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OPTIMIZATION METHOD FOR ENHANCEMENT OF LASER RADAR IMAGES BY A CLASS OF PIECEWISE LINEAR TRANSFORMATION OF GRAY LEVELS

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ABSTRACT

The present paper is concerned with an optimization method for the enhancement of laser radar images with a criterion of a histogram entropy in a class of piecewise linear transformations of gray scales. Images of laser radars for meteorological measurements need enhancement by gray scale transformations to have outputs of good quality. Existing methods of the enhancement such as the histogram flattening use nonlinear transformations. The laser radar images, however, prefer linear transformations, since the nonlinearities may induce unexpected deformations on the enhanced image and the outputs become unreliable. Here the enhancement problem is formulated as an optimization in a class of piecewise linear transformations by considering the information maximization property of the histogram flattening. Moreover a generalized criterion for general histograms is considered. Although the present method is developed for the laser radar images, it is applicable to many other pictures in scientific measurements.

INTRODUCTION

Images by scientific measurements are usually recognized by specialists. In their examinations they need enhancement of desired part of their pictures to clarify many characteristics of the images. Moreover the scientists must understand the types of the transformations for the enhancement and their effects on the transformed image. Therefore they frequently dislike nonlinear transformations, since the nonlinearities may induce unexpected effects or deformations on the enhanced image and the outputs become unreliable.

The histogram flattening has been an elementary technique in the image enhancement: it is easy to see that the image with the flat histogram has maximum amount of information [1]. In spite of their effectiveness in many areas of picture processings, the applicability of the histogram modifications is severely restricted when images by a laser radar for meteorological measurements should be processed. The histogram modifications require nonlinear transformations, whereas the images by the laser radar prefer linear transformations as is stated above.

In this paper a method of the enhancement by a class of piecewise linear transformations is considered. An entropy criterion is maximized by using two parameters which describe the line segments of the piecewise linear transformations. This method is applied to an image obtained by a large-scale laser radar at the National Institute for Environmental Studies, Japan [2]. Moreover generalized criteria which are applicable to general histograms are considered.

Although the method here has been developed for the laser radar images, it is applicable to many other types of pictures, since it is based on the general principle of maximum entropy.

ENHANCEMENT OF LASER RADAR IMAGES

For twenty years the laser radar has been used for observing various features in atomospheric pollutions [3]. Generally signals of the laser radar are represented as two dimensional pixel data in which intensity of a pixel represents that of a signal at a specified spot in a two dimensional region. As is usual for many measurement data, an image by the laser radar is different from the ordinary pictures in that many features in the image can be grasped or recognized not at a glance but though careful examination of a specialist.

In representing the signals of the laser radar as a picture and analyzing it, the following should be noted.

(a) Unnecessary objects such as clouds or noises which have high intensities of the signals may haze necessary portions of the image and the significant parts frequently have lower contrasts.

(b) An absolute value of the observed signal does not mean a specific level of a pollution. That is, two signals of the same intensity in different images may well correspond to different levels of a pollution. On the other hand, relative difference in one image is in general meaningful.

Enhancement of an image by the laser radar is a routine task in order to see various features in it. Ordinarily one can not obtain a good picture by a simple linear transformation of a signal intensity into a gray level as shown by the dashed line in

Fig. 1, which transforms the strongest intensity into the highest gray level H and the weakest signal into the lowest gray level L, due to the reason (a) above mentioned.

In such a case a piecewise linear transformation with three line segments as the solid line in Fig. 1 is considered to improve the contrast, which transforms signals below a level a into the same gray level L and those above b into one gray level H. The threshold a and b are determined after several trials so that significant information in the image is visible. This transformation means that the values of the signals above b and below a are useless and ignored. Note that the values of the thresholds can not be determined beforehand, due to the reason (b). In fact, this method of the trials has been used for the laser radar images obtained at the National Institute for Environmental Studies, Japan, which is a huge human task.

Here a problem of automatic enhancement of the images arises. One of the elementary techniques for this is the histogram flattening. Unfortunately this kind of the histogram modifications uses nonlinear transformations which have serious drawbacks. The piecewise linear transformations produces the outputs which can be seen like a contour map except the highest and the lowest gray levels, that is, comparison of the intensities at different spots in a relative scale in one image is possible. In case of the air pollution, this leads to an overview of the status of the pollution of a specific region. On the other hand, the nonlinear transformations induce unexpected deformations on the output, which make the interpretations of a result more difficult or lead to a misunderstanding by showing a

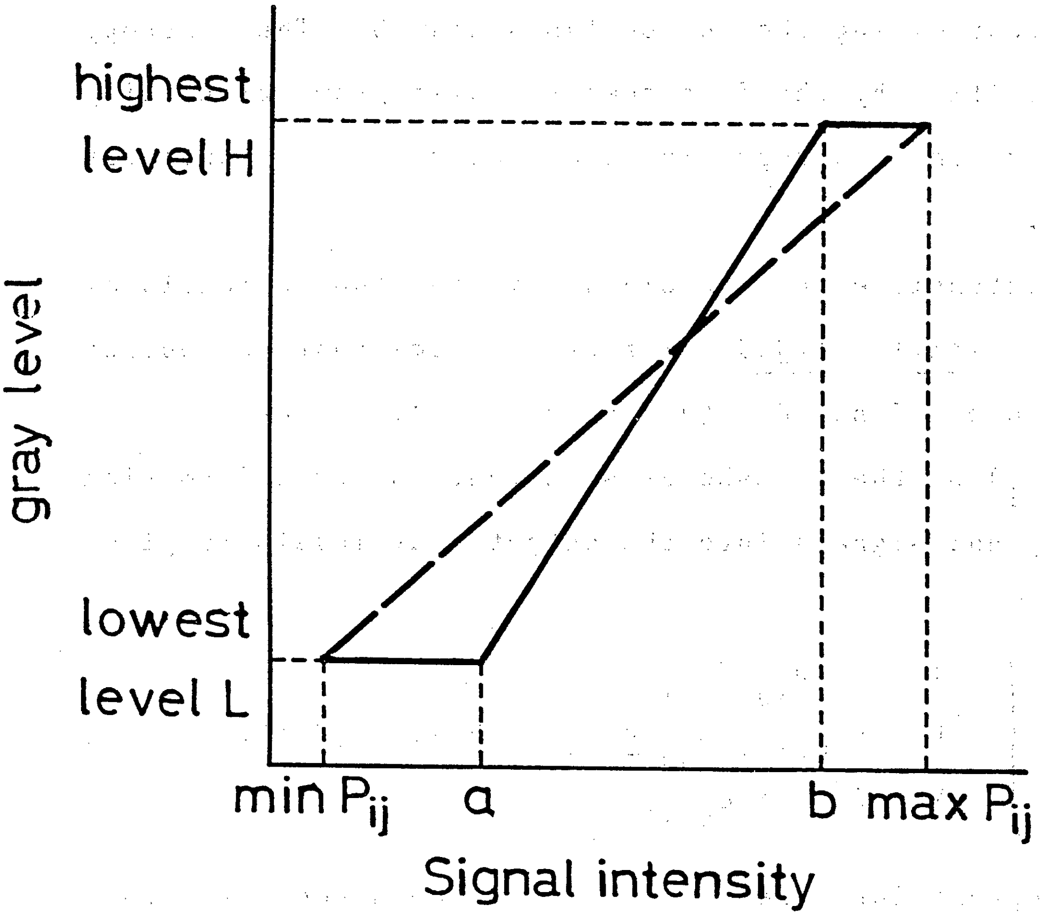

Fig. 1 Linear and piecewise linear transformations from input pixel data to output gray-levels

contour map with unequal length of the intervals. Moreover nonlinear transformations exclude higher order processings such as the filtering and the smoothing which are applicable after the piecewise linear transformations.

In this paper a method with the above piecewise linear transformations which takes advantages in the histogram modifications is developed. That is, an entropy criterion is maximized by controlling the thresholds a and b. The entropy criterion is justified by the fact that the histogram flattening gives the maximum entropy in the class of nonlinear transformations.

Let the intensities of the signals in the two dimensional array be (p_{ij}), $1 \leq i \leq I$, $1 \leq j \leq J$. Moreover assume that the output gray levels are denoted by $(1,2,...,K)$. Let $c = (c_1, c_2, ..., c_{K-1})$ be the thresholds such that the transformation T from the original signals into the output gray levels is given by

$$T(p_{ij}) = \begin{cases} 1 , & (p_{ij} < c_1) \\ k , & (c_{k-1} \leq p_{ij} \leq c_k , \quad k=2,...,K-1). \\ K , & (p_{ij} \geq c_{K-1}) \end{cases}$$

Let the histogram for the gray levels obtained by this transformation be $(h_1, h_2, ...h_K)$: h_k is the number of pixels having the gray level k on the output. Since the histogram depends on the thresholds, the components can be written as $h_k = h_k(c)$.

An optimization problem is considered by controlling the thresholds c. Two admissible classes of the thresholds are given:

(i) Monotone class: $M=\{ c \mid c_1 < c_2 < \ldots < c_{K-1} \}$.

(ii) The class with intervals of an equal length except the both ends α and β :

$$M_1(\alpha, \beta) = \{ c \mid c_1 = \alpha, c_{K-1} = \beta, c_2 - c_1 = c_3 - c_2 = \ldots = c_{K-1} - c_{K-2} > 0 \}.$$

It is easy to see that the transformation T with the class M_1 corresponds to the piecewise linear transformations in Fig. 1. The class M is the largest one in which every threshold is independently changeable, whereas in M_1 only two parameter can be controlled.

The criterion to be maximized is the entropy of the histogram:

$$H(c) = - \sum_{k=1}^{K} \frac{h_k(c)}{h} \log \frac{h_k(c)}{h},$$

where h is the total number of the pixels: $h = h_1 + h_2 + \ldots + h_K$.

It is clear that the solution of

$$\max_{c \in M} H(c)$$

is nothing but the histogram flattening, since the entropy takes its maximum value when $h_1 = h_2 = \ldots = h_K$. (See [1].)

As was noted earlier, we consider the piecewise linear transformation in Fig.1, which leads to the optimization

$$\max_{c \varepsilon M_1(\alpha, \beta)} H(c) . \tag{1}$$

MODIFIED CRITERIA FOR GENERAL HISTOGRAMS

As the histogram flattening was generalized to the histogram hyperbolization and to modification to adjust arbitrary histograms, the present method of optimization can be generalized to adjust the histogram to any shape in class M_1. For this purpose a modified entropy criterion is considered.

Let the desired histogram be $f = (f_1, f_2, \ldots, f_K)$ such that $f_1+f_2+\ldots+f_K= h$.

Let

$$m_i = (\max_i f_i) - f_i, \qquad i=1,2,\ldots,K.$$

Then consider a criterion

$$Hm(c) = -\sum_{i=1}^{K} \frac{h_i(c) + m_i}{K \max_i f_i} \log \frac{h_i(c) + m_i}{K \max_i f_i} .$$

Note that Hm(c) is reduced to H(c) if histogram is flat $f_1 = f_2 = \ldots = f_K$. Moreover it is easy to see that the optimal solution of

$$\max_{c \in M} Hm(c)$$

is $h_i = f_i$, $i=1,2,\dots,K$. Therefore we consider

$$\max_{c \in M_1(\alpha, \beta)} Hm(c) \tag{2}$$

as modified optimization problems for general histograms in the class of piecewise linear transformations.

The above defined criterion Hm is not a heuristic one. On the contrary it is an almost unique choice as the generation of the histogram entropy H for the present purpose, which is justified by the following.

Let us consider a family of generalized entropies Hg with given histogram $f = (f_1, f_2, \dots, f_K)$:

$$Hg(c) = -\sum_{i=1}^{K} q_i(h_i(c), f) \log q_i(h_i(c), f),$$

where $q_i(x,y)$, $(x,y) \in [0,1] \times [0,\infty)^K$ is a continuous and differentiable function such that $q_i(x,y) \geq 0$ for $(x,y) \in [0,1] \times [0,\infty)^K$.

For the present purpose the following conditions should be considered,

(A) $q_i(x_i, f) = 1$ for all $x_i \in [0,1]$ such that $x_1 + x_2 + \dots + x_K = h$.

(B) Optimal solution of

$$\max_{c \in M} Hg$$

is $h_i(c) = f_i, \quad 0 \leq i \leq K.$

(C) If $f_1 = f_2 = \dots = f_K$, i.e., $f_i = h/K$, $1 \leq i \leq K$, then $Hg(c) = H(c)$.

Then we have the following theorem.

Theorem In order that the function $Hg(c)$ satisfies the conditions (A),(B),(C), it is necessary and sufficient that

$$q_i(x_i, f) = \frac{x_i + \bar{f} - f_i}{K \bar{f}}$$

where $\bar{f}$ is a constant which is in general dependent on the histogram f ($\bar{f} = \bar{f}(f)$) and satisfies

(D) $\bar{f} \geq f_i$ for $1 \leq i \leq K$.

(E) If $f_1 = f_2 = \dots = f_K = h/K$, then $\bar{f} = h/K$.

Proof We prove the necessity; sufficiency follows immediately from simple calculations.

Let

$$L = \sum_i q_i(x_i, f) - \lambda\left(\sum_i x_i - h\right)$$

with a multiplier λ. then from the condition (A) $\partial L / \partial x_i =$

$\partial q_i/\partial x_i - \lambda = 0$ is satisfied for all x_i such that $\sum_i x_i = h$
Integrating the above equation, we have

$$q_i(x_i, f) = \lambda x_i + a_i,$$

where λ and a_i are constants which are in general dependent on the histogram. With a slight modification we consider

$$q_i(x_i, f) = \lambda (x_i - f_i) + b_i .$$

Note that the condition (B) is equivalent to $q_i(f_i, f) = 1/K$, whence $b_i = 1/K$. Then putting $\bar{f} = 1/K\lambda$, we have

$$q_i(x_i, f) = \frac{K \lambda (x_i - f_i) + 1}{K} = \frac{x_i + 1/K\lambda - f_i}{1/\lambda}$$

$$= \frac{x_i + \bar{f} - f_i}{K \bar{f}} .$$

To guantee the positivity of q_i ($q_i \geq 0$), it is necessary and sufficient that $\bar{f} \geq f_i$ for all i.

The last condition (E) follows from (C). That is, if we put

$$q_i(x_i, h/K) = \frac{x_i + \bar{f} - h/K}{K \bar{f}} = \frac{x_i}{h},$$

we have $\bar{f} = h/K$, and vice versa.

Q.E.D.

Just one step from the above theorem leads to the criterion Hm(c). Namely, the simplest choice of $\bar{f}$ such that the conditions (D) and (E) are satisfied is $\bar{f} = \max_i f_i$.

APPLICATION TO A LASER RADAR IMAGE

In this example images show measurement data on vertical direction, where the horizontal direction corresponds time variation of the data. Therefore the images show time variation of air status at one spot. Details of semantics of the images are omitted here.

Figure 2 shows a laser radar image with ten gray levels on the output. Thresholds c are given by the equal length intervals:

$$c_1 - \min_{i,j} p_{ij} = c_2 - c_1 = \dots = c_9 - c_8 = \max_{i,j} p_{ij} - c_9 > 0. \quad (3)$$

The thin area of higher gray levels in the lower part of the image shows particles suspended in air such as dust and vapors. Figure 3 shows the results of the optimization (1), where the triangular domain

$$\{(\alpha,\beta) \mid \min_{i,j} p_{ij} < \alpha < \max_{i,j} p_{ij}, \ \min_{i,j} p_{ij} < \beta < \max_{i,j} p_{ij}, \ \alpha < \beta\}$$

was divided by two sets of 20 lines of equal intervals which are parallel to the α axis and the β axis, then each grid was examined to find the maximum value of the entropy.

Figure 4 shows the result of the optimization (2) of Hm(c), where hyperbolic histogram $f_i = 1/((i-1)K + d)$ is considered. In Fig. 4 the constant is set to be d = 0.1. It seems that Fig. 4 exhibits a sharper result than Fig. 3. The ultimate choice for the histograms should, however, wait futher experiences.

Remark. The areas of higher intensities which are near to the ground implies the mixture layer [3]: The mixture layer hight is decisive parameter of the status of air pollutions.

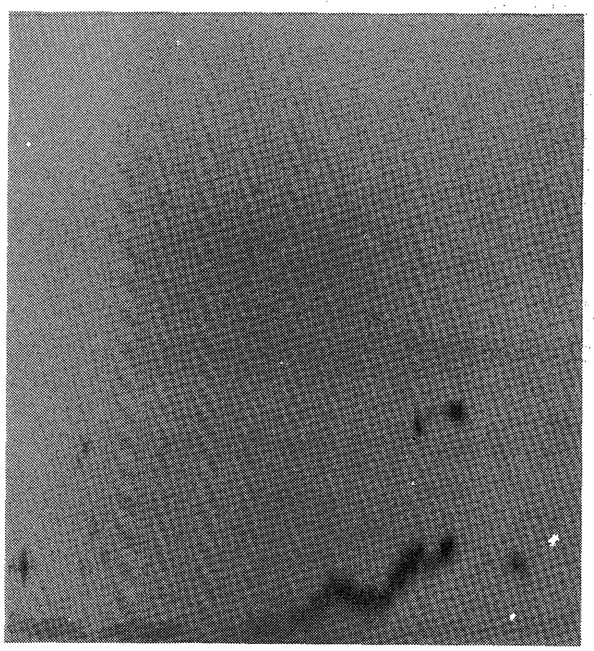

Fig.2 Original figure with intervals (3).

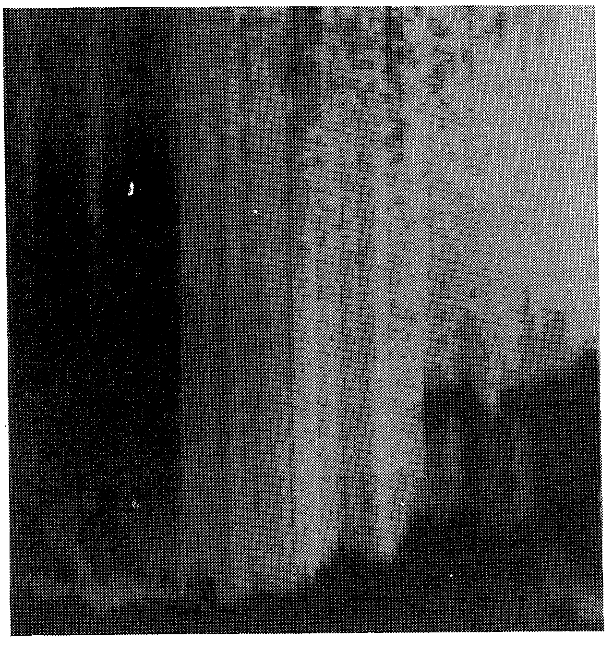

Fig. 3 Result of the optimization (1) by M_1.

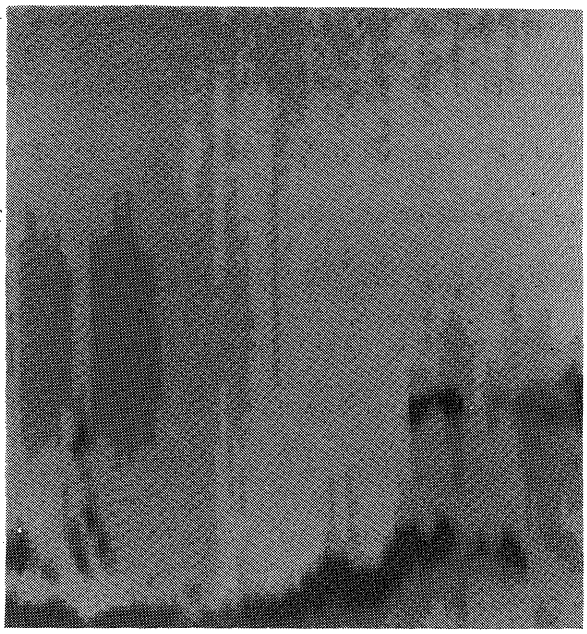

Fig. 4 Result of the optimization (2) by M_1.

CONCLUSION

In summary, the above method is based on the following consideration:

(a) The measurement data by the laser radar require the piecewise linear transformations.

(b) The method of histogram flattening maximizes the entropy of the histogram in class of nonlinear transformations.

(c) Therefore, maximization of the entropy in the class of the piecewise linear transformations should be considered.

The piecewise linear transformation in Fig. 1 is frequently used in image processings of the measurement data. Empirical choices or the trials in the determination of the thresholds a and b are usual. The optimization of the entropy is a simple method and the computation of the solution is not difficult, therefore it is applicable to many types of the pictures in the measurement in science and engineering.

All numerical calculations were performed on HITAC M180 at the National Institute for Environmental Studies, Japan.

REFERENCES

[1] A. Rosenfeld and A. C. kak, Digital Picture Processing, New York: Academic Press, 1976.

[2] N. Takeuchi, H.Shimizu, and M. Okuda, "Synchronous lidar observation of atomospheric pollution and meteorological parameters at NIES", 8th Int. Laser Radar Conf. 23, Philadelphia, June, 1977.

[3] R. Perry and R. J. Young, eds. Handbook of Air Pollution Analysis, London: Chapman and Hall, 1977.

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SUPPLEMENTARY NOTES