n),c)) \rightarrow (n,s,e,c)$ $-(n,(m,s),e,(Add,c)) \longrightarrow (n+m,s,e,c)$ $-(n,(m,s),e,(\mathbf{Sub},c)) \longrightarrow (n-m,s,e,c)$ $-(n,(m,s),e,(Mult,c)) \longrightarrow (n * m,s,e,c)$ - $(n, (m, s), e, (Div, c)) \rightarrow (n / m, s, e, c)$ $-(0,s,e,((\mathbf{Test}(i,j)),c)) \longrightarrow (0,s,e,i c)$ $-(n, s, e, ((\text{Test}(i, j)), c)) \rightarrow (n, s, e, j, c)$ if n is a number different from 0

An irreducible term is a tuple where the fourth component—the contents of the code register—is empty. If i is a sequence of instructions and if the term (0, [], [], i) reduces to an irreducible term of the form $(V, _, _, [])$, then we say that V is the result of the execution of i, and we write $i \Rightarrow V$.

4.4 Compilation of PCF

We can now give the compilation rules for PCF

- $|\mathbf{x}|_e$ = Search n where n is the position of x in the environment e - $|\mathbf{t} u|_e$ = Pushenv, $|u|_e$, Push, $|\mathbf{t}|_e$, Apply, Popenv - $|\text{fun } \mathbf{x} \rightarrow \mathbf{t}|_e$ = Mkclos $|\mathbf{t}|_{e,_,x}$ - $|\text{fixfun } \mathbf{f} \mathbf{x} \rightarrow \mathbf{t}|_e$ = Mkclos $|\mathbf{t}|_{e, f, x}$ - $|n|_e$ = Ldi n - $|\mathbf{t} + u|_e$ = $|u|_e$, Push, $|\mathbf{t}|_e$, Add - $|\mathbf{t} - u|_e$ = $|u|_e$, Push, $|\mathbf{t}|_e$, Sub - $|\mathbf{t} * u|_e$ = $|u|_e$, Push, $|\mathbf{t}|_e$, Mult - $|\mathbf{t} / u|_e$ = $|u|_e$, Push, $|\mathbf{t}|_e$, Div - $|\text{ifz } \mathbf{t}$ then u else $v|_e$ = $|\mathbf{t}|_e$, Test($|u|_e, |v|_e$)