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Hisao Kameda* and Eitan Altman†

Abstract

There exist many systems that consist of a common-pool resource shared by multiple users, each of whom strives to optimize its own utility noncooperatively by determining its input to the common-pool resource. Such systems are regarded as noncooperative games. The situations where every user has attained its optimization coincidently but noncooperatively are Nash equilibria. In this article, first, it is seen that, in a fairly general framework of such systems, Nash equilibria are strongly Pareto inefficient, that is, for each of these Nash equilibria, there exists another situation where all users have better utilities than in it. Then, a number of examples of such situations in communication networking are presented, which are similar to the 'tragedy of the commons' in economics. In particular, this article considers a noncooperative flow-control problem for communication networks with multiple ports of entry and of exit, where each user decides its throughput, as its input, so as to optimize its own performance objective as its utility. As such an objective, we mainly consider the power which is the quotient of the throughput over the expected packet-passage time. The existence of a Nash equilibrium is given. It is shown that this Nash equilibrium has strong Pareto inefficiency. It is also shown that for a category of networks, the degree of Pareto inefficiency increases as the number of users increases, and it can increase without bound in some cases. We also show a flow-control version of the Braess-like paradox. Furthermore, we consider another flow-control setting with additive costs and the power control in wireless communications.

Keywords— Braess paradox, common-pool resource, flow control, Nash equilibrium, noncooperative game, Pareto optimum and inefficiency, power, power control, tragedy of the commons.

1 Introduction

There exist many systems where multiple independent users, or players, may strive to optimize each own utility by determining its input to a common-pool resource, which can be regarded as noncooperative games. The situation where each user attains its own optimum coincidently is a Nash equilibrium. For example, communication networks like the Internet are becoming more and more widespread and are having more important roles in societies. As the scale of a communication network increases, the number of independent users or organizations, like Internet service providers, that join the network tends to increase. It is natural that these independent users seek their own benefits or utilities noncooperatively. Thus, such systems are regarded as noncooperative games.

Nash equilibria may, however, be Pareto inefficient, that is, there may exists another situation of a system where no users have less benefits and some more benefits than in the Nash equilibrium of the system. Dubey (1986) showed that Nash equilibria may generally be Pareto inefficient, but it appears to be difficult to obtain the concrete cases of inefficient Nash equilibria from his result. In particular, we call a situation of a system *strongly Pareto inefficient* if all users have more benefits in another situation than the situation. As for the communication and transportation networks, however, examples of such strong Pareto inefficiencies have been shown with respect to noncooperative routing, first by Braess (1968), and a number of related studies

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followed (Murchland, 1970; Frank, 1981; Cohen and Kelly, 1990; Kelly, 1991; Cohen and Jeffries, 1997; Korilis *et al.*, 1995; Korilis *et al.*, 1999; Roughgarden, 2001). As for the non-cooperative load balancing in distributed computer systems, the existence of paradoxes similar to Braess's that appear only in the case of a finite number of players but not in the case of infinitesimal players, in the same environment, has been shown (Kameda *et al.*, 2000; Kameda and Pourtallier, 2002). Note that load balancing and routing have mutually similar logical structures that are different from those of the systems considered here (Tantawi and Towsley, 1985; Kameda *et al.*, 1997; Li and Kameda, 1998; Altman and Kameda, 2001).

On the other hand, flow control is also a very important issue in communication networks (Hsiao and Lazar, 1991; Parekh and Gallager, 1993; Chakravorti, 1994; Korilis and Lazar, 1995; Altman and Başar, 1998; Ching, 1999). It is necessary to keep the adequate amount of flow through and the proper congestion-level of a communication network by controlling the admission rates of packets to the network. It is natural to think of noncooperative optimal flow control and of the Nash equilibrium concept therein. It appears, however, that few studies have addressed the issue of Pareto inefficiency of Nash equilibria in noncooperative flow control. This article addresses mainly this issue. In addition, we note that the Nash equilibrium concept has been discussed with respect to the power control in wireless communications (Famolari *et al.*, 1999; MacKenzie and Wicker, 2001; Saraydar *et al.*, 2001; Saraydar *et al.*, 2002; Ji and Huang, 1998). We slightly touch on it. The examples of such systems as considered here have been studied in social science under the name, 'Tragedy of the Commons' (see, Hardin (1968), Roemer (1989), Roemer and Silvester (1993), Funaki and Yamamoto (1999), *etc.*).

This article first shows a fairly general framework of strongly Pareto-inefficient Nash equilibria. Also mentioned are some concrete examples of noncooperative games, all of which are shown to have strongly inefficient Nash equilibria. In particular, this article considers a flow-control problem for communication networks with multiple ports of entry and of exit, where each user decides its throughput, that is, the rate of its packets to inject into a network so as to optimize its own performance objective. As such an objective, we mainly consider the power which is the quotient of the throughput over the expected delay, that is, the expected time for a packet to pass through the network. The optimized situation is a Nash equilibrium, the existence of which is proved here. (We note that Korilis and Lazar showed the existence of Nash equilibria for networks of one pair of ports of entry and exit where each user optimizes noncooperatively its thoughput within its response time constraint (Korilis and Lazar, 1995).) We call a situation of a system *strongly Pareto inefficient* if all users have more benefits in another situation than the situation. We show that our Nash equilibrium is always strongly Pareto inefficient and what is Pareto superior to it. (We note, in passing, that Mazumdar *et al.* (1991) discussed *cooperative* power optimization, that is, the Nash arbitration scheme, in Jackson networks extensively, but mentioned briefly the Pareto inefficiency of the Nash equilibrium without showing its existence. We also note that Dubey (1986) showed that Nash equilibria may generally be Pareto inefficient if they exist, but it appears to be difficult to obtain from his result the concrete description of the way how each Nash equilibrium is inefficient, for example, which state is Pareto superior to the Nash equilibrium in question.)

Furthermore, we show that for a category of noncooperative networks, the degree of inefficiency increases as the number of users increases, and it can increase without bound in some cases. On the other hand, we may have a Pareto-optimal solution that achieves the solution of the Nash equilibrium proportionately for the category of networks. It is shown that this Nash equilibrium is always strongly Pareto inefficient. It is also shown that for a category of networks, the degree of inefficiency increases as the number of users increases, and it can increase without bound in some cases. On the other hand, we may have a Pareto-optimal solution that achieves the solution of the Nash equilibrium proportionately for the category of networks. We also show a flow-control version of the Braess-like paradox. That is, adding connections to a noncooperative flow-control system may leads to the degradation of the power of every user. Furthermore, we present another flow-control setting with additive costs (instead of the power criterion) as well as power-control problems in wireless communications as examples of the general framework of strongly Pareto-inefficient Nash equilibria.

Organization of this paper

The rest of this paper is organized as follows. Section 2 discusses a general framework of strongly Pareto-inefficient Nash equilibria. Section 3 discusses a flow-control problems, and Subsections 3.2, and also 3.3, show that the Nash equilibrium of noncooperative flow-control on the network considered is always strongly Pareto inefficient. Subsection 3.2.1 presents more detailed estimates on the Pareto efficiency for a category of networks. Subsection 3.2.2 presents numerical examples of some cases where the degree of inefficiency of Nash equilibrium can increase without bound as the number of users increases. Subsection 3.2.3 presents a flow-control version of the Braess-like paradox. In Subsection 3.3, we present inefficiency results for the flow control with additive costs. Section 4 presents another example of the general noncooperative game, the power control in wireless communications, which has a strongly inefficient Nash equilibrium. Section 5 concludes this article. The Appendix presents a proof of the existence of a Nash equilibrium for the noncooperative flow control presented in Subsections 3.2 and 3.3.

2 A General Framework of Pareto-inefficient Nash Equilibria

Consider a noncooperative game that has n players each of whom decides the value of $\lambda_i \geq 0$, that is, the strategy space consists of real nonnegative numbers. Denote $\boldsymbol{n} = (1, 2, \ldots, n)$. Thus, the strategy profile is presented by a vector, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$. Let $U_i(\boldsymbol{\lambda})$ denote the utility that player i strives to maximize. Let $\boldsymbol{L}$ be the product of the strategy spaces. Denote by $\boldsymbol{C}$ ($\subset \boldsymbol{L}$) the set of feasible values of $\boldsymbol{\lambda}$. The definition of feasibility may depend on the system concerned. For example, for a stochastic system, such $\boldsymbol{\lambda}$ as leads the system to statistical equilibrium is feasible. $\boldsymbol{C}$ may have boundaries. Let $\tilde{\boldsymbol{\lambda}}$ ($\in \boldsymbol{C}$) denote a strategy profile that presents a Nash equilibrium (with finite utilities). Denote $\tilde{\boldsymbol{n}} = \{i \mid \tilde{\lambda}_i$ is not a boundary value of $\boldsymbol{C}\}$. Introduce the following assumptions on a Nash equilibrium $\tilde{\boldsymbol{\lambda}}$:

Assumption $\Psi1$. *For a Nash equilibrium $\tilde{\boldsymbol{\lambda}}$, the partial derivatives of $U_i(\boldsymbol{\lambda})$ for $i \in \tilde{\boldsymbol{n}}$ exist at $\boldsymbol{\lambda} = \tilde{\boldsymbol{\lambda}}$ and either of the following two holds:*
(1) *The utility, U_i, of player i is decreasing in λ_j, $j \neq i$, that is,*

$$\frac{\partial U_i}{\partial \lambda_j} < 0 \ \ \textit{for all} \ \ j \in \tilde{\boldsymbol{n}} \ (j \neq i).$$

(2) *The utility, U_i, of player i is increasing in λ_j, $j \neq i$, that is,*

$$\frac{\partial U_i}{\partial \lambda_j} > 0 \ \ \textit{for all} \ \ j \in \tilde{\boldsymbol{n}} \ (j \neq i).$$

Assumption $\Psi2$. *For a Nash equilibrium $\tilde{\boldsymbol{\lambda}}$, more than one element of $\tilde{\boldsymbol{\lambda}}$ is not a boundary value, but is an interior value. Then, $\tilde{\boldsymbol{n}}$ has more than one element. That is, the strategies $\tilde{\lambda}_i$ of at least two users are of interior values.*

Assumption $\Psi2$ implies:

$$\left.\frac{\partial U_i}{\partial \lambda_i}\right|_{\boldsymbol{\lambda}=\tilde{\boldsymbol{\lambda}}} = 0 \ \text{ for } \ i \in \tilde{\boldsymbol{n}}. \tag{1}$$

Theorem 1 *If Assumptions* $\Psi1$ *and* $\Psi2$ *hold for a Nash equilibrium in $\boldsymbol{C}$, it is strongly Pareto inefficient.*

[Proof] Consider the situation where reducing the values of all elements $\lambda_i(t)$, $i \in \tilde{\boldsymbol{n}}$, of $\boldsymbol{\lambda}(t)$ from $\tilde{\lambda}_i$ except elements $\lambda_i(t)$, $i \notin \tilde{\boldsymbol{n}}$. Then, by noting that $d\lambda_j/dt = 0$, $j \notin \tilde{\boldsymbol{n}}$, we have for $i \in \boldsymbol{n}$,

$$\frac{dU_i}{dt} = \frac{\partial U_i}{\partial \lambda_i}\frac{d\lambda_i}{dt} + \sum_{q \neq i, q \in \tilde{\boldsymbol{n}}} \frac{\partial U_i}{\partial \lambda_q}\frac{d\lambda_q}{dt}. \tag{2}$$

Consider the case (1) of Assumption $\Psi1$. Note, for all $i \in \tilde{\boldsymbol{n}}$, that $\left.\frac{\partial U_i}{\partial \lambda_i}\right|_{\lambda(t)=\tilde{\lambda}} = 0$ (since $\tilde{\lambda}$ is a Nash equilibrium) by Assumption $\Psi2$ (eq. (1)), and that the coefficient $\left.\frac{\partial U_i}{\partial \lambda_q}\right|_{\lambda(t)=\tilde{\lambda}}$ of $\frac{d\lambda_q}{dt}$ for all $q \in \tilde{\boldsymbol{n}}$ ($q \neq i$) is negative by Assumption $\Psi1$. Therefore, $\left.\frac{dU_i}{dt}\right|_{\lambda(t)=\tilde{\lambda}} > 0$ for all i, if $\frac{d\lambda_i}{dt} < 0$ for all $i \in \tilde{\boldsymbol{n}}$, that is, all λ_i, $i \in \tilde{\boldsymbol{n}}$ are being reduced from $\tilde{\lambda}_i$. This implies that there exists a value of λ such that $U_i(\lambda) > U_i(\tilde{\lambda})$ for all i.

Similarly for the case (2) of Assumption $\Psi1$. □

Remark 1 Note that, in the case where $\tilde{\boldsymbol{n}}$ has only one element i, reducing λ_i from $\tilde{\lambda}_i$, while keeping all other $\lambda_j = \tilde{\lambda}_j$, $j \neq i$, may decrease the utility U_i, although all other utilities U_j, $j \neq i$, may increase as seen from (2). This comes from the definition of the Nash equilibrium. □

Example 1 Consider the case where the utility function for player i consists of two components, one depends on λ and the other only on λ_i as follows:

$$U_i(\lambda) = R_i(\lambda_i) - T_i(\lambda). \tag{3}$$

Note that (3) includes most of the cases where $U_i(\lambda) = R_i(\lambda_i)/T_i(\lambda)$, since, then,

$$\log U_i(\lambda) = \log R_i(\lambda_i) - \log T_i(\lambda). \tag{4}$$

If R_i is increasing in λ_i and if $T_i(\lambda)$ is increasing in λ_j for all j ($\neq i$), the above assumptions $\Psi1$ and $\Psi2$ hold. Such utilities have already appeared in the literature (see for example, Haurie and Marcotte (1985)). □

3 Flow Control in Networks

3.1 Assumptions on Networks

Consider a communication network modeled by an open product-form network of m state-independent queues, $k = 1, 2, \ldots, m$ (that model communication links, or, simply, links) (Baskett *et al.*, 1975). Define $\boldsymbol{m} = (1, 2, \ldots, m)$. The vertices or nodes connected by links model the routers of the communication network. There are n independent users, $1, 2, \ldots, n$ as before. User i decides the rate λ_i of packets to pass through a communication network so that the utility, U_i, of the user i may be maximum. $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$. T_i is the average passage time of the packets in control of user i.

μ_{ik} is the state-independent service rate of user-i packets at link k. In this article, it is assumed that each router (or, node) has a sufficient capacity of storing packets, and, thus, losses of packets may not occur.

q_{ik} is the resulting visit rate of user-i packets to link k. That is, q_{ik}, for all i, k, is the solution of the following system of equations:

$$q_{ik} = p^i_{0k} + \sum_l q_{il} p^i_{lk},$$

where p^i_{lk} and p^i_{0l}, respectively, are the probabilities that a user-i packet goes to link k after leaving link l and when entering the system. Then, if user i injects the rate λ_i of packets into the network, user-i packets visit link i at the rate of $q_{ik}\lambda_i$. User i injects the rate, $p^i_{0k}\lambda_i$, of packets into link k from the outside of the network. User-i packets departing from link k leave the network at the frequency (or, probability) q^i_{k0}. That is, the network has multiple ports of entry and of exit. Consider the case where the mean response time for a user-i packet to pass through link k, $T^{(k)}_i$ is

$$T^{(k)}_i = \mu^{-1}_{ik} T^{(k)} \text{ and } T^{(k)} = \frac{1}{1 - s_l \sum_p q_{pk}\lambda_p/\mu_{pk}}, \tag{5}$$

where s_l is 1 for a link modeled by a single-server, $1/h$ for a link consisting of h parallel channels each of which is chosen with probability $1/h$ and is modeled by a single server, and 0 for a link modeled by an infinite server, for $1 - s_l \sum_p q_{pk}\lambda_p/\mu_{pk} > 0$ (Baskett *et al.*, 1975). Denote $\boldsymbol{k} = \{l | s_l \neq 0\}$. Then, using the Little's result,

$$T_i(\lambda) = \sum_{l \in \boldsymbol{k}} \frac{Q_{il}}{1 - s_l \sum_p Q_{pl}\lambda_p} + \sum_{l \in \boldsymbol{m}-\boldsymbol{k}} Q_{il}, \text{ if } 1 - s_l \sum_p Q_{pl}\lambda_p > 0 \text{ for all } l, \text{ otherwise infinite,} \tag{6}$$

$$\text{where } Q_{il} = \frac{q_{il}}{\mu_{il}}.$$

Clearly, $T_i(\lambda)$ is increasing in λ. Since $\sum_{l \notin \boldsymbol{k}} Q_{il}$ is constant and independent of the strategy, we only consider the case where all links are in $\boldsymbol{k}$. In order that the statistical equilibrium of this network should be attained, it must hold that $\lambda \in \boldsymbol{C}$, where the feasible region $\boldsymbol{C}$ is

$$\boldsymbol{C} = (\lambda \mid \lambda_i \geq 0, i \in \boldsymbol{n}, 1 - s_l \sum_p Q_{pl}\lambda_p > 0, l \in \boldsymbol{k}). \tag{7}$$

Furthermore, define regions $\bar{\boldsymbol{C}}$ and $\hat{\boldsymbol{C}}$ such that

$$\bar{\boldsymbol{C}} = (\lambda \mid \lambda_i > 0, i \in \boldsymbol{n}, 1 - s_l \sum_p Q_{pl}\lambda_p > 0, l \in \boldsymbol{k}), \tag{8}$$

$$\hat{\boldsymbol{C}} = (\lambda \mid \lambda_i \geq 0, i \in \boldsymbol{n}, 1 - s_l \sum_p Q_{pl}\lambda_p \geq 0, l \in \boldsymbol{k}). \tag{9}$$

$\hat{\boldsymbol{C}} - \bar{\boldsymbol{C}}$ comprises the boundary consisting of $n + k$ hyperplanes each with $(n-1)$-dimensions, n from $\lambda_i = 0$, $i \in \boldsymbol{n}$, and k from $1 - s_l \sum_p Q_{pl}\lambda_p = 0$, $l \in \boldsymbol{k}$. We call the part of the boundary consisting of $\lambda_i = 0$, the $(i-0)$ policy boundary, and the part of boundary which is not any of $(i-0)$ policy boundary, $i \in \boldsymbol{n}$, the capacity boundary. We also define for convenience,

$$A_{il} = 1 - s_l \sum_{p \neq i} Q_{pl}\lambda_p. \tag{10}$$

3.2 Noncooperative Flow Control with Power as the Objective

The power is defined as $P_i = \lambda_i/T_i$ for a user-i packet. In this subsection, we consider the case where the utility, U_i, of user i depends only on and increases monotonically in its power, P_i. From (6), $P_i(\lambda)$ is defined for all $\lambda \in \boldsymbol{L}$, and $P_i(\lambda) = 0$ for $\lambda \in \boldsymbol{L} - \bar{\boldsymbol{C}}$ and $i \in \boldsymbol{n}$. Furthermore, from (6), for $\lambda_i > 0$ and $1 - s_l \sum_p Q_{pl}\lambda_p > 0$ for all l,

$$P_i^{-1}(\lambda) = \frac{T_i(\lambda)}{\lambda_i} = \sum_l \frac{Q_{il}}{\lambda_i(1 - s_l \sum_p Q_{pl}\lambda_p)} \tag{11}$$

$$= \sum_l \Big(\frac{1}{\lambda_i} + \frac{s_l Q_{il}}{1 - s_l \sum_p Q_{pl}\lambda_p}\Big)\frac{Q_{il}}{1 - s_l \sum_{p \neq i} Q_{pl}\lambda_p}. \tag{12}$$

Clearly, P_i^{-1} is convex in λ. The partial differential coefficients of P_i^{-1} are the following:

$$\frac{\partial}{\partial \lambda_i} P_i^{-1} = -\frac{1}{\lambda_i^2}\Big(\sum_l \frac{Q_{il}}{1 - s_l \sum_{p \neq i} Q_{pl}\lambda_p}\Big) + \sum_l \frac{s_l^2 Q_{il}^3}{(1 - s_l \sum_{p \neq i} Q_{pl}\lambda_p)(1 - s_l \sum_p Q_{pl}\lambda_p)^2} \tag{13}$$

$$= -\frac{1}{\lambda_i^2}\Big(\sum_l \frac{Q_{il}}{A_{il}}\Big) + \sum_l \frac{s_l^2 Q_{il}^3}{A_{il}(A_{il} - s_l Q_{il}\lambda_i)^2}. \tag{14}$$

(13) is derived from (12). It can be seen that, since both the first and second terms of the right-hand side of (14) is increasing in λ_i, given λ_j, for all $j \neq i$, there is a unique value, $\check{\lambda}_i$, of λ_i that makes (14) to be zero and, thus, maximizes the power P_i, that is, from (14),

$$\sum_l \frac{Q_{il}}{A_{il}} = \sum_l \frac{Q_{il}}{A_{il}}\Big(\frac{A_{il}}{s_l Q_{il}\check{\lambda}_i} - 1\Big)^{-2}, \tag{15}$$

$$0 < \check{\lambda}_i < \min_l\{A_{il}/(s_l Q_{il})\}\}. \tag{16}$$

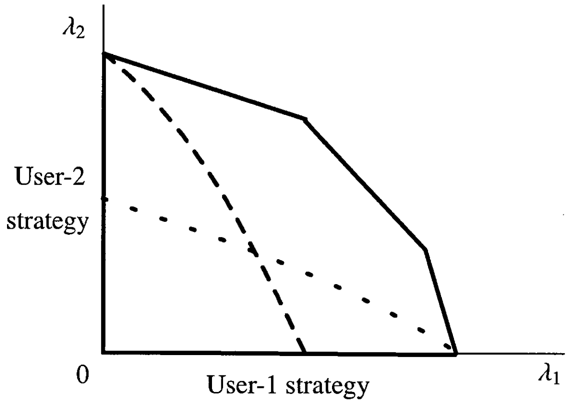

Figure 1: A Nash equilibrium point

It is evident that such a value of $\check{\lambda}_i$ satisfies $1 - s_l \sum_{p \neq i} Q_{pl}\lambda_p - s_l Q_{il}\check{\lambda}_i > 0$ for all l. Denote by $\check{\lambda}_i(\lambda^{(-i)})$ the value of λ_i that maximizes P_i given the values of the other λ_j for all $j \neq i$. Then, from this, we see that $(\check{\lambda}_i(\lambda^{(-i)}), \lambda^{(-i)}) \in \boldsymbol{C}$. Furthermore, from (15),

$$\sum_l \frac{Q_{il}A'_{il}}{A_{il}^2}\left\{1 + \frac{1 + \dfrac{A_{il}}{s_l Q_{il}\check{\lambda}_i}}{\left(\dfrac{A_{il}}{s_l Q_{il}\check{\lambda}_i} - 1\right)^3}\right\} = \left\{\sum_l \frac{2/s_l}{\left(\dfrac{A_l}{s_l Q_{il}\check{\lambda}_i} - 1\right)^3}\right\}\frac{\check{\lambda}'_i}{\check{\lambda}_l^2}. \tag{17}$$

Then, from (10) and (15), we see that as any of other λ_j, $j \neq i$, increases, $\check{\lambda}_i$ decreases.

Clearly, the value of $\tilde{\lambda} = (\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_n)$ that satisfies the above for all i is a Nash equilibrium. Denote by $\mathbf{0}$ a vector for which all the elements have the value, zero. We can see that $(\check{\lambda}_i(\lambda^{(-i)}), \lambda^{(-i)})$, $\lambda \in \boldsymbol{C}$, comprises an $(n-1)$-dimensional hyper-surface that connects the point $(\check{\lambda}(\mathbf{0}^{(-i)}), \mathbf{0}^{(-i)})$ and $(n-2)$-dimensional hyper-surface that is the intersection of the $(i-0)$ policy boundary and the capacity boundary. In total, there exist n of these $(n-1)$-dimensional hyper-surfaces, one for each $i (\in \boldsymbol{n})$, and the intersection of all these hyper-surfaces will be one (and possibly only one) point, which is a Nash equilibrium. Fig. 1 illustrates a case where $n = 2$ and $k = 3$. The solid lines show the boundary. The dashed curve consists of the points $(\check{\lambda}_1, \lambda_2)$ where each $\check{\lambda}_1$ is the strategy optimal to user 1, given the strategy of user 2, λ_2. The dotted curve consists of the points $(\lambda_1, \check{\lambda}_2)$ where each $\check{\lambda}_2$ is the strategy optimal to user 2, given the strategy of user 1, λ_1. The intersection of the dashed and dotted curves shows a Nash equilibrium point. A proof of the existence of a Nash equilibrium, $\tilde{\lambda} \in \boldsymbol{C}$, based on the Kakutani fixed-point theorem (Kakutani, 1941) is presented in the Appendix.

Theorem 2 *Nash equilibria for flow control where each user optimizes its power are always srtongly Pareto inefficient.*

[Proof] Consider the situation where reducing the values of all elements of $\lambda(t)$ from $\tilde{\lambda}$. Note that, from (11),

$$\frac{d}{dt}P_i^{-1} = \frac{\partial P_i^{-1}}{\partial \lambda_i}\frac{d\lambda_i}{dt} + \sum_{q \neq i} \frac{\partial P_i^{-1}}{\partial \lambda_q}\frac{d\lambda_q}{dt}, \tag{18}$$

$$\text{where } \frac{\partial P_i^{-1}}{\partial \lambda_q} = \sum_l \left\{\frac{s_l Q_{il} Q_{ql}}{\lambda_i (1 - s_l \sum_p Q_{pl}\lambda_p)^2}\right\}.$$

Note, for all i, that $\left.\dfrac{\partial P_i^{-1}}{\partial \lambda_i}\right|_{\lambda(t)=\tilde{\lambda}} = 0$ (since $\tilde{\lambda}$ is a Nash equilibrium), and that the coefficient, $\dfrac{\partial P_i^{-1}}{\partial \lambda_q}$, of $\dfrac{d\lambda_q}{dt}$ for all $q \neq i$ in the second term of (18) is positive. If $(-P_i^{-1})$ is considered as the utility, U_i, of player i, then Assumptions $\Psi 1$ and $\Psi 2$ hold. Therefore, from Theorem 1 follows this theorem. □

Remark 2 It is to be noted that the powers of all users improve by reducing the throughputs, λ, of all users from the Nash equlibrium coincidently. □

3.2.1 Flow Control in a Category of Networks

Consider, in particular, the case where q_{ik}/μ_{ik} $(= Q_{ik}) = \gamma_k/\mu_i$ for some γ_k, for all i, k. This can be satisfied, for example, when there exist δ_{ik} such that $\mu_{ik} = \delta_{ik}\mu_i$ and $q_{ik} = \delta_{ik}\gamma_k$, for all i, k. Define $\rho_i = \lambda_i/\mu_i$ and $\rho = \sum_p \rho_p$. Then, from the assumptions on networks given in Subsection 3.1, we have

$$T_i = \frac{1}{\mu_i} D(\rho), \qquad D(\rho) = \sum_{l \in k} \frac{\gamma_l}{1 - s_l \gamma_l \rho}. \tag{19}$$

Then, we have the following property.

Lemma 1 *For $D(\rho)$ given by* (19) *and* $0 \leq \rho < \min_k\{1/(s_k\gamma_k)\}$, *$D(\rho)/D'(\rho)$ is decreasing in ρ.*

[Proof] Note that

$$D'(\rho) = \sum_{l \in k} s_l \left(\frac{\gamma_l}{1 - s_l \gamma_l \rho}\right)^2. \tag{20}$$

Without losing generality, $k = 1, 2, \ldots, m$, can be renumbered such that $s_1\gamma_1 = \min_{l \in k}\{s_l\gamma_l\}$. Then,

$$D(\rho) = \sum_{l \in k} \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2 \frac{1 - s_l\gamma_l\rho}{\gamma_l} + \sum_{l \notin k} \gamma_l$$

$$= \frac{1 - s_1\gamma_1\rho}{s_1\gamma_1} \sum_{l \in k} s_l \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2 + \sum_{l \in k, l \neq 1} s_l \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2 \left(\frac{1}{s_l\gamma_l} - \frac{1}{s_1\gamma_1}\right) + \sum_{l \notin k} \gamma_l.$$

Therefore,

$$\frac{D(\rho)}{D'(\rho)} = \frac{\sum_{l \in k} \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2 \frac{1 - s_l\gamma_l\rho}{\gamma_l} + \sum_{l \notin k} \gamma_l}{\sum_{l \in k} s_l \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2}$$

$$= \frac{1 - s_1\gamma_1\rho}{s_1\gamma_1} + \frac{\sum_{l \in k, l \neq 1} s_l \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2 \left(\frac{1}{s_l\gamma_l} - \frac{1}{s_1\gamma_1}\right) + \sum_{l \notin k} \gamma_l}{\sum_l s_l \left(\frac{\gamma_l}{1 - s_l\gamma_l\rho}\right)^2}.$$

Thus, by noting that $s_1\gamma_1 \leq s_k\gamma_k$ for all k, we see that $D(\rho)/D'(\rho)$ is decreasing in ρ. □

From the above, we have

Proposition 1 *For $D(\rho)$ given by* (19) *and* $0 \leq \rho < \min_{l \in k}\{1/(s_l\gamma_l)\}$, $\log D(\rho)$ *is increasing and convex in ρ.*

[Proof] From (19) and Lemma 1, follows this proposition. □

Then, the following assumption is satisfied.

Assumption Π1 *Given λ, T_i is given by a function $D(\rho)$ as follows:*

$$T_i = \frac{1}{\mu_i} D(\rho), \quad \textit{where } \rho = \sum_p \rho_p, \quad \textit{and } \rho_i = \frac{\lambda_i}{\mu_i}. \tag{21}$$

$D(\rho)$, defined for $\rho \geq 0$, satisfies the following: $D(0) = 1$, $D(\rho)$ is increasing, and $\log D(\rho)$ *is convex.*

Noncooperative flow control with power as the objective If Assumption Π1 holds, additional properties can be derived. From Assumption Π1,

$$P_i = \frac{\lambda_i}{T_i} = \frac{\lambda_i \mu_i}{D(\rho)} = \frac{\mu_i^2 \rho_i}{D(\rho)}. \tag{22}$$

From (22)

$$\frac{\partial}{\partial \rho_i}(\log P_i) = \frac{1}{\rho_i} - \frac{D'(\rho)}{D(\rho)}, \quad \text{for } \rho_i > 0. \tag{23}$$

Thus, the set of the values $\tilde{\rho}_i$ of ρ_i s.t. $\tilde{\rho}_i = D(\tilde{\rho})/D'(\tilde{\rho})$, for all i, is a Nash equilibrium, where

$$\tilde{\rho} = \frac{nD(\tilde{\rho})}{D'(\tilde{\rho})}. \tag{24}$$

Then, the noncooperative optimum flow for user i is $\tilde{\lambda}_i = \mu_i \tilde{\rho}_i$.

Cooperative flow control Consider an overall measure, $O = \sum_i \mu_i^{-2} P_i$. Then, from (22),

$$O = \sum_i \mu_i^{-2} P_i = \frac{\rho}{D(\rho)}, \tag{25}$$

$$\frac{\partial}{\partial \rho}(\log O) = \frac{1}{\rho} - \frac{D'(\rho)}{D(\rho)}, \quad \text{for } \rho > 0. \tag{26}$$

Then, an overall optimum for this overall measure O is given by such a value $\hat{\rho}$ of ρ that

$$\hat{\rho} = \frac{D(\hat{\rho})}{D'(\hat{\rho})}. \tag{27}$$

There are distinct sets of flows for users that results in $\hat{\rho}$ and achieves this cooperative optimum. One set of flows for users that gives the cooperative optimum is given by $\lambda_i = \mu_i \hat{\rho}/n$ and $\hat{\rho}_i = \hat{\rho}/n$.

Noncooperative vs. cooperative flow control Denote by $\tilde{P}_i$ and $\hat{P}_i$ the powers of user i in noncooperative and cooperative flow control, respectively. The following property holds in the setting of the model.

Theorem 3 *There exists a unique Nash equilibrium of noncooperative flow control, and it is always strongly Pareto inferior to the cooperative optimum defined above, that is, for all i, $\tilde{P}_i < \hat{P}_i$. The power of each user in the cooperative optimum is proportionate to that in the Nash equilibrium, that is, $\hat{P}_i = K\tilde{P}_i$, for some constant $K > 1$, for all i.*

[Proof] Since $\log D(\rho)$ is convex and increasing by Assumption Π1, $D'(\rho)/D(\rho)$ is nondecreasing in ρ. Note also that $D'(\rho)/D(\rho) > 0$ for $\rho \geq 0$, from the assumption Π1. Thus, $D(\rho)/D'(\rho) > 0$ is nonincreasing for $\rho \geq 0$. From (23) and (26), respectively, follows that there exist unique $\tilde{\rho}$ and $\hat{\rho}$. Clearly, from (24) and (27), $\tilde{\rho} > \hat{\rho}$, and $\tilde{\rho}$ and thus $\tilde{\rho}/\hat{\rho}$ increases as n increases. $\tilde{P}_i = \mu_i^2/D'(\tilde{\rho})$ and $\hat{P}_i = \mu_i^2/D'(\hat{\rho})$. Then, for all i,

$$\hat{P}_i = K\tilde{P}_i, \quad K = \frac{D'(\tilde{\rho})}{D'(\hat{\rho})} > 1. \ \square$$

Remark 3 It may be said that the cooperative optimum achieves the Nash equilibrium proportionately. □

Define $K_i \triangleq \hat{P}_i/\tilde{P}_i$. Then, $K_i = K = D'(\tilde{\rho})/D'(\hat{\rho})$. K is regarded as the degree of Pareto superiority of the cooperative optimum over the Nash equilibrium.

From (24), as n increases, $\tilde{\rho}$, and, thus, $D'(\tilde{\rho})$ increases, while $\hat{\rho}$ and, thus, $D'(\hat{\rho})$ remain the same, as seen from (27). Thus, $K_i = K$ increases as n increases, which means the following.

Proposition 2 *The degree of Pareto superiority of the cooperative flow control over the Nash equilibrium of noncooperative flow control increases as the number of independent users increases.*

3.2.2 A Special Case: Series-Parallel Channels

Consider the case where the network is regarded as σ parallel paths each of which consists of a series of κ identical links, that is, series-parallel queues. A new random choice of a path is made by each user for each packet with an equal probability $1/\sigma$ ($\kappa\sigma = m$) where choices are made independently of past choices.

$$D(\rho) = \frac{\kappa}{1 - \rho/\sigma}. \tag{28}$$

Clearly, the $D(\rho)$ given by (28) satisfies the assumption Π1. Then, from (28), $D/D' = \sigma - \rho$. Then, for the noncooperative flow control,

$$\tilde{\rho} = \sigma n/(n+1),\ \tilde{\rho}_i = \sigma/(n+1).$$
$$\text{Therefore, } \tilde{P}_i = \mu_i^2\sigma/(n+1)^2.$$

For the cooperative flow control,

$$\hat{\rho} = \sigma/2. \text{ Then, the optimum can be}$$
$$\text{achieved by } \hat{\rho}_i = \sigma/(2n). \text{ Then } \hat{P}_i = \mu_i^2\sigma/(4n).$$

Thus, $K_i = \hat{P}_i/\tilde{P}_i = (n+1)^2/(4n)$, and $K_i = K > 1$ for $n \geq 2$, $K \to \infty$ $(n \to \infty)$. Note, in passing, that

$$\tilde{\rho}_i = \frac{\sigma}{n+1},\ \hat{\rho}_i = \frac{\sigma}{2n},\ \text{ and, therefore, } \frac{\hat{\lambda}_i}{\tilde{\lambda}_i} = \frac{n+1}{2n} < 1,$$

$$D(\tilde{\rho}) = \kappa(n+1), D(\hat{\rho}) = 2\kappa,\ \text{ and, therefore, } \frac{\hat{T}_i}{\tilde{T}_i} = \frac{2}{n+1} > 1.$$

Thus, in the cooperative optimum, each user injects less flow and has better responsiveness than in the Nash equilibrium. Some numerical examples are as in the following. Recall that n is the number of users.

For 9 users, in the Nash equilibrium, each user injects the rate of packets of 1.8 times, and receives the average packet-passage time of 5 times and the power of 0.36 times as large as those in the cooperative optimum.

For 99 users, in the Nash equilibrium, each user injects the rate of packets of 1.98 times, and receives the average packet-passage time of 50 times and the power of 0.0396 times as large as those in the cooperative optimum.

For 999 users, in the Nash equilibrium, each user injects the rate of packets of 1.998 times, and receives the average packet-passage time of 500 times and the power of 0.003996 times as large as those in the cooperative optimum.

3.2.3 A Flow-Control Version of the Braess-like Paradox

By the paradox, we mean that situation where the benefit of every user in a Nash equlibrium after adding connections to a system is less than that before adding it (Braess, 1968). Note that the original Braess paradox has been considered in the context of network routing. We show here a paradox similar to the Braess paradox can also occur in flow control.

Consider the system where there are n users and n paths. Consider two cases [A] and [B]: In the case [B] (before adding connections) each of n users uses only one path dedicated to it. In the case [A], n users shares the use of n paths. Intuitively, the performance of the system in the case (A) may not be worse than in the case (B), at least, but, in fact, it may be so in noncooperative flow control as shown in the following. Note that case [A] is identical to the case of Subsection 3.2.2 where $\sigma = n$ paths are commonly used by n users, and that case [B] is the situation where $\sigma = n$ paths are separated with each path being used by one user only. Clearly, the power of user i in the case [B] is, $\tilde{P}_i = \mu_i^2/4$, which happens to be the same as the situation of the cooperative flow control. The power of user i in the case [A] is $\tilde{P}_i = \mu_i^2\sigma/(\sigma+1)^2$.

That is, for all users, the ratio, $1/K$, of the power after adding connections ([A]) to that before ([B]) is $4\sigma/(\sigma+1)^2$, which is 8/9 for $\sigma = 2$, is 3/4 for $\sigma = 3$, is 0.36 for $\sigma = 9$, is 0.0396 for $\sigma = 99$, *etc.* Therefore, each user has less power after adding connections ([A]) than before adding them ([B]), which may look paradoxical.

3.3 Noncooperative Flow Control with Additive Costs

In this subsection, we briefly touch on another case of each user's objective. That is, a common utility function used in flow control is the sum of two components: The first corresponds to some function of the throughput, and the second to some cost. More precisely, consider the network described in Section 3.1, and assume that the cost per packet over link k is given by the function $(1/\mu_{ik})T^{(k)}(\rho_k)$ (given by (5)) where

$$\rho_k = \sum_p \rho_{pk}, \quad \rho_{ik} = Q_{ik}\lambda_i.$$

The total cost payed by player i is thus

$$J_i(\lambda) = \lambda_i T_i = \sum_l \rho_{il} T^{(l)}(\rho_l).$$

The utility for player i is then given by

$$U_i(\lambda) = R_i(\lambda_i) - a_i J_i(\lambda),$$

where R_i is concave in its argument and a_i is a positive constant. Utilities with the above structure are common in telecommunication networks (see, for example, Alpcan and T. Başar (2002; 2003) that study special cases of such utilities).

Clearly, given the strategies, $\lambda^{(-i)}$, of other users, user i optimizes U_i by choosing its strategy $\check{\lambda}_i$, which is unique given $\lambda^{(-i)}$, and

$$0 \le \check{\lambda}_i < \min_l \{A_{il}/(s_l Q_{il})\}. \tag{29}$$

If we have $\tilde{\lambda}$ such that, given $\tilde{\lambda}^{(-i)}$ as $\lambda^{(-i)}$, $\check{\lambda}_i = \tilde{\lambda}_i$ holds for all i, $\tilde{\lambda}$ is a Nash equilibrium. In the Appendix, we show the existence of a Nash equilibrium, $\tilde{\lambda} \in \boldsymbol{C}$, based on the Kakutani fixed-point theorem.

Since $T^{(k)}$ is strictly increasing in its argument for all k, then Ψ1 holds. Therefore, from Theorem 1, it is seen that, if more than one user has the positive $\tilde{\lambda}_i$ in a Nash equilibrium, Ψ2 holds as well, and it is strongly Pareto inefficient.

4 Another Example: Uplink Power Control in CDMA

This application is taken from Alpcan *et al.* (2002). There are n mobiles, and mobile i has to determine its transmission power λ_i, $i = 1, 2, \ldots, n$. The utility is additive with two components: the first is a utility that is a function of the signal to interference ratio, and the second is proportional to the consumed power. More precisely, it is a function of the ratio between the power received at the base station from station i and the total noise received: the interference from other mobiles plus a thermal noise. Thus, the utility is given by

$$U_i(\lambda) = f_i(\gamma_i) - \alpha_i\lambda_i, \quad \text{where } \gamma_i = L\frac{h_i\lambda_i}{\sum_{j\ne i} h_j\lambda_j + \sigma^2}.$$

Here, h_j is the attenuation between mobile j and the base station, L is called the spreading gain factor and σ^2 is the thermal noise. f_i is assumed to be increasing in its argument. Alpcan *et al.* (2002) consider the case where

$$f_i(\gamma_i) = u_i \ln(1+\gamma_i). \tag{30}$$

With this choice of f_i, this utility is proportional to the Shannon capacity for user i (if we make the simplifying assumption that the noise plus the interference of all other users constitute an independent Gaussian noise) and can thus be interpreted as the throughput that user i can achieve with a given power. The existence of a Nash equilibrium for this system has been shown (Alpcan *et al.*, 2002).

In this system, $\Psi 1$ is clearly satisfied. Therefore, Theorem 1 shows that, if more than one mobile has the positive power in a Nash equilibrium, $\Psi 2$ holds, and it is strongly Pareto inefficient.

Similar models of the power control in wireless communications can be found (Famolari *et al.*, 1999; Saraydar *et al.*, 2001; Saraydar *et al.*, 2002; Ji and Huang, 1998). We can see that the models of these papers satisfy Assumption $\Psi 1$. Therefore, Theorem 1 implies that, in the situation where Assumption $\Psi 2$ holds for them, the Nash equilibria of these models are Pareto inefficient.

5 Concluding Remarks

In this article, a general framework of strongly Pareto-inefficient Nash equilibria in noncooperative games competing common-pool resources is presented. Some examples of such noncooperative games in communication networking are given. In particular, it is shown that the noncooperative flow control for which each user optimizes its power has the strongly Pareto-inefficient Nash equilibrium, the existence of which is shown on the basis of the Kakutani fixed-point theorem. Furthermore, it is also shown that, in some flow-control games, the degree of Pareto inefficiency of the Nash equilibrium in noncooperative flow control can increase without bound as the number of users increases. These observations anticipate the possibility that the paradox like the Braess one may also occur in such systems modeled by the noncooperative game including flow control, for which we show an example. Examples examined include another flow-control setting with additive costs and the power control in wireless communications.

Appendix. A Proof of the Existence of a Nash Equilibrium in Noncooperative Flow Control

In this appendix, we give a proof of the existence of a Nash equilibrium in noncooperative flow control as given in Subsections 3.2 and 3.3. We consider the utility function $\hat{U}_i(\lambda) = \exp\{U_i\} = \exp\{R_i(\lambda_i) - a_i J_i(\lambda)\}$ for Subsection 3.3. Then, both utility functions, $P_i(\lambda)$ for Subsection 3.2 and $\hat{U}_i(\lambda)$ for Subsection 3.3, have non-negative finite values for $\boldsymbol{\lambda} \in \boldsymbol{C}$ and the value zero for $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}} - \boldsymbol{C}$. In the following part 1), we first show that there exist a Nash equilibrium in region $\hat{\boldsymbol{C}}$ with the above utility functions. Then, in part 2), we show that such a Nash equilibrium is in region $\boldsymbol{C}$, which is a really feasible region considering the achievability of the statistical equilibrium of the systems considered.

1) Consider the following function, ϕ_i, for arbitrary i, whose domain is $\hat{\boldsymbol{C}}$, defined as follows. Given $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}}$, the function ϕ_i gives the $\check{\lambda}_i$ as follows with other $\boldsymbol{\lambda}^{(-i)}$ being unchanged: $\check{\lambda}_i$ is uniquely given, if $A_{il} > 0$ for all l, by (15) for Subsection 3.2 and by the statement above (29) for Subsection 3.3. and, $\check{\lambda}_i = 0$, if $A_{il} = 0$ for some l (that is, $\lambda_i = 0$ and $1 - s_l \sum_p Q_{pl}\lambda_p$, for some l). Note that, in the case where $A_{il} > 0$ for all l, $\check{\lambda}_i$ is determined to be the same value, regardless of whether $\hat{U}_i$ or U_i is used for the utility of user i for Subsection 3.3. In the case where $A_{il} = 0$ for some l, $\check{\lambda}_i$ is determined independently of the shape of the utility function of user i.

From (15) for Subsection 3.2 and from the statement above (29) for Subsection 3.3. it is clearly seen that, for $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}}$, such that $A_{il} > 0, l \in \boldsymbol{k}$, ϕ_i is a continuous function of $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}}$ to $(\check{\lambda}_i, \boldsymbol{\lambda}^{(-i)}) \in \hat{\boldsymbol{C}}$. Furthermore, from (16) for Subsection 3.2 and (29) for Subsection 3.3, as $A_{il} \to 0$ for an arbitrary l with $\boldsymbol{\lambda}$ remaining in $\hat{\boldsymbol{C}}$, $\check{\lambda}_i \to 0$. For such $\boldsymbol{\lambda}$ that $A_{il} = 0$ for some l, $\check{\lambda}_i$ keeps to be 0 while $\boldsymbol{\lambda}$ remains in $\hat{\boldsymbol{C}}$. Therefore, ϕ_i is a continuous function of $\boldsymbol{\lambda}$, for $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}}$ and $i \in \boldsymbol{n}$. Thus, ϕ_i is a continuous function of $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}}$ into $\hat{\boldsymbol{C}}$.

Consider a function $\phi = \phi_1 \circ \phi_2 \circ \cdots \circ \phi_n$. From the above, we see that ϕ is a continuous function and maps $\boldsymbol{\lambda} \in \hat{\boldsymbol{C}}$ to $\check{\boldsymbol{\lambda}} \in \hat{\boldsymbol{C}}$. By noting that $\hat{\boldsymbol{C}}$ is a closed compact set, from the Kakutani fixed point theorem (Kakutani, 1941), the function ϕ has a fixed point $\tilde{\boldsymbol{\lambda}}$ such that $\phi(\tilde{\boldsymbol{\lambda}}) = \tilde{\boldsymbol{\lambda}}$, $\tilde{\boldsymbol{\lambda}} \in \hat{\boldsymbol{C}}$. We can easily see that, if $\phi(\tilde{\boldsymbol{\lambda}}) = \tilde{\boldsymbol{\lambda}}$, then $\phi_1(\tilde{\boldsymbol{\lambda}}) = \phi_2(\tilde{\boldsymbol{\lambda}}) = \cdots = \phi_n(\tilde{\boldsymbol{\lambda}}) = \tilde{\boldsymbol{\lambda}}$. Thus, if $\phi(\tilde{\boldsymbol{\lambda}}) = \tilde{\boldsymbol{\lambda}}$, $\tilde{\boldsymbol{\lambda}}$ is a Nash equilibrium.

2) Clearly, λ, such that $1 - s_l \sum_p Q_{pl}\lambda_p = 0$ for some l, cannot be such a fixed point. Indeed, such λ gives zero utilities ($P_i = 0$ and $\hat{U}_i = 0$ for all i), and user i such that $\lambda_i > 0$ could increase its utility by decreasing its λ_i (that is, if $\lambda_i > 0$, then it must hold that $A_{il} > 0$ for all l. Then, from (16) for Subsection 3.2 and (29) for Subsection 3.3, $\check{\lambda}_i$ mapped from λ must be such that $1 - s_l \sum_{p \neq i} Q_{pl}\lambda_p - s_l Q_{il}\check{\lambda}_i = A_{il} - s_l Q_{il}\check{\lambda}_i > 0$, and, thus, is less than λ_i). Therefore, a fixed point of ϕ, $\tilde{\lambda}$, exists and $\tilde{\lambda} \in \boldsymbol{C}$, which is a Nash equilibrium of the noncooperative flow control.

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