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ABSTRACT - A token ring is the most suitable candidate for a high-speed optical fiber LAN. In this paper the performence model based on queueing theory for a higth-speed token ring LAN with a heterogeneous environment is set up. The related calculating formulas for system parameters such as the mean turn-around delay time and the mean packet length at a station are derived.

Key word: Analysis, higth speed, token ring, optical fiber LAN, heterogeneous environment.

1 Introduction

Optical fibers can be used to provide a high-speed (more than 100 Mbit / s) channel with good noise immunity and higth bandwidth for use in local area neworks. A selection of topologies that might be appropriate [(1),(2)] has been considered and it has been found that star and ring networks are the most suitable candidates for an optical fiber LAN. Especially, the results of the analysis[(3)] show that the token ring offers the best ferformance over a wide range of packet size and line rate. At the same time the token ring network is regarded as superior in that a major problem of all optical fiber networks, namely optical fiber coupler has been removed because only point-point links are required. therefore it is the most suitable choice for a high-speed optical fiber LAN.

Many papers [(4),(5)] have been presented for analysis of ring LAN so far, but they have been limited to lower bit-rates with a homogeneous environment due to simplifying models. The motivation for this paper is to present analysis for the high-speed optical fiber LAN with a heterogeneous environment by means of queueing theory, and to derive related calculating formulas.

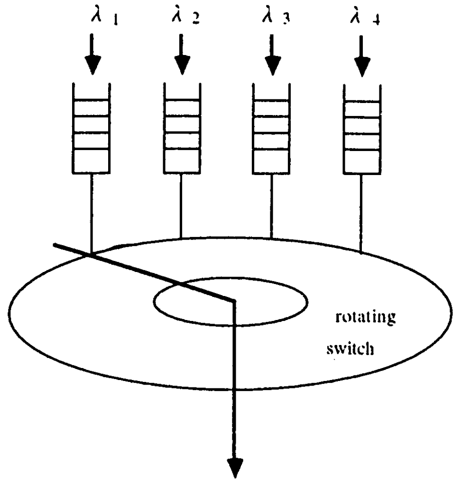

Fig.1 Performance model of single-server system for token ring

2 Performance modeling of a token ring

A single token system is considered. In a token ring the token circulates in one direction around the ring. Access to the transmission channel is controlled by passing a permission token. When the system is initialized a designated station generates a free token which travels around the ring. Then a station ready to transmit changes it to busy and put its packets onto the ring.The packet size can, in principle, be of arbitray length.The sending station is resposible for removing its own packets from the ring. At the end of its transmission, it passes the access permission to the next station by generating a new free token.This operation can be discribed by the performance model as shown in Fig.1 which is a single-server queueing model with as many queues as stations attached to the ring. The queues are serviced in a cyclic manner symbolized by the rotating switch which sdands for the free token. With respect to the order of service, here, a limited number of packets denoted by l is serviced per access possibility, which is called " nonexhaustive service ". In this order, when the number of queueing packets at a station is less than l it will not be serviced.

In this analysis, several assumptions for the performance model of a token ring are described as follows

1) Poisson arrival distribution
2) General service time distribution
3) First-come-first -sevice (FCFS) queueing discipline
4) Message independance

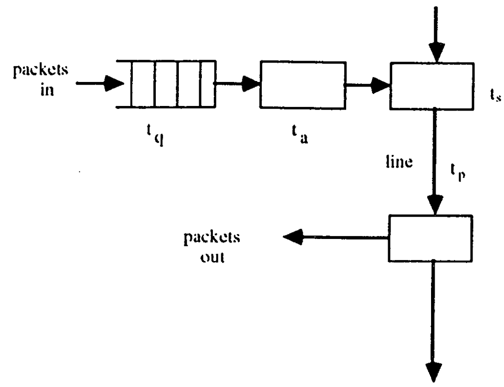

Fig. 2 The overall time delay for a packet

As shown in Fig.2, the overall time delay t_o, for a packet consists of four components: the queueing delay t_q, the access delay t_a, the service time t_s, and the propagation delay t_p, thus,

$t_o = t_q + t_a + t_s + t_p$.

The parameters used in the analysis are as follows

N: Number of stations

λ_i: Average arrival rate

$Q_t(t)$: Token circulation period distribution, that is the time for token to travels completely around the ring

H_i (t): Time distribution including service time and propagation delay

Q_{iz} (t): Token returning time distribution, that is, the time for token to spend utill the next arrival after leaving station i

$W_{iq}(t)$: Packet waiting time distribution including the queueing delay and the access delay

r_n: State probability of packet number denoted by n, queueing at the station i when the token arrives at the station i

r_{id}: Probability that the packet number is b<l, $r_{id} = \sum_{b=0}^{l-1} r_{ib}$ when the token arrives at the station i

r_{is}: Probability that the packet number is c≧l, $r_{is} = \sum_{c=l}^{\infty} r_{ic}$, when the token arrives at the station i

r_{in}: Probability that the packet number is n when the token arrives at the station i

$R_i(z)$: Generating function of r_n , $R_i(z) = \sum_{n=0}^{\infty} r_n z^n$

Note that the lower notation "i" in above notations represents the station i.

In addition, the following relationships exist.

$Q_i(t) = H_i(t) * Q_{iz}(t)$

$Q_i(s) = H_i(s) * Q_{iz}(s)$

$r_{is} = 1 - r_{id}$

where the notation " * " represents convolution operation and $Q_i(s)$, $H_i(s)$ and $Q_{iz}(s)$ represent the Laplace-Stieltjes transform of $Q_i(t)$, $H_i(t)$ and $Q_{iz}(t)$, respectively.

3 Mathematical analysis

Assume the the following analysis is done under a steady state of the system.

When the token arrives at the station i two cases are considered.

A) The packet number at station i (denoted by b) is less than l (i.e. b<l) and therefore the token directly passes by, then when it arrives at the station i again the packets have increased from b to n. In this case, the state probability of the packet number denoted by r_n' is

$$r_n' = \sum_{b=0}^{l-1} r_{ib} \int_0^{\infty} [(\lambda_i t)^{n-b} e^{-\lambda it} / \{(n-b)!\}] \, dQ_{iz}(t) \tag{1}$$

B) The packet number at the station i (denoted by c) is equal to or larger than l (i.e. c = or > l), and the token leaves station i after l packets are sent. When it arrives at the same station again the packets have increased from (c - l) to n. In this case ,the state probability of packet number denoted by r_n'' is

$$r_n'' = \sum_{c=l}^{n+l} r_{ic} \int_0^{\infty} (\lambda_i t)^{n-c+l} e^{-\lambda it} / \{(n-c+l)!\} \quad d\,Q_i(t) \tag{2}$$

Regarding $r_n = r_n' + r_n''$, Eq.(1) and (2), we obtain

$$r_n = \sum_{b=0}^{l-1} r_{ib} \int_0^{\infty} [(\lambda_i t)^{n-b} e^{-\lambda it} / \{(n-b)!\}] dQ_{iz}(t)$$

$$+ \sum_{c=0}^{n+1} r_{ic} \int_0^{\infty} [(\lambda_i t)^{n-c+1} e^{-\lambda it}/\{(n-c+1)!\}] d\, Q_t(t) \tag{3}$$

Furtherly, regarding $R_i(z) = \sum_{n=0}^{\infty} r_n z^n$ and

$$\sum_{n=b}^{\infty} (\lambda_i t)^{n-b} z^{n-b}/\{(n-b)!\} = e^{\lambda itz}$$

and

$$\sum_{n=c-1}^{\infty} (\lambda_i t)^{n-c+1} z^{n-c+1}/\{(n-c+1)!\} = e^{\lambda itz}$$

We have

$$R_i(z) = \sum_{b=0}^{l-1} r_{ib} z^b \int_0^{\infty} e^{-(\lambda i - \lambda i)t} d\, Q_{iz}(t) + (\sum_{c=1}^{\infty} r_{ic} z^{c-1}) \int_0^{\infty} e^{-(\lambda i - \lambda iz)t} dQ_t(t) \tag{4}$$

Let

$$Q_{iz}(s) = \int_0^{\infty} e^{-(\lambda i - \lambda iz)t} dQ_{iz}(t) \tag{5}$$

$$Q_t(s) = \int_0^{\infty} e^{-(\lambda i - \lambda iz)t} dQ_t(t) \tag{6}$$

where $s = \lambda_i - \lambda_i z$, $Q_{iz}(s)$ and $Q_t(s)$ are just the Laplace - Stieltjes transforms of $Q_{iz}(t)$ and $Q_t(t)$, respectively. Thus, Eq.(4) becomes

$$R_i(z) = \sum_{b=0}^{l-1} r_{ib} z^b Q_{iz}(s) + z^{-1}\{R_i(z) - \sum_{b=0}^{l-1} r_{ib} z^b\} Q_t(s) \tag{7}$$

Regarding $Q_t(s) = H_i(s)\, Q_{iz}(s)$ and $r_{id} = \sum_{b=0}^{l-1} r_{ib}$ solve Eq.(7) for $R_i(z)$

$$R_i(z) = (r_{id} z^b)\{Q_{iz}(s) - z^{-1}Q_t(s)\} / \{1 - z^{-1}Q_t(s)\} \quad (8)$$

and

$$R_i(z) = (r_{id} z^b)[Q_{iz}(s)\{z^{-1}H_i(s) - 1\}] / \{1 - z^{-1}Q_{iz}(s)H_i(s)\} \quad (9)$$

By applying $R_i(z)_{z=1} = (\sum_{n=0}^{\infty} r_n)_{z=1} = 1$ Eq.(8) becomes

$$[r_{id}\{Q_{iz}(s) - z^{-1}Q_t(s)\} / \{1 - z^{-1}Q_t(s)\}]_{z=1} = 1 \quad (10)$$

Applying Robbida-Theoream we obtain

$$[r_{id}\{Q_{iz}'(s) + 1 - Q_t'(s)\} / \{1 - Q_t'(s)\}]_{z=1} = 1$$

where the notation " ' " represents differential operation. Since $Q_{iz}'(s)_{z=1} = \lambda_i \int_0^{\infty} t \, dQ_{iz}(t)$, $Q_t'(s)_{z=1} = \lambda_i \int_0^{\infty} t \, dQ_t(t)$ and $\int_0^{\infty} t \, dQ_{iz}(t)$ denoted by q_{iz} represents the average of $Q_{iz}(t)$ and $\int_0^{\infty} t dQ_t(t)$ denoted by q_t represents the average of $Q_t(t)$. r_{id} results in

$$r_{id} = (1 - \lambda_i q_t) / (\lambda_i q_{iz} + 1 - \lambda_i q_t) \quad (11)$$

Let h_i denote the average of $H_i(t)$, and regarding $q_t = h_i + q_{iz}$, r_{id} becomes

$$r_{id} = (1 - \lambda_i h_i - \lambda_i q_{iz}) / (1 - \lambda_i q_{iz}) = 1 - \lambda_i q_{iz} / (1 - \lambda_i h_i) \quad (12)$$

and

$$r_{is} = 1 - r_{id} = \lambda_i q_{iz} / (1 - \lambda_i h_i) \quad (13)$$

where h_i and q_{iz} can be obtained by means of Eq.I - (5) and Eq.I - (7) of appendix I. Thus,

$$r_{id} = 1 - \lambda_i q_{iz} / \{1(1 - \lambda_i h_{ir})\} \quad (14)$$

and

$$r_{is} = \lambda_i q_{iz} / \{ l(1 - \lambda_i h_{ir})\} \tag{15}$$

and

$$q_{iz} = r_{is} / \{ \lambda_i l (1 - \beta_i - \beta_{ip}) \tag{16}$$

where β_i and β_{ip} denote utilization factors for station i to send packets and for packets of station i to propagate on the ring respectively.

Comparing Eq.(16) and Eq.I - (7) of appendix I we have

$$r_{is}\, l (1 - \beta_i - \beta_{ip}) = \lambda_i \left(\sum_{j=1}^{N} q_{jb} + 2Nq_k + \sum_{j=1}^{N} r_{js} h_i \right) \tag{17}$$

r_{is} (i = 1,2,...N) in Eq.(17) can be solved by a matrix.

$$r_{is} = \lambda_i \left(\sum_{j=1}^{N} q_{jb} + 2Nq_k \right) / [l \{ 1 - \sum_{j=1}^{N} (\beta_j + \beta_{jp})\}] \tag{18}$$

On the other hand, under the same condition, i.e., when the token arrives at the station i there are c (c > or = l)packets, the probability denoted by p_{in} that the packet number changes from c to n after l packets are sent is

$$p_{in} = \sum_{c=1}^{n+1} (r_{ic} / r_{is}) \int_0^{\infty} [\{ (\lambda_i t)^{n-c+1} e^{-\lambda i t} \} / \{(n - c + 1)!\}]dH_i(t) \tag{19}$$

Let $P_i(z)$ denote the generating function of p_{in}, i.e.,$P_i(z) = \sum_{n=0}^{\infty} p_{in} z^n$

Thus we have

$$P_i(z) = (1/r_{is}) \sum_{c=1}^{\infty} r_{ic} z^{c-1} \int_0^{\infty} [e^{-\lambda i t} \sum_{n=c-1}^{\infty} [\{(\lambda_i t)^{n-c+1} (z^{n-c+1})\}/\{(n - c + 1)!\}]]dH_i(t)$$

$$= [1 / (r_{is} z^l)] \{ (\sum_{c=1}^{\infty} r_{ic} z^c) + \sum_{b=0}^{l-1} r_{ib} z^b - \sum_{b=0}^{l-1} r_{ib} z^b \} \int_0^{\infty} e^{-\lambda i t} e^{-\lambda i t z}\, dH_i(t)$$

$$= [1 / (r_{is} z^l)]\{R_i(z) - \sum_{b=0}^{l-1} r_{ib} z^b \} H_i(s) \quad (20)$$

where $s = \lambda_i - \lambda_i z$. Substituting $R_i(z)$ into Eq.(20) we have

$$P_i(z) = \{ \sum_{b=0}^{l-1} (r_{ib} z^b / r_{is})\}[H_i(s)\{Q_{iz}(s) - 1\}] /\{z^l - H_i(s) Q_{iz}(s)\} \quad (21)$$

Under a steady state the probability that the station i generats n packets during the overall time delay for a packet including the queueing delay, the access delay, the service time and the propagation delay is

$$p_{in} = \int_0^\infty [\{(\lambda_i t)^n e^{-\lambda i t} z^n / (n!)\}] d\{W_i q(t) * H_i(t)\} \quad (22)$$

and the generating function of p_{in} denoted by $P_i(z)$ is

$$P_i(z) = \int_0^\infty [e^{-\lambda i t} \sum_{n=0}^{\infty} [\{(\lambda_i t)^n z^n\} / (n!)]]d\{W_i q(t) * H_i(t)\}$$

$$= \int_0^\infty e^{-(\lambda i - \lambda i z) t} d\{W_{iq}(t) * H_i(t)\}$$

$$= W_{iq}(s) H_i(s) \quad (23)$$

Substituting $z = (\lambda_i - s)/\lambda_i$ into Eq.(23) we have

$$W_{iq}(s) = P_i\{(\lambda_i - s)/ \lambda_i\} /H_i(s)$$

$$= (1/r_{is}) \sum_{b=0}^{l-1} r_{ib}\{(\lambda_i - s) / \lambda_i\}^b [\lambda_i^l\{Q_{iz}(s) - 1\} / \{(\lambda_i - s)^l - \lambda_i^l H_i(s) Q_{iz}(s)\}] \quad (24)$$

Let w_{iq} denote the average of $W_{iq}(t)$ then

$$w_{iq} = \{ - dW_{iq}(s) / ds\}_{s=0}$$

$$= q_{iz}^{(2)} /(2q_{iz}) + \{ \lambda_i(q_{iz}^{(2)} + 2q_{iz} h_i + h_i^{(2)}\} / [2\{1 - \lambda_i(q_{iz} + h_i)\}]$$

$$- \{l(l-1)\} / [2\lambda_i\{1-\lambda_i(q_{iz}+h_i)\}] + \sum_{b=0}^{l-1} br_{ib}(1-\lambda_i h_i) / [\lambda_i \{1-\lambda_i(q_{iz}+h_i)\}] \quad (25)$$

where the $h_i^{(2)}$ is the 2nd moment of $H_i(t)$ and the $q_{iz}^{(2)}$ is the 2nd moment of the $Q_{iz}(t)$.
The average of overall time delay for a packet at the station i denoted by w_i is

$$w_i = w_{iq} + h_i \quad (26)$$

and the average of packet length at the station i denoted by L_i is

$$L_i = w_i \lambda_i \quad (27)$$

Conclusion

The token ring is the most suitable candidate for a high-speed optical fiber LAN.This paper presented the analysis for the token ring with the "nonexhaustive service" dicipline and heterogeneous environment by means of the single-server queueing model set up in this paper and derived the related calculating formulas of the system parameters.This analysis and related formulas will provide an important means for system peformance evaluation of the high-speed token ring LAN with a heterogeneous environment.

The system simulation about the high-speed token ring LAN with a heterogeneous environment will be done as the next research works.

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References

(1) P.Davis and F.Chani: A access protocol for an optical - fiber ring network,Computer Communication ,Vol.6,No.4,1983 pp.185-191.

(2) P.A.Davis and F.C.Abdul: Optical - fiber computer communication networks , Proc.Melecow,83.Athens,Greece(24 - 26, May 1983) pp A1.10-11.

(3) D.D.Clsrk,K.T.Progran, and D.P.Reed: "An introduction to local area networks", Proc.IEEE, Vol.66, pp.1497 - 1517,1978.

(4) P.Zatiroplo: Performance evaluation of reliability improvement techniquesfor single loop

communication systems,IEEE Trans.Commun, Vol.22,No.6(June 1974)pp742 - 751.

(5) W.Bux and H.L.Truong: Mean-Delay Approximation for Cycic-Service Queueing systems,North Holland Performance Evaluation 3 [1983] 187 - 196.

(6) L. Kleinrock : Queueing systems, Vol.1:Theory,Univ. of Colifornia,1975.

(7) L. Kleinrock: Queueing systems, Vol.2:Computer Applications, Univ. of Colifornia,1975.

Appendix I

Assumptions

1) Time distribution of packet length at station i is known and is denoted by $Q_{iw}(t)$

2) Propagation delay distribution for a packet between ith and (i + 1)th station is known and is denoted by $Q_{ia}(t)$

3) Probability transmiting messages from station i to any other station j (j is not equal to i) on the ring is known and is denoted by r_{ij}

Difinitions of parameters

$Q_{ip}(t)$: Loop propagation delay distribution for a packet

$Q_{k}(t)$: Staying time distribution for the token at each station

L_{k}: Token length

With respect to the difinition of $H_{ir}(t)$, the following relationship exists

$$H_{ir}(t) = Q_{iw}(t) * Q_{ip}(t) \qquad \text{I - (1)}$$

By taking Laplace-Stieltjes transform for Eq. I - (1) we can obtain

$$H_{ir}(s) = Q_{iw}(s)Q_{ip}(s)$$

$$= Q_{iw}(s) \{ \sum_{j=i+1}^{N} r_{ij} \sum_{k=i}^{j-1} Q_{ka}(s) + \sum_{j=1}^{i-1} r_{ij} \prod_{k=i}^{N} Q_{ka}(s) \prod_{k=1}^{j-1} Q_{ka}(s) \} \qquad \text{I-(2)}$$

Let h_{ir} denote the average of $h_{ir}(t)$, q_{iw} denote the average of $Q_{iw}(t)$, and q_{ia} denote the average of $Q_{ia}(t)$ then

$$h_{ir} = -\{ dH_{ir}(s) / ds \}_{s=0}$$

$$= q_{iw} + \sum_{j=i+1}^{N} r_{ij} \sum_{k=i}^{j=1} q_{ka} + \sum_{i=1}^{j-1} r_{ij} \{ \sum_{k=i}^{N} q_{ka} + \sum_{k=1}^{j-1} q_{ka} \} \qquad \text{I - (3)}$$

The time distribution transmiting l packets is

$$H_i(t) = H_{ir}^{*l}(t) \qquad \text{I - (4)}$$

where the notation "*l" denotes the convolution operation of l times. The average of $H_i(t)$ denoted by h_i is

$$h_i = lh_{ir} \qquad \text{I - (5)}$$

According to the difinition of $Q_{iz}(t)$, the Laplace - Stieltjes transform of $Q_{iz}(t)$ denoted by $Q_{iz}(s)$ is

$$Q_{iz}(s) = Q_{1a}(s)Q_{2a}(s)\ldots Q_{Na}(s)Q_k^N(s)\{r_{is}\, h_{ir}(s) + (1 - r_{is})\}^l\{r_{2s}H_{2s}(s) + (1 - r_{2s})\}^l\ldots$$

$$\{r_{i-1,s}H_{i-1,r}(s) + (1 + r_{i-1,s})\}^l\{r_{i+1,s}\, H_{i+1,r}(s) + (1 - r_{i+1,s})\}^l\ldots\{r_{Ns}H_{Nr}(s)+(1-r_{Ns})\}^l$$

$$= \{\prod_{j=1}^{N} Q_{ja}(s)\}Q_k^N(s)\prod_{\substack{j=1\\(j\neq i)}}^{N}\{r_{ij}H_{jr}(s) + (1-r_{js})\}^l \qquad \text{I - (6)}$$

The average of $Q_{iz}(t)$ denoted by q_{iz} is

$$q_{iz} = -\{dQ_{iz}(s) / ds\}_{s=0} = \sum_{j=1}^{N} q_{ia} + 2Nq_k + \sum_{\substack{j=1\\(j\neq i)}}^{N} r_{is}\, h_i \qquad \text{I - (7)}$$

where q_{ia}, q_k and h_i denote the average of $Q_{ia}(t)$, $Q_k(t)$ and $H_i(t)$ respectively.

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SUPPLEMENTARY NOTES