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Successive Pursuit with a Restricted Detection Domain

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Abstract

We study a coplanar model of the successive pursuit of two evaders with unlimited turn-rates of all players and a restricted detection domain of pursuer. We describe two guaranteed pursuit strategies that include a two-stage strategy to shorten to a specified quantity the distance to the nearer evader, and a two-stage strategy to search and capture the other. The strategies are distinguished by their search plans. First coalition is pursued as a whole. Then pursuer approaches the first evader using the strategy of successive pursuit with unmoved second evader at the last observed position. Subsequently pursuer moves directly to that position of the second evader or, according to the more complex plan, alternates between traversing a straight line and turns of logarithmic spirals, and after detection captures the remaining evader using the simple pursuit strategy. We also provide some numerical results for a set of parameters of the game.

Introduction

The problem of pursuit of two evaders in succession is very complex indeed. So far merely the models in the frame of differential games with perfect information have been investigated (see, e.g., Refs.1-4). We describe here a game of the coplanar successive pursuit with imperfect visibility of pursuer. We assume that

- turn-rates of E_1,E_2,P are unlimited;
- E_1 ,E_2's identical maximal speed is less then P's;
- P knows a certain evader's position only if the evader is somewhere inside the disk of a finite radius centered at P;
- E_1 and E_2 continuously know P's position;
- P's goal is to shorten to a specified quantity the distance to E_1 and then to capture E_2, and coalition $\{E_1, E_2\}$ strives to escape.

Even starting with both evaders inside the detection domain and having the advantage of being faster, P can not retain them there for a long time. At boundary states where P still succeeds the following information stages might arise:

1. P knows instantaneous positions of both evaders;
2. P knows instantaneous positions of E_1 and only the last observed position of E_2;
3. P knows the last observed position of E_2 and searches for new detection;
4. P knows instantaneous positions of E_2 again.

We set up the problem as a game of kind and study it for the boundary initial states backwards. At the last stage P's behaviour is quite obvious. At the stage 3 we make use of the guaranteed strategies described in Ref.6. Then we prove that in a subsidiary game of degree at the stage 2 the optimal pursuit strategy is the same as in the game of successive pursuit $P \rightarrow E_1 \rightarrow E_2$ with fixed E_2 at the last observed position (see Ref.3). Finally, we set up a subsidiary game of the stage 1 and study it in the same manner as the game of successive pursuit of three evaders in minimum total time (see Ref.4). We also provide some numerical results for a set of parameters of the game.

The problem

We study the game of kind with the state equation

$$\dot{\bar{z}} = \bar{u}, \qquad (1)$$

and initial condition

$$\bar{z}(0) = \bar{z}^0, \quad \bar{z}^0 \in \mathcal{D}, \qquad (2)$$

where [1]

[1] $v = |\bar{v}| = \sqrt{v_x^2 + v_y^2}$ if $\bar{v} = (v_x, v_y)$.

- $\bar{z} = (\bar{z}_1, \bar{z}_2, \bar{z}_p)$ is the state vector;
- $\bar{z}_i = (z_{ix}, z_{iy})$, $i = 1, 2, p$, are cartesian coordinates of players in a plane;
- $\bar{u} = (\bar{u}_1, \bar{u}_2, \bar{u}_p)$ is the control vector and $u_k \le \beta, k = 1, 2, \ u_p \le 1, \ \beta < 1$;
- $\mathcal{D} = \{\bar{z}^0 \mid \ R \le |\bar{z}_1^0 - \bar{z}_p^0| \le |\bar{z}_1^0 - \bar{z}_p^0| \le r\}$ is the admissable set of initial states, $0 \le R < r$.

Let U_1, U_2, U_p and $\{\bar{z}_i(t; \bar{z}^0; U_1, U_2, U_p), t \ge 0\}$, $i = 1, 2, p$, be strategies and corresponding trajectories of players. [2] If there exist the instants $t = t_1$ and $t = t_2$ that

$$0 \le t_1 \le t_2 < +\infty,$$

$$|\bar{z}_1(t_1; \bar{z}^0; U_1, U_2, U_p) - \bar{z}_p(t_1; \bar{z}^0; U_1, U_2, U_p)| = R, \tag{3}$$

$$\bar{z}_2(t_2; \bar{z}^0; U_1, U_2, U_p) = \bar{z}_p(t_2; \bar{z}^0; U_1, U_2, U_p), \tag{4}$$

then P's cost function $G(\bar{z}_0; U_1, U_2, U_p)$ equals 0, otherwise 1. The game is assumed to be zero-sum, so that coalition $\{E_1, E_2\}$ has G as the gain function.

For the boundary initial states where P still succeeds the game can be divided into the four stages:

1. P knows instantaneous positions of both evaders and approaches coaliton as a whole;
2. P knows instantaneous positions of E_1 and only the last observed position of E_2 and shortens to R the distance to E_1;
3. P knows the last observed position of E_2 and searches for new detection inside an expanding disk;
4. P knows instantaneous positions of E_2 again and captures her.

The stage 1 ends at the first instant $t = t_r$ when

$$|\bar{z}_2^{t_r} - \bar{z}_p^{t_r}| = r, \tag{5}$$

and P seeks to get the best position for the following actions using a strategy

$$U_p^{(1)} = U_p^{(1)}(\bar{z}_1, \bar{z}_2, \bar{z}_p).$$

The stage 2 continues until the first instant $t = t_1$ when (see (3))

$$|\bar{z}_1^{t_1} - \bar{z}_p^{t_1}| = R, \tag{6}$$

and P not only captures E_1 but also maximizes the chance to detect E_2 afterwards using a strategy

$$U_p^{(2)} = U_p^{(2)}(\bar{z}_1, \bar{z}_2^{t_r}, \bar{z}_p).$$

At the stage 3 P forms a search plan

$$U_p^{(3)} = U_p^{(3)}(\bar{z}_2^{t_r}, \bar{z}_p, t - t_r),$$

to shorten to r the distance to E_2 taking into account the last position of E_2 observed at the stage 1 and the elapsed time. At the last stage P captures E_2 (see (4)) using a strategy

$$U_p^{(4)} = U_p^{(4)}(\bar{z}_2, \bar{z}_p).$$

[2] Further we'll ommit some arguments in the notations if the functions don't depend upon them.

Analysis

We analyse the problem backwards, from the fourth to first stage.

Stage 4. P captures E_2 in a finite time using, for example the simple pursuit strategy (see,e.g.,Ref.5)

$$U_p^{(4)} = (\bar{z}_2 - \bar{z}_p)/|\bar{z}_2 - \bar{z}_p|.$$

Stage 3. The optimal search plan in an expanding disk is unknown, but there are two guaranteed strategies described in Ref.6. According to the first plan $\hat{U}_p^{(3)}$ P simply traverses the straight-line segment $P^{t_1}E_2^{t_r}$, and if

$$(t_1 - t_r + |\bar{z}_2^{t_r} - \bar{z}_p(t_1; \bar{z}^{t_r}; U_1^{(2)}, U_p^{(2)})|)/r \leq 1/\beta, \tag{7}$$

then P detects E_2 without fail because the set of accesible positions of E_2 is covered by the detection domain by the instant of arrival P at $E_2^{t_r}$ (see Fig.1).

The other search plan is more complex. Let $U_{r,\beta}$ be the following two-step strategy from Ref.6:

1. Traverse the straight-line segment $P^{t_1}E_2^{t_r}$ until the instant $t = t_{ic}$ of internal contact of r-circle centered at P and $\beta(t - t_r)$-circle centered at $E_2^{t_r}$ (see Fig.2a).

2. Choose a direction (clockwise or anticlockwise) and traverse the turn of the logarithmic spiral with radial projection of the velocity equal β (see Fig.2b).

$\check{U}_p^{(3)}$ consists of traversing the straight-line segment $P^{t_1}E_2^{t_r}$ until the instant $t = t_{ec}$ of external contact of the $r-$ and $\beta(t - t_r)-$circles, and successive applications of $U_{r,\beta}$. If

$$(t_1 - t_r + |\bar{z}_2^{t_r} - \bar{z}_p(t_1; \bar{z}^{t_r}; U_1^{(2)}, U_p^{(2)})|)/r < f(\beta)/\beta, \tag{8}$$

where $f(\beta) = 1 + 2/[exp(2\pi\beta/\sqrt{1-\beta^2}) - 1]$, then detection of E_2 is guaranteed (Ref. 6).

Stage 2. Success of both above-mentioned search plans depends on the value of the coinciding left-hand parts of (7) and (8). So that for the boundary states a guaranteed pursuit strategy can be found as a solution of the subsidiary zero-sum game of P and E_1 with the state equation (1), initial condition satisfying (5), P's gain function

$$G^{(2)}(\bar{z}^{t_r}; U_1, U_p) = (t_1 - t_r + |\bar{z}_2^{t_r} - \bar{z}_p(t_1; \bar{z}^{t_r}; U_1, U_p)|)/r$$

and terminal condition (6).

Proposition 3.1. The optimal strategy of P in the subsidiary game of the stage 2 is the same as in the game of successive pursuit of two evaders in minimum time with fixed E_2 at the point $E_2^{t_r}$, and the value $V^{(2)}$ of the game is described by expression

$$V^{(2)}(\bar{z}^{t_r}) = \zeta(\bar{z}^{t_r}, \psi_1),$$

where ψ_1 is a solution of the equation $\mathcal{F}(\bar{z}^{t_r}, \psi_1) = 0$,

$$\zeta(\bar{z}^{t_r}, \psi_1) = \sin(\psi_1 + \varphi - \alpha + \gamma)/\sin\psi_1,$$

$$\sigma = R/|\bar{z}_1^{t_r} - \bar{z}_p^{t_r}|,$$

$$
\begin{aligned}
\alpha &= \sin^{-1} \sigma \sin \psi_1, \\
\varphi &= \sin^{-1} \beta \sin \psi_1, \\
\psi_2 &= 2\psi_1 + \varphi, \\
\cos \gamma &= (\bar{z}_1^{t_r} - \bar{z}_p^{t_r}) \cdot (\bar{z}_2^{t_r} - \bar{z}_p^{t_r})/(r|\bar{z}_1^{t_r} - \bar{z}_p^{t_r}|), \\
\mathcal{F}(\bar{z}^{t_r}, \psi_1) &= \begin{cases} |\bar{z}_1^{t_r} - \bar{z}_p^{t_r}| \sin(\psi_1 - \alpha) \sin 2\psi_1 - \\ r \sin(\psi_1 - \varphi) \sin(\psi_2 - \alpha + \gamma) & \text{if } V_{\bar{z}_1}^{(2)} \neq 0, \\ \\ \varphi - \alpha + \gamma & \text{otherwise.} \end{cases}
\end{aligned}
\tag{9}
$$

Proof. Let $\bar{e}_1$ and $\bar{e}_p$ be unit vectors parallel to the instantaneous velocities of E_1 and E_p and $\bar{e}_2$ be the unit vector directed from P^{t_1} to $E_2^{t_r}$. It follows from the main equation that $\bar{e}_1$ and $\bar{e}_p$ are parallel to $V_{\bar{z}_1}^{(2)}$ and $-V_{\bar{z}_p}^{(2)}$ correspondingly, and

$$
\beta|V_{\bar{z}_1}^{(2)}| - |V_{\bar{z}_p}^{(2)}| + 1 = 0 .
$$

Furthermore it is easy verify that $V_{\bar{z}_1}^{(2)}$ and $-V_{\bar{z}_p}^{(2)}$ are constants and

$$
\bar{z}_p^{t_1} = \bar{z}_1^{t_1} - R\bar{e}_1,
$$

$$
|V_{\bar{z}_1}^{(2)}|\bar{e}_1 + \bar{e}_2 - |V_{\bar{z}_p}^{(2)}|\bar{e}_p = 0 .
$$

Therefore, if the direction from $\bar{z}_p^{t_r}$ to $\bar{z}_1^{t_r} - R\bar{e}_1$ is adopted as the angular reference and $\psi_1, \psi_2 + \pi, \varphi$ denote the angles of $\bar{e}_1, \bar{e}_2, \bar{e}_p$ (see Fig.3),then $\varphi = \sin^{-1} \beta \sin \psi_1$ and $\psi_2 = 2\psi_1 + \varphi$. These relations coincide with the relations for corresponding angles in the game of successive pursuit (see Refs.1-2 for $R = 0$, Ref.3 for $R \geq 0$). So that we can use the results of Ref.3 to get remained dependencies of (9).

Fig.4a and 4b show the surface of $V^{(2)}$ and its cross-sections in a reduced space for fixed $E_2^{t_r}$, P^{t_r}, $|P^{t_r}E_1^{t_r}| = 10$, and various $E_1^{t_r}$ inside the annulus $1 \leq |P^{t_r}E_1^{t_r}| \leq 10$, $\beta = 0.4$, $R = 1$, $r = 10$. E_1's polar coordinates (ρ_1, φ_{21}) are given in the system with the pole at P^{t_r} and the polar axis directed to $E_2^{t_r}$, i.e. $\rho_1 = |P^{t_r}E_1^{t_r}|$ and φ_{21} is the angle that $P^{t_r}E_1^{t_r}$ makes with $P^{t_r}E_2^{t_r}$.

Stage 1. To find out a guaranteed pursuit strategy at this stage, we study one more subsidiary game of degree of P and coalition $\{E_1, E_2\}$ with the state equation (1), initial condition (2), P's gain function

$$
G^{(1)}(\bar{z}^0; U_1, U_2, U_p) = V^{(2)}(\bar{z}^{t_r}),
$$

and terminal condition (5). Let $V^{(1)}(\bar{z}^0)$ be the value of the game and $\xi(\bar{z}^0, \psi_1)$ be an expression for $V^{(1)}$ containing the variable ψ_1 (see Proposition 1). Let $\bar{s} = (\bar{s}_1, s_2, \bar{s}_p, s_{\psi_1})$ be a vector of the parameters and

$$
\begin{aligned}
\bar{z}_1 &= \bar{s}_1, \\
\bar{z}_2 &= \bar{s}_p + r\bar{e}(s_2), \\
\bar{z}_p &= \bar{s}_p, \\
\psi_1 &= s_{\psi_1},
\end{aligned}
$$

be a parametric representation of the terminal manifold (see Fig.5), where [3]

$$\mathcal{F}(\bar{s}_1, \bar{s}_p + r\bar{e}(s_2), \bar{s}_p, s_{\psi_1}) = 0.$$

Let $\nu(\bar{s}) = \xi(\bar{s}_1, \bar{s}_p + r\bar{e}(s_2), \bar{s}_p, s_{\psi_1})$ be the value at the terminal points, and $\alpha_1, \alpha_2, \alpha_p$ be the optimal angles measured from the direction from P^0 to E_1^0 (see Fig.5).

Proposition 3.2. $\alpha_1, \alpha_2, \alpha_p$ are constants satisfying the following system of equations

$$\begin{aligned} \bar{e}(\alpha_1) &= \frac{(\nu_{\bar{s}_1} - \nu_{s_{\psi_1}}\mathcal{F}_{\bar{z}_1}/\mathcal{F}_{\psi_1})}{|\nu_{\bar{s}_1} - \nu_{s_{\psi_1}}\mathcal{F}_{\bar{z}_1}/\mathcal{F}_{\psi_1}|}, \\ \bar{e}(\alpha_p) &= \frac{\nu_{s_{\psi_1}}(\mathcal{F}_{\bar{z}_2} + \mathcal{F}_{\bar{z}_p})/\mathcal{F}_{\psi_1} - \nu_{\bar{s}_p} + d_2\bar{e}(\alpha_2)}{|\nu_{s_{\psi_1}}(\mathcal{F}_{\bar{z}_2} + \mathcal{F}_{\bar{z}_p})/\mathcal{F}_{\psi_1} - \nu_{\bar{s}_p} + d_2\bar{e}(\alpha_2)|}, \\ \beta(|\nu_{\bar{s}_1} - \nu_{\bar{s}_\psi}\mathcal{F}_{\bar{z}_1}/\mathcal{F}_{\psi_1}| + d_2) &- |\nu_{s_{\psi_1}}(\mathcal{F}_{\bar{z}_2} + \mathcal{F}_{\bar{z}_p})/\mathcal{F}_{\psi_1} - \nu_{\bar{s}_p} + d_2\bar{e}(\alpha_2)| = 0, \end{aligned} \tag{10}$$

where

$$d_2 = (\nu_{\bar{s}_2} - r\nu_{s_{\psi_1}}/\mathcal{F}_{\psi_1}\bar{e}_\perp(s_2)\cdot\mathcal{F}_{\bar{z}_2})/(r\bar{e}_\perp(s_2)\cdot\bar{e}(\alpha_2)),$$

and all the functions are estimated at the terminal point corresponding to the chosen angles.

Proof. The main equation for ξ is

$$\min_{\bar{u}_p}\max_{\{\bar{u}_1,\bar{u}_2\}}\sum_{i=1,2,p}(\xi_{\bar{z}_i} - \mathcal{F}_{\bar{z}_i}\xi_{\psi_1}/\mathcal{F}_{\psi_1})\cdot\bar{u}_i = 0. \tag{11}$$

Hence the optimal directions are parallel to $\xi_{\bar{z}_k} - \mathcal{F}_{\bar{z}_k}\xi_{\psi_1}/\mathcal{F}_{\psi_1}, k = 1, 2$, and $-(\xi_{\bar{z}_p} - \mathcal{F}_{\bar{z}_p}\xi_{\psi_1}/\mathcal{F}_{\psi_1})$ correspondingly, and

$$\sum_{k=1,2}\beta|\xi_{\bar{z}_k} - \mathcal{F}_{\bar{z}_k}\xi_{\psi_1}/\mathcal{F}_{\psi_1}| - |\xi_{\bar{z}_p} - \mathcal{F}_{\bar{z}_p}\xi_{\psi_1}/\mathcal{F}_{\psi_1}| = 0. \tag{12}$$

Functions $\xi_{\psi_1}/\mathcal{F}_{\psi_1}$ and $\xi_{\bar{z}_i} - \mathcal{F}_{\bar{z}_i}\xi_{\psi_1}/\mathcal{F}_{\psi_1}, i = 1, 2, p$, do not vary along the optimal trajectories, so that

$$\begin{aligned} \xi_{\psi_1} &= d_{\psi_1}\mathcal{F}_{\psi_1}, \\ \xi_{\bar{z}_i} &= d_{\psi_1}\mathcal{F}_{\bar{z}_i} + \bar{d}_i, \quad i = 1, 2, p, \end{aligned}$$

for some constant $d_{\psi_1}, \bar{d}_1, \bar{d}_2, \bar{d}_p$. The transversality conditions for (11) are

$$\nu_{\bar{s}_1} = d_{\psi_1}\mathcal{F}_{\bar{z}_1} + \bar{d}_1, \tag{13}$$

$$\nu_{\bar{s}_1} = r\bar{e}_\perp(s_2)\cdot(d_{\psi_1}\mathcal{F}_{\bar{z}_2} + \bar{d}_2), \tag{14}$$

$$\nu_{\bar{s}_p} = d_{\psi_1}\mathcal{F}_{\bar{z}_2} + \bar{d}_2 + d_{\psi_1}\mathcal{F}_{\bar{z}_p} + \bar{d}_p, \tag{15}$$

$$\nu_{s_{\psi_1}} = d_{\psi_1}\mathcal{F}_{\psi_1}. \tag{16}$$

Expressing d_{ψ_1} from (16) and d_2 from (14) and substituting them into (13),(15),(12) we get (10).

Corollary 3.1. Along the optimal trajectories of the subsidiary games of the first two stages E_1 traverses the same straight line.

Fig.6a and 6b show the boundaries of the regions where P guarantees successive approach of two evaders using the four-stage strategies with two different search plans, $\hat{U}_p^{(3)}$ and $\check{U}_p^{(3)}$ (dashed line), for fixed $E_1^0, P^0, |P^0E_1^0| = 7$, and various E_2^0 inside the annulus $7 \leq |P^0E_2^0| \leq 10$, $\beta = 0.4, R = 1, r = 10$. In Fig.6a E_2's polar coordinates (ρ_1, φ_{12}) are given in the system with the pole at P^0 and the polar axis directed to E_1^0, i.e. $\rho_2 = |P^0E_2^0|$ and φ_{12} is the angle that $P^0E_2^0$ makes with $P^0E_1^0$.

[3] We use the following notations for some unit vectors: $\bar{e}(\theta) = (\cos\theta, \sin\theta)$ and $\bar{e}_\perp(\theta) = (-\sin\theta, \cos\theta)$.

Conclusion

We have investigated here a model of successive pursuit of two evaders by pursuer with imperfect visibility. We have described two four-stage guaranteed strategies distinguished by their search plans to approach the nearer evader and then capture the other. In order to refine the model, one might want to avoid such assumptions as, for example

- at initial moment both evaders are inside the detection domain;
- P does not observe E_2's positions right after the moment $t = t_r$ (see (5)).

We have studied the game merely for the boundary initial states where P still succeeds. For some parameters of the game and initial states the structure of the strategies can be different. There might also exist the situations where P retains E_2 inside the detection domain during approaching E_1.

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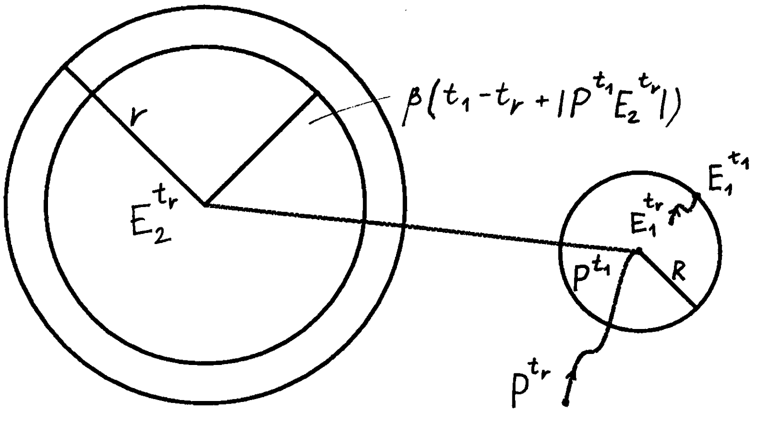

Fig.1. Guaranteed detection in case $\beta(t_1 - t_r + |P^{t_1}E^{t_r}|) < r$.

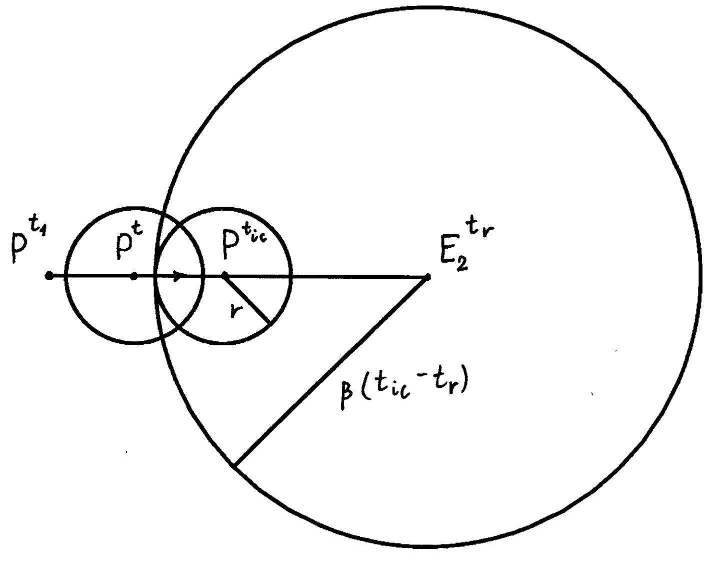

Fig.2a. Strategy $U_{\beta,r}$: traversing $P^{t_1}E_2^{t_r}$ until internal contact of the r- and $\beta(t-t_r)$-circles.

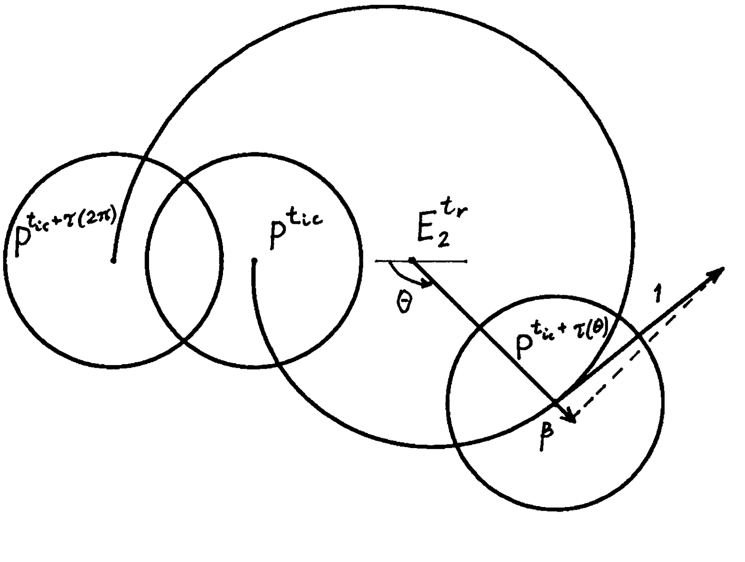

Fig.2b. Strategy $U_{\beta,r}$: traversing a turn of the logarithmic spiral.

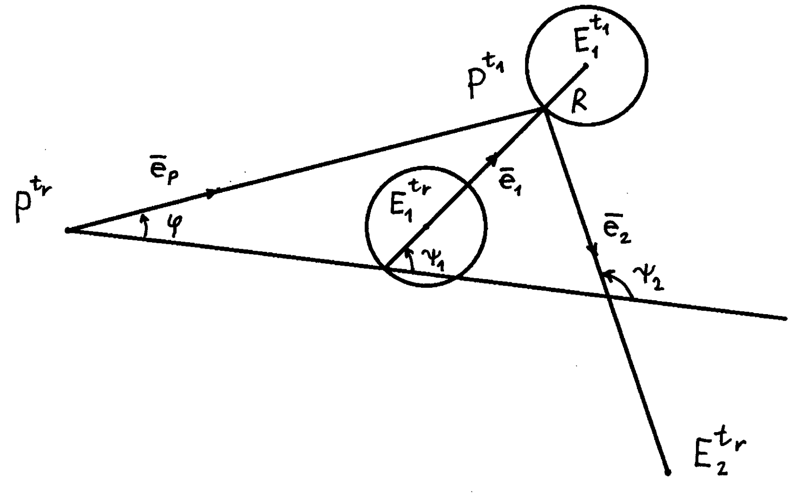

Fig.3. Optimal directions in the subsidiary game of the stage 2.

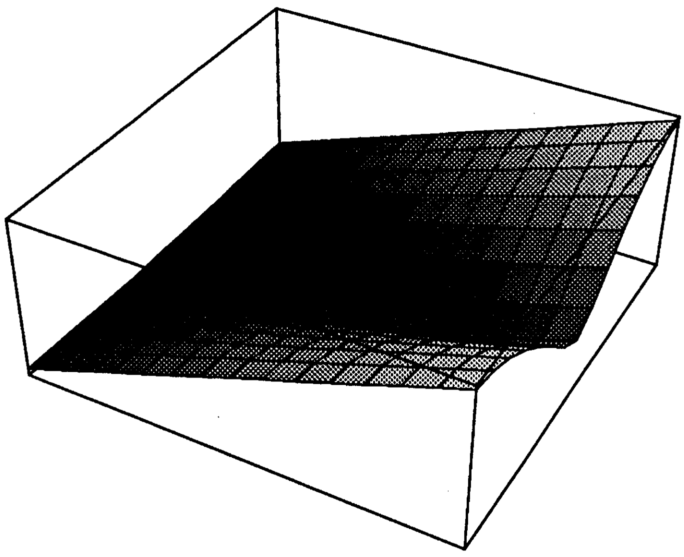

Fig.4a. Surface of $V^{(2)}$.

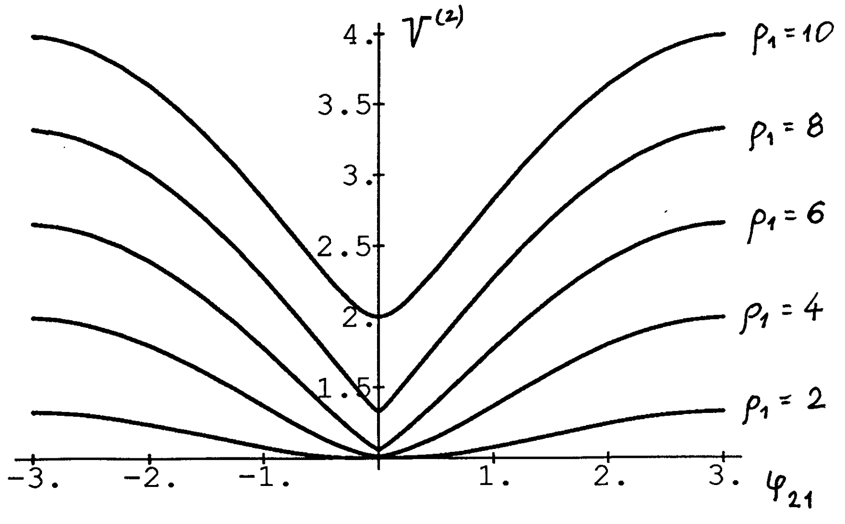

Fig.4b. Cross-sections of $V^{(2)}$.

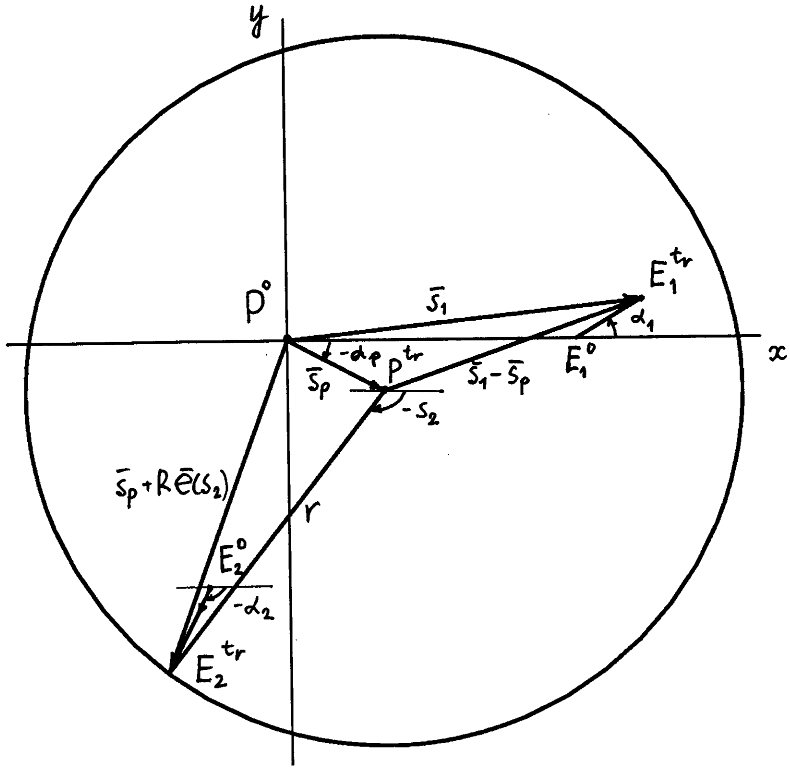

Fig.5. Geometry of the optimal pursuit in the subsidiary game of the stage 1.

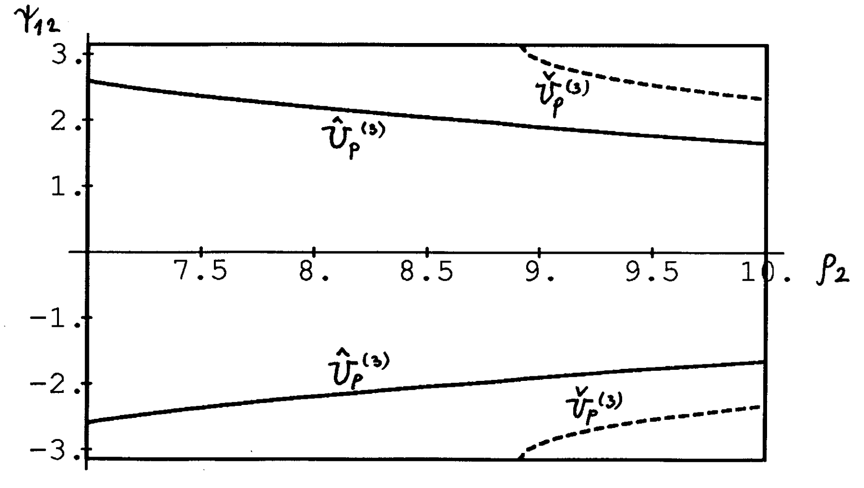

Fig.6a. Boundaries of the regions with guaranteed successive approach in the reduce space.

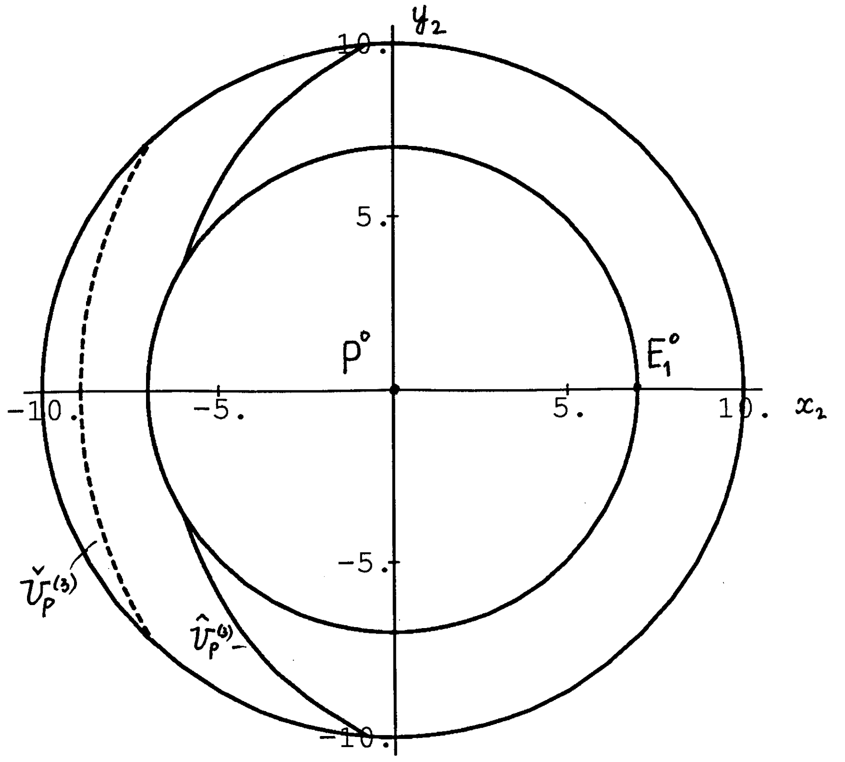

Fig.6a. Boundaries of the regions with guaranteed successive approach in the realistic space.

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ABSTRACT

We study a coplanar model of the successive pursuit of two evaders with unlimited turn-rates of all players and a restricted detection domain of pursuer. We describe two guaranteed pursuit strategies that include a two-stage strategy to shorten to a specified quantity the distance to the nearer evader, and a two-stage strategy to search and capture the other. The strategies are distinguished by their search plans. First coalition is pursued as a whole. Then pursuer approaches the first evader using the strategy of successive pursuit with unmoved second evader at the last observed position. Subsequently pursuer moves directly to that position of the second evader or, according to the more complex plan, alternates between traversing a straight line and turns of logarithmic spirals, and after detection captures the remaining evader using the simple pursuit strategy. We also provide some numerical results for a set of parameters of the game.

SUPPLEMENTARY NOTES