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SPECTRAL MAPPING THEOREMS THROUGH ELEMENTARY APPROACH

by

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ABSTRACT

Spectral mapping theorems and their applications, despite of practical importance, seldom find their way into introductory textbooks of linear algebra and matrix theory. The reason appears to be traceable to the way the function of a single matrix variable is defined, as is in Gantmacher [1].

In this note we derive spectral mapping theorems from another point of view by observing a simple yet beautiful correspondence between a holomorphic function $f(z)$ and the special upper-triangular matrix $f^{*}(z)$ whose (i, j)-entry equals $f^{(j-i)}(z)/(j - i)!$, $j \geq i$.

1. INTRODUCTION

Let $f(z)$ be a holomorphic function of a complex variable z defined on a region G of the complex plane and let A be a complex square matrix of order n. If a contour C in G encloses the spectrum of A (i.e., the set of all eigenvalues of A), then $f(A)$ is defined by

$$f(A) = \frac{1}{2\pi i} \int_C f(t)\ (tI - A)^{-1}\, dt \qquad (1)$$

where the right-hand side is independent of a particular contour C so long as C encloses the spectrum of A. See Dunford & Schwartz [2, Chapter 7]. It is well-known that if $f(z)$ is a polynomial or a rational function whose denominator does not vanish on the spectrum of A or a power series in z converging for any finite z, then the definition of $f(A)$ by (1) and the matrix obtained by substituting A into z in $f(z)$ agree. For example, taking $f(z) = z$, we have:

$$A = \frac{1}{2\pi i} \int_C t\ (tI - A)^{-1}\, dt \qquad (2)$$

where C encloses the spectrum of A.

The purpose of this paper is to give the aforementioned fact an alternative approach that is more direct and transparent, using only the Jordan canonical form theorem and an algebraic homomorphism (see Theorem 1 in the next section) from which yet another hidden feature of the Jordan block emerges (see Example 1 in the next section). The arguments given in the following sections solely depends on the spectral analysis, or a class of theorems which analyses the transformation of spectral structure under the transformation $A \to f(A)$ where $f(z)$ denotes a polynomial or a rational function or a power series in z converging for any finite z, and where $f(A)$ denotes the matrix obtained by substituting A into z in $f(z)$ in an obvious way.

The topics covered in this note seldom find their way into introductory textbooks of matrix theory, in a form accessible to readers with moderate mathematical background in spite of their significance in applications of eigenvalue problems. We will show in this note that this can be done. The key is given by the systematic use of the simple mapping $f(z)$ to $f^*(z)$, an upper-triangular matrix whose (i, j)-entry is defined by $f^{(j-i)}(z)/(j - i)!$, $j \geq i$. It happens that the mapping represents a homomorphism from the algebra of functions defined on a complex region to the commutative algebra of upper-triangular matrices of the type just mentioned. The matrix $f^*(z)$ is a well-known matrix appearing in various contexts in matrix theory, see, e.g., Gantmacher [1, Chapter 5]. Moreover our approach enables us to dispense with the interpolation theory as seen, e.g., in Gantmacher [1, Chapter 5], and leads us in a natural way to the integral representation of matrix function $f(A)$ as given in Dunford & Schwartz [2, Chapter 7]: See Section 4 below.

2. MATRIX REPRESENTATION OF LAWS FOR DIFFERENTIATION

Let $f(z)$ be holomorphic in a region in the complex plane. Let n denote a positive integer. By the symbol $f^*(z)$, or simply f^*, we mean the n by n matrix defined by:

$$f^*(z) = \begin{bmatrix} f(z) & f'(z) & f''(z)/2! & \cdots & f^{(n-1)}(z)/(n-1)! \\ & f(z) & f'(z) & \ddots & \vdots \\ & & \ddots & \ddots & \\ & 0 & & & f'(z) \\ & & & & f(z) \end{bmatrix} \qquad (3)$$

When the order n of the square matrix $f^*(z)$ must be indicated explicitly, we use the symbol $[f^*(z)]_n$ for the case.

We have the following theorem.

THEOREM 1.

1° $(f + g)^* = f^* + g^*$

2° $(c\, f)^* = c\, f^*$, where c is a constant

3° $(f\, g)^* = f^*\, g^* = g^*\, f^*$

4° $(f/g)^* = f^*(g^*)^{-1} = (g^*)^{-1} f^*$, $g \neq 0$

5° $(1/g)^* = (g^*)^{-1}$, $g \neq 0$

Proof. 1° and 2° are obvious. 3° is proven by Leibniz's law:

$$\frac{(f\, g)^{(k)}}{k!} = \sum_{r=0}^{k} \frac{f^{(r)}}{r!} \frac{g^{(k-r)}}{(k-r)!}$$

4°. By 3° we have

$$f^* = \left(\frac{f}{g}\, g\right)^* = \left(\frac{f}{g}\right)^* g^* = g^* \left(\frac{f}{g}\right)^*$$

where g^* is nonsingular since $g \neq 0$, which proves 4°.

5°. Let $f = 1$ in 4° and use the fact $f^* = I$ (identity matrix). □

The mapping $f \to f^*$ is an algebraic homomorphism; sum and product of functions, scalar multiple of a function correspond to sum and product of matrices, scalar multiple of a matrix, respectively.

The following example illustrate a few immediate applications of Theorem 1.

Example 1. Compute the p-th power of an n by n Jordan block

$$J = \begin{bmatrix} z & 1 & & O \\ & \ddots & \ddots & \\ O & & z & 1 \end{bmatrix}$$

Let $f(z) = z$. Then $f^*(z) = z^* = J$, and by 3° we have:

$$J^p = (z^*)^p = (z^p)^* = \begin{bmatrix} z^p & \binom{p}{1} z^{p-1} & \cdots & \binom{p}{n-1} z^{p-n+1} \\ & \ddots & \ddots & \vdots \\ & & & \binom{p}{1} z^{p-1} \\ O & & & z^p \end{bmatrix}$$

where

$$\binom{p}{k} = \frac{p!}{k!\,(p-k)!}$$

Example 2. Find the inverse of a Jordan block J (= z^*). By 5°

$$(z^*)^{-1} = (1/z)^*, \quad z \neq 0$$

yielding

$$J^{-1} = \begin{pmatrix} z^{-1} & -z^{-2} & z^{-3} & \cdots & (-1)^{n-1} z^{-n} \\ & z^{-1} & \ddots & & \vdots \\ & & \ddots & \ddots & -z^{-2} \\ & 0 & & & z^{-1} \end{pmatrix}$$

Example 3. Find the inverse of the following n by n matrix A:

$$A = \begin{pmatrix} a_0 & a_1 & \cdots & a_{n-1} \\ & a_0 & \ddots & \vdots \\ & & \ddots & a_1 \\ 0 & & & a_0 \end{pmatrix}, \quad a_0 \neq 0$$

Let $f(z) = a_0 + a_1 z + \cdots + a_{n-1} z^{n-1}$. Then A is represented as:

$$A = \begin{pmatrix} f(0) & f'(0) & \cdots & f^{(n-1)}(0)/(n-1)! \\ & f(0) & \ddots & \vdots \\ & & \ddots & f'(0) \\ 0 & & & f(0) \end{pmatrix} = f^*(z)\Big|_{z=0}$$

By 5°

$$A^{-1} = [f^*(z)\Big|_{z=0}]^{-1} = [f^{-1}(z)]^*\Big|_{z=0}$$

and thus the (i, j)-entry of A^{-1} is given by:

$$\frac{1}{(j-i)!}\frac{d^{j-i}}{dz^{j-i}}\left(\frac{1}{f(z)}\right)\Bigg|_{z=0}, \quad i \leq j$$

3. SPECTRAL MAPPING THEOREMS

Unless otherwise stated, we interpret henceforth in this note that A is an n by n matrix and its Jordan form is given by:

$$J = V^{-1}A\,V = \begin{bmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_m \end{bmatrix} \qquad (4)$$

A typical Jordan block J_i is assumed to be of order k_i and has the form

$$J_i = \begin{bmatrix} \lambda_i & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & \lambda_i \end{bmatrix}, \quad i = 1, \dots, m \qquad (5)$$

where $\lambda_1, \dots, \lambda_m$ are not necessarily distinct.

THEOREM 2 (Spectral mapping theorem). Let h(z) be a rational function; viz. $h(z) = f(z)/g(z)$, where

$$f(z) = a_0 z^p + a_1 z^{p-1} + \cdots + a_{p-1} z + a_p, \quad a_0 \neq 0, \; p \geq 0$$

$$g(z) = b_0 z^q + b_1 z^{q-1} + \cdots + b_{q-1} z + b_q, \quad b_0 \neq 0, \; q \geq 0$$

1° If $g(\lambda_i) \neq 0$, $i = 1, \ldots, m$, then $g(A)$ is nonsingular and

$$V^{-1} h(A)\, V = \begin{bmatrix} [h^*(\lambda_1)]_{k_1} & & & 0 \\ & [h^*(\lambda_2)]_{k_2} & & \\ & & \ddots & \\ 0 & & & [h^*(\lambda_m)]_{k_m} \end{bmatrix} \qquad (6)$$

where

$$h(A) = f(A)\,[g(A)]^{-1} = [g(A)]^{-1} f(A)$$

$$[h^*(\lambda_i)]_{k_i} = \begin{bmatrix} h(\lambda_i) & h'(\lambda_i) & \cdots & h^{(k_i-1)}(\lambda_i)/(k_i-1)! \\ & h(\lambda_i) & \ddots & \vdots \\ & & \ddots & h'(\lambda_i) \\ 0 & & & h(\lambda_i) \end{bmatrix} \qquad (7)$$

2° If $h'(\lambda_i) \neq 0$, then a Jordan form of $[h^*(\lambda_i)]_{k_i}$ has a single Jordan block. If $h'(\lambda_i) = 0$, then a Jordan form of $[h^*(\lambda_i)]_{k_i}$ has two or more Jordan blocks.

3° $\det[h(A) - \lambda I] = [h(\lambda_1) - \lambda]^{k_1} \cdots [h(\lambda_m) - \lambda]^{k_m}$

Proof. 1°. We have:

$$V^{-1} f(A)\, V = f(J) = \begin{bmatrix} f(J_1) & \ddots & 0 \\ 0 & & f(J_m) \end{bmatrix}$$

$$V^{-1} g(A)\, V = g(J) = \begin{bmatrix} g(J_1) & \ddots & 0 \\ 0 & & g(J_m) \end{bmatrix}$$

If $g(\lambda_i) \neq 0$, $i = 1, \ldots, m$, then $g(J)$ is nonsingular and so is $g(A)$. Then:

$$V^{-1} h(A) V = V^{-1} f(A) V [V^{-1} g(A) V]^{-1}$$

$$= \begin{bmatrix} f(J_1)[g(J_1)]^{-1} & & 0 \\ & \ddots & \\ 0 & & f(J_m)[g(J_m)]^{-1} \end{bmatrix} = \begin{bmatrix} h(J_1) & & 0 \\ & \ddots & \\ 0 & & h(J_m) \end{bmatrix}$$

For 1° one must show that $h(J_i) = [h^*(\lambda_i)]_{k_i}$. By Theorem 1

$$h^*(z) = f^*(z) [g^*(z)]^{-1}$$

$$= [a_0(z^*)^p + \cdots + a_p 1^*][b_0(z^*)^q + \cdots + b_q 1^*]$$

Jordan block J_i is represented as:

$$J_i = [z^*]_{k_i}\Big|_{z=\lambda_i}$$

Thus we have:

$$[h^*(\lambda_i)]_{k_i} = f(J_i) [g(J_i)]^{-1} = h(J_i)$$

2° . If $h'(\lambda_i) \neq 0$, then $[h^*(\lambda_i)]_{k_i}$ has a single linearly independent eigenvector and thus a Jordan form of $[h^*(\lambda_i)]_{k_i}$ has a single Jordan block. If $h'(\lambda_i) = 0$, then $[h^*(\lambda_i)]_{k_i}$ has at least two linearly independent eigenvectors and thus a Jordan form of $[h^*(\lambda_i)]_{k_i}$ has two or more Jordan blocks.

3°. The proof is obvious and is omitted. □

Example 4. Let A be diagonalizable; $V^{-1}AV = \text{diag}[\lambda_1, \ldots, \lambda_m]$ where $\lambda_1, \ldots, \lambda_m$ are not necessarily distinct. For rational $h(z) = f(z)/g(z)$ with $g(\lambda_i) \neq 0$, $i = 1, \ldots, n$, $g(A)$ is nonsingular and we have:

$$V^{-1}h(A)V = \text{diag}[h(\lambda_1), \ldots, h(\lambda_m)]$$

THEOREM 3 (Spectral mapping theorem). Let $h(z)$ be an infinite power series:

$$h(z) = a_0 + a_1 z + a_2 z^2 + \cdots = \sum_{k=0}^{\infty} a_k z^k$$

whose radius of convergence is strictly greater than the spectral radius of A. Then

$$V^{-1}h(A)V = \begin{pmatrix} [h^*(\lambda_1)]_{k_1} & & 0 \\ & \ddots & \\ 0 & & [h^*(\lambda_m)]_{k_m} \end{pmatrix} \quad (8)$$

(the same form as (6) for rational h(z)).

Proof. Let

$$h_r(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_r z^r, \; r = 1, 2, \ldots$$

By Theorem 2 we have:

$$V^{-1}h_r(A)V = \begin{pmatrix} [h_r^*(\lambda_1)]_{k_1} & & 0 \\ & \ddots & \\ 0 & & [h_r^*(\lambda_m)]_{k_m} \end{pmatrix} \quad (9)$$

Since each λ_i is in the circle of convergence of h(z), taking the limit as $r \to \infty$ in (9) yields (8). □

Example 5. Let $h(z) = e^{tz}$, where t is an arbitrary complex number. Then we have:

$$V^{-1} e^{tz} V = \begin{bmatrix} E_1 & & 0 \\ & \ddots & \\ 0 & & E_m \end{bmatrix}$$

where

$$E_i = e^{t\lambda_i} \begin{bmatrix} 1 & t & \cdots & t^{k_i - 1}/(k_i - 1)! \\ & 1 & \ddots & \vdots \\ & & \ddots & t \\ 0 & & & 1 \end{bmatrix}$$

4. APPLICATIONS OF SPECTRAL MAPPING THEOREMS

As immediate applications of spectral mapping theorems, we prove two theorems, one giving a necessary and sufficient condition for equality of two matrix functions and the other a Cauchy type integral representation of a matrix function.

THEOREM 4 (Identity theorem). Let each of $\phi(z)$ and $\psi(z)$ be a rational function; viz.

$$\phi(z) = f_1(z)/g_1(z), \quad \psi(z) = f_2(z)/g_2(z)$$

where $f_1(z)$, $g_1(z)$, $f_2(z)$ and $g_2(z)$ are polynomials. We assume that $g_1(\lambda_i) \neq 0$ and $g_2(\lambda_i) \neq 0$, i = 1, ..., m, where $\lambda_1, \ldots, \lambda_m$

are eigenvalues of A which are not necessarily distinct. Let $\mu_1, \ldots, \mu_r$ denote distinct eigenvalues of A.

1°. For $\phi(A) = \psi(A)$, it is necessary and sufficient that

$$\phi^{(i)}(\mu_j) = \psi^{(i)}(\mu_j), \; i = 0, 1, \ldots, m_j - 1; \; j = 1, \ldots, r \qquad (10)$$

where m_j denotes the order of the largest Jordan block corresponding to μ_j.

2°. For $\phi(A) = \psi(A)$, it is sufficient that

$$\phi^{(i)}(\mu_j) = \psi^{(i)}(\mu_j), \; i = 0, 1, \ldots, p_j - 1; \; j = 1, \ldots, r \qquad (11)$$

where p_j denotes the algebraic multiplicity of μ_j.

Proof. 1°. $\phi(A) = \psi(A)$, if and only if

$$V^{-1}\phi(A)\, V = V^{-1}\psi(A)\, V \qquad (12)$$

By Theorem 2, (12) is expressed equivalently as follows:

$$[\phi^*(\lambda_i)]_{k_i} = [\psi^*(\lambda_i)]_{k_i}, \; i = 1, \ldots, m$$

or

$$[\phi^*(\mu_j)]_{m_j} = [\psi^*(\mu_j)]_{m_j}, \; j = 1, \ldots, r$$

which yields (10).

2°. Since $m_j \le p_j$, the proof follows immediately from 1°. □

Example 6 (Special case of Hermite-Birkhoff interpolation problem). Suppose a rational function $\phi(z)$ is given. Then a polynomial $\psi(z)$ can be constructed so that $\phi(A) = \psi(A)$, where $\psi(z)$ must satisfy (10). If one restrict the degree of $\psi(z)$ be at most $m_1 + m_2 + \cdots + m_r - 1$, then $\psi(z)$ is uniquely determined. The Lagrange-Sylvester interpolation polynomial gives a representation of such $\psi(z)$. A practically more useful expression, however, is given as the following Newton-type form:

$$\psi(z) = \sum_{j=1}^{r} \{ \prod_{k=1}^{j-1} (z - \mu_k)^{m_k} \}\{ \sum_{k=0}^{m_j} a_{jk} (z - \mu_j)^k \}$$

Example 7. Let

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}, \quad b \neq 0$$

Compute A^p, where p is any integer (if $p < 0$ we assume that A is nonsingular). Let

$$\phi(A) \equiv A^p = c_0 I + c_1 A \equiv \psi(A) \qquad (13)$$

where c_0 and c_1 are unknown constants to be determined. For (13) it is necessary and sufficient that

$$\lambda_i^p = c_0 + c_1 \lambda_i , \; i = 1, 2 \qquad (14)$$

where $\lambda_1 = a + b$ and $\lambda_2 = a - b$. Solving (14) for c_0 and c_1 yields:

$$A^p = \frac{1}{2}\begin{bmatrix} \lambda_1^p + \lambda_2^p & \lambda_1^p - \lambda_2^p \\ \lambda_1^p + \lambda_2^p & \lambda_1^p + \lambda_2^p \end{bmatrix}$$

Example 8. Find a general term expression for the following difference equation:

$$y_{n+1} - 2y_n - y_{n-1} = 0, \quad n = 1, 2, \ldots \qquad (15)$$

where y_0 and y_1 are assumed to be given. Since

$$\begin{pmatrix} y_{n+1} \\ y_n \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} y_n \\ y_{n-1} \end{pmatrix}$$

or

$$\begin{pmatrix} y_{n+1} \\ y_n \end{pmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{pmatrix} y_1 \\ y_0 \end{pmatrix}$$

the problem is reduced to that of computing the power

$$\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^n$$

The method in the previous example applies and y_n is given as:

$$y_n = \frac{\lambda_1^n - \lambda_2^n}{2\sqrt{2}} y_1 + \frac{\lambda_1^{n-1} - \lambda_2^{n-1}}{2\sqrt{2}} y_0$$

THEOREM 5 (Integral representation of h(A)). Let h(z) be either a rational function or an infinite power series. Let C be a simple closed curve in the complex plane. Assume that h(z) is holomorphic on C and in the interior of C. Assume further that every eigenvalue of a given square matrix A of order n locates in the interior of C. Then

$$h(A) = \frac{1}{2\ i} \int_C h(t)\ (tI - A)^{-1}\ dt \tag{16}$$

Proof. Theorem 2 or 3 gives:

$$V^{-1} h(A) V = \begin{pmatrix} [h^*(\lambda_1)]_{k_1} & & 0 \\ & \ddots & \\ 0 & & [h^*(\lambda_m)]_{k_m} \end{pmatrix}$$

Since

$$h^{(k)}(\lambda_i) = \frac{k!}{2\pi i} \int_C \frac{h(t)}{(t - \lambda_i)^{k+1}} dt \ , \quad k = 0, 1, \ldots$$

by Cauchy's integral formula, we have:

$$[h^*(\lambda_i)]_{k_i} = \frac{1}{2\pi i} \int_C h(t) \begin{bmatrix} \frac{1}{t-\lambda_i} & \frac{1}{(t-\lambda_i)^2} & \cdots & \frac{1}{(t-\lambda_i)^k} \\ & \ddots & \ddots & \vdots \\ & & & \frac{1}{(t-\lambda_i)^2} \\ 0 & & & \frac{1}{t-\lambda_i} \end{bmatrix} dt$$

$$= \frac{1}{2\pi i} \int_C h(t) \begin{bmatrix} t-\lambda_i & -1 & & 0 \\ & \ddots & \ddots & \\ & & & -1 \\ 0 & & & t-\lambda_i \end{bmatrix}^{-1} dt \quad \text{(by Example 2)}$$

$$= \frac{1}{2\pi i} \int_C h(t) \, (tI - J_i)^{-1} \, dt$$

Thus we have:

$$V^{-1} h(A)\, V = \frac{1}{2\pi i}\int_C h(t) \begin{bmatrix} (tI - J_1)^{-1} & & O \\ & \ddots & \\ O & & (tI - J_m)^{-1} \end{bmatrix} dt$$

$$= \frac{1}{2\pi i}\int_C h(t)\,(tI - J)^{-1}\, dt,$$

which proves (16). □

Based on the result in Theorem 5, we define $h(A)$ for a general $h(z)$: Let C be a simple closed curve in the complex plane. $h(z)$ is assumed to be holomorphic on C and in the interior of C. Further assumed is that every eigenvalue of a given square matrix A of order n is in the interior of C. Then we define $h(A)$ as follows:

$$h(A) = \frac{1}{2\pi i}\int_C h(t)\,(tI - A)^{-1}\, dt \qquad (17)$$

We know that the right-hand side of (17) is determined merely by $h(z)$ and A, irrespective of a particular C. This fact is verified by observing the relation

$$\frac{1}{2\pi i}\int_C h(t)\,(tI - A)^{-1}\, dt = V \begin{bmatrix} h(\lambda_1) & & O \\ & \ddots & \\ O & & h(\lambda_m) \end{bmatrix} V^{-1}$$

It is also clear that the right-hand side of (17) does not depend on a particular nonsingular matrix V which yields a Jordan form of A.

By the generalization (17) of a matrix function, the necessary and sufficient condition for $\phi(A) = \psi(A)$ is given the same as (10) even for a case where $\phi(z)$ and $\psi(z)$ are holomorphic functions on a region that contains every eigenvalue of A in its interior. We further know that the polynomial p(z) of degree at most n is uniquely determined so that $\phi(A) = \psi(A)$.

5. CONCLUSION

We have provided a method for deriving a class of spectral mapping theorems by observing the correspondence between a holomorphic function f(z) and an upper-triangular matrix $f^*(z)$ whose (i, j)-entry is defined by $f^{(j-i)}(z)/(j-i)!$, $j \geq i$. Based on the spectral mapping theorems and our introduced correspondence between f(z) and $f^*(z)$, we have also shown a way for reaching a Dunford's integral representation of f(A) for a given square matrix A.

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In this note we derive spectral mapping theorems from another point of view by observing a simple yet beautiful correspondence between a holomorphic function $f(z)$ and the special upper-triangular matrix $f^*(z)$ whose (i, j)-entry equals $f^{(j-i)}(z)/(j - i)!$, $j \geq i$.

SUPPLEMENTARY NOTES