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by

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Abstract

A fuzzy control has recently been introduced in numerical computations of a control problem governed by partial differential equations(PDE), and its validity has been reported. Considering its implementation to real systems governed by PDE or its application as a numerical method to problems governed by PDE, reduction of the computational time in fuzzy reasoning seems to be serious.

In the paper parallelization of the fuzzy control is considered. Two types of parallelization are proposed. One is synchronous parallelization(SP). Another is full parallelization(FP). As for SP, its programing is easily carried out, so evaluation of fuzzy reasoning is easily carried out by numerical simulations before real implimentation . Futhermore, SP is partially performed on vector machines. On the other hand, as for FP, evaluation of fuzzy reasoning before real implimentation is not easily carried out numerically without parallel processing machines. Development of numerical evaluation of FP is a future problem for us. FP is superior than SP in speed, therefore its implementation is intensively expected.

1. Introduction

It has been already recognized that fuzzy control to systems governed by ordinary differential equations(ODE systems) is valid [4-6]. However, it is not so easy to apply fuzzy control to systems governed by partial differential equations(PDE systems). Main problems before real implimentation are as follows.

1) Connection between sampling points and control points

In PDE systems there are several sampling points and control points in general even if the number of governing equations is one(see Figure 1). Then it is a problem to make adequate connecting networks between these points[3]. Connection between sampling points is a priori given through governing PDE. Connection between control points and between both control points and sampling points should be determined considering both controlability and simplicity. Simple networks may not be adequate from the control view points, while complicated networks may not be adequate from the view point of economy or real implimentation.

2) Table of fuzzy reasoning

After determining a connecting network it is a problem to consider adequate fuzzy reasoning, i.e. to make adequate tables of fuzzy reasoning[3]. These tables must be determined by considering the property of governing PDE. This table making may need mathematical sense and enough experiences.

There is another problem in real implimentation. This is parallelization of fuzzy control. For speed is required in real implimentation. Therefore above problems 1) and 2) should be also considered along parallelization. Furthermore, some parallelization may be necessary in numerical simulations of fuzzy control, because evaluation of connecting networks and tables of fuzzy reasoning should be evaluated numerically before real implimentation. Hence parallelization of fuzzy control is considered in the next section.

2. Parallel processing of fuzzy control

In this section parallel processing of fuzzy control is considered. Two types of parallelization is mainly considered. One is synchronous parallelization(SP), another is full parallelization (FP).

(i) Synchronous Parallelization(SP)

SP is parallelization with data flow synchronization.

There are three data flow at a control point. First one is that from sampling points. Second one is that from the same point. Last one is that from other control points. In these three data flow the last one is restricted for parallelization, i.e. reference of next value at other control points is prohibited. Then next value at every control points is determined by fuzzy reasoning at the same time.

SP is of the most practical use now. This is because validity of fuzzy reasoning is easily evaluated numerically before real implimentation. Data synchronization at control points is automatically carried out in normal calculations of PDE. This means that fuzzy control with SP is easily programed. Therefore validity of fuzzy reasoning is easily evaluated numerically on normal computing machines.

Futhermore, SP is partially performed on vector machines. Vectorization in the program is carried out as follows. At first every case in a fuzzy reasoning table is indicated by some index. Hereafter this index is called case index. Then computations at fuzzy reasoning are decomposed into two steps.

a) At the first step, next values at every control point are computed for all cases in the reasoning table. At the same time case index of every control points is determined along the reasoning table.

b) At the second step next value at a control point is chosen following the case index.

(ii) Full Parallelization(FP)

FP is full parallelization, i.e. data flow synchronization is not necessary in FP. Then FP has no restriction in principle.

FP is superior than SP in speed, then its implimentation to real systems is intensively expected. However, FP is not of practical use now because evaluation before real implimentation is not easily carried out numerically without parallel processing machines. Development of numerical evaluation of FP is our next purpose.

3. Numerical simulations

In this section fuzzy control with SP is applied to an ill-posed control problem governed by a Laplace equation and it is simulated numerically.

The problem is defined on a rectangular which is located in the xy plane and is parallel to the xy axes. Sampling points are distributed on the center line parallel to the y-axis. The same number of control points are distributed on the right-hand segment parallel to the y-axis. Boundary values are given on other parts of the boundary. Boundary values at control points are controled to attain desired values at sampling points. Numerical calculations are carried out by discretizing the problem. 20×20 mesh points are used here.

For this control problem a valid connecting network(see Figure 2) and a valid table of fuzzy reasoning have been investigated [1]. Then SP of this control is partially performed on a vector machine. The program is made along a guideline mentioned in the above section. Numerical results are shown in Table 3. CPU times spent in both solving PDE and fuzzy reasoning are presented for two cases that vectorized processing of fuzzy reasoning is carried out or not. Calculations for solving PDE are vectorized in both cases. CPU time spent in fuzzy reasoning is improved about 3 times by vectorization. CPU time for PDE may be improved using sparseness of the matrix. On the other hand, CPU time for fuzzy reasoning may increase if more complicated fuzzy reasoning is used.

These numerical results show that fuzzy control may be available as a fast method for numerical computations of control problem governed by PDE. As such examples, one can easily consider time dependent mesh generation in numerical computations of time evolutional PDE. One can also consider determination of parameters which attain minimun of cost functions. However, one of the most effective applications is fast and stable numerical computations of ill-posed optimal control problems (e.g. control problems concerned with Cauchy problems for elliptic operators[2]).

4. Conclusions

Two parallelizations SP and FP are considered here for the fuzzy control system governed by partial differential equations.

Numerical implimentation of SP are is carried out here. Numerical results show that SP is of most practical use now. Because its programing is easy and before its implimentation to real systems validity of fuzzy reasoning is easily evaluated by numerical simulations using normal computing machines. Numerical results also show that fuzzy control with SP may be available as a simple and fast numerical method for control problems. Because SP is partially performed on vector machines.

FP is superior than SP in speed, then its implimentation to real systems is intensively expected. However, it is not of practical use now because evaluation before real implimentation is not easily carried out numerically without parallel processing machines. Development of numerical evaluation of FP is our next purpose.

5. Acknowledgments

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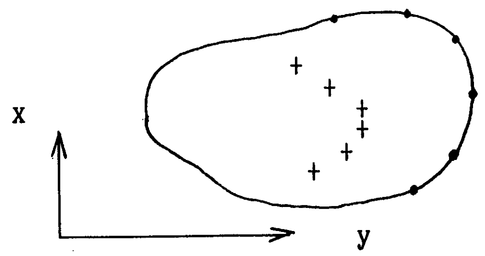

Figure 1. Distribution of sampling points and control points.

+ : sampling points.

· : control points.

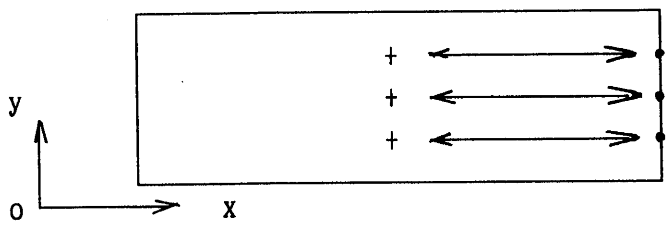

Figure 2. Distribution of sampling points and control points in an ill-posed control problem.

+ : sampling points.

· : control points.

	CPU time for fuzzy reasoning (× 10^{-2} sec)	CPU time for PDE (× 10^{-2} sec)
Scalar	0.70	3.21
Vector	0.24	

Table 3. CPU time spent in numerical simulations of SP.

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SUPPLEMENTARY NOTES