A PARADOX IN DISTRIBUTED OPTIMIZATION OF PERFORMANCE

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A Paradox in Distributed Optimization of Performance

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Abstract

We report the existence of paradoxical cases, in load balancing, where adding the communication capacity to the system leads to unlimitedly large performance degradation in an intermediately distributed performance optimization. In these cases such paradoxical performance degradation occurs neither in the completely centralized optimization nor in the completely distributed optimization. The degradation reduces and finally disappears as the optimization decision becomes more and finally completely distributed. We study a simple model of two parallel identical servers each of which has its own queue and the identical arrival. It is notable that we can find paradoxes that may bring unlimitedly large performance degradation in such a simple and common model.

keywords Distributed decision, Braess paradox, Nash equilibrium, Wardrop equilibrium, performance optimization, parallel queues, load balancing.

1 Introduction

We may have various objectives for performance optimization in many systems including communication networks, distributed computer systems, transportation flow networks, etc. Among them, we have three typical objectives or optima depending on the degree of the distribution of decision in performance optimization:

(1) [Completely centralized decision] The system optimizes the total cost or the mean response time of the entire system as a single performance measure. This optimized situation is called the system optimum, overall optimum, or social optimum. We call it the *overall optimum* here.

(2) [Completely distributed decision] Each of infinitely many individuals, users, or jobs optimizes its own cost or the expected response time for itself independently of others. In this optimized situation each of infinitely many individuals cannot receive any further benefit by changing its own decision. It is further assumed that the decision of a single individual has a negligible impact on the performance of other individuals. This optimized situation is called the individual optimum, Wardrop equilibrium, or user optimum (by some people). We call it the *individual optimum* or *Wardrop equilibrium* here.

(3) [Intermediately distributed decision] Each of a finite number ($N(>1)$) of users, classes, or players optimizes its own cost or the expected response time only for jobs

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of the class non-cooperatively. In this optimized situation each of a finite number of users, classes, or players cannot receive any further benefit by changing its decision. This optimized situation is called the class optimum, Nash non-cooperative equilibrium, or user optimum (by some other people). We call it the *class optimum* or *Nash equilibrium* here.

Note that (2) is reduced to (1) when the number of players reduces to 1 ($N = 1$) and approaches (3) when the number of players becomes infinitely many ($N \to \infty$) [6].

Intuitively, we can think that the total processing capacity of a system will increase when the capacity of a part of the system increases, and so we expect improvements in performance objectives accordingly in that case. The famous Braess paradox tells us that this is not always the case; i.e., increased capacity of a part of the system may sometimes lead to the degradation in the benefits of all users in an individual optimization or Wardrop equilibrium [3, 4, 5, 6]. We can expect that, in the class optimum or the Nash equilibrium a similar type of paradox occurs (with large N), whenever it occurs for the Wardrop equilibrium ($N \to \infty$). Indeed, Korilis et al. found examples wherein the Braess-like paradox appears in a Nash equilibrium where all user classes are identical in the same topology for which the original Braess paradox (for the Wardrop equilibrium) was in fact obtained [13, 14].

As it is known that the Nash equilibrium converges to the Wardrop equilibrium as the number of users becomes large [6], it is natural to expect the same type of paradox in the Nash equilibrium context (for a large number of players), whenever it occurs for the Wardrop equilibrium, although it never occurs in the overall optimum where the total cost is minimized.

Kameda et al. [8] have obtained, however, numerical examples where a paradox similar to Braess's appear in the Nash equilibrium but does not occur in the Wardrop equilibrium in the same environment. These cases look quite strange if we note that such a paradox should never occur in the overall optimum and if we regard the Nash equilibrium as an intermediate between the overall optimum and the Wardrop equilibrium. In particular, the numerical examples show that the increased capacity of a part of a system would degrade the benefits of all classes up to a few 10 percent, in a class optimum (Nash equilibrium) whereas it should not degrade the benefits of all classes at the same time in a Wardrop equilibrium in the same environment. (In the background of this work, it has been observed that increased capacity of a part of a system may lead to somewhat awkward behavior in terms of a system-wide measure, in a model of distributed computer system [8, 9, 20]. The methods and algorithms for obtaining the optima and the equilibria are described in [9, 11, 12, 15, 19].)

In this paper, we present an analytic study of a simple model of static load balancing between two identical servers each of which has an identical arrival and its own queue. Although the model and its analysis look simple, we would like to present the results since they look quite counter-intuitive to us and show that the ratio of the performance degradation in the paradoxical cases can be *unlimitedly large*. In the model studied, each server (or processor) has the identical arrival of jobs or customers and a communication means for forwarding jobs to be processed by the other server. It is intuitively clear that in the overall optimum, no forwarding of jobs should occur. In the individual optimum, no forwarding of jobs occurs also. In the class optimum, no forwarding of jobs occurs for some parameter setting. For some other parameter setting, however, in the class optimum (which is unique) mutual forwarding does occur, and the performance (the mean response time for each class) can be unlimitedly many times of that of no mutual forwarding. The ratio of performance degradation decreases and finally disappears as the number of classes increases unlimitedly. These situations look quite paradoxical and surprising to us, although we know the existence of the prisoners' dilemma and although it has been already shown that Nash equilibria of games with smooth payoff functions are generally Pareto-inefficient [2].

2 The Model and Assumptions

We consider a model consisting of two identical servers (nodes) and a communication means that connects both servers. Servers are numbered 1 and 2 (Fig. 1). Jobs (or customers) are classified into $2n$ classes $R_{ik}, i = 1, 2, k = 1, 2, \cdots, n$. Jobs of class R_{ik} arrive only at server i with identical rate scaled down to $1/n$, Out of each class arrival, the rate x_{ik} of jobs are forwarded upon arrival through the communication means to the other server j $(i \neq j)$ to be processed there. Therefore the remaining rate $1/n - x_{ik}$ of class R_{ik} jobs are processed at server i. We have $0 \leq x_{ik} \leq 1/n, \quad i = 1, 2$. We denote the vector $(x_{11}, x_{12}, \cdots, x_{1n}, x_{21}, x_{22}, \cdots, x_{2n})$ by $\boldsymbol{x}$. We denote the set of $\boldsymbol{x}$'s that satisfy the constraints by $\boldsymbol{C}$. Within these constraints, a set of values of x_{ik} $(i = 1, 2, k = 1, 2, \cdots, n)$ are chosen to achieve optimization. Thus the load β_i on server i is given by

$$\beta_i = 1 - \sum_l x_{il} + \sum_l x_{jl}, (i \neq j). \tag{1}$$

Then, the expected processing (including queueing) time $D_i(\beta_i)$ of a job that is processed at server i (or the cost function at server i), is

$$D_i(\beta_i) = \frac{1}{\mu - \beta_i} \text{ for } \beta_i < \mu \text{ (otherwise it is infinite)}. \tag{2}$$

(We have a simple assumption of the external time-invariant Poisson arrival for each class, and the exponentially distributed service times for each class jobs with identical service rate μ at both servers.)

As to the communication means, we consider two communication lines 1 and 2 separately for each server. One line i is used for forwarding of a job that arrives at server i. The expected communication time of a job arriving at server i and being processed at server j $(\neq i)$ is expressed simply as t, i.e. independent of the traffic and the job class and with no queueing delay.

We refer to the length of time between the instant when a job arrives at a server and the instant when a job leaves one of the servers after all processing and communication, if any, are over as *the response time* for the job.

Thus the expected response time of a class R_{ik} job that arrives at server i is

$$T_{ik}(\boldsymbol{x}) = n\{(\frac{1}{n} - x_{ik})T_{iik}(\boldsymbol{x}) + x_{ik}T_{ijk}(\boldsymbol{x})\}, \tag{3}$$

where

$$\begin{aligned} T_{iik}(\boldsymbol{x}) &= D_i(\beta_i), \quad \text{and} &(4)\\ T_{ijk}(\boldsymbol{x}) &= D_j(\beta_j) + t, \text{ for } j \neq i. &(5) \end{aligned}$$

(The above expressions hold, again, only for positive values of denominators, and are otherwise infinite.)

Then, the overall expected response time of a job that arrives at the system is

$$T(\boldsymbol{x}) = \frac{1}{2n} \sum_{i,k} T_{ik}(\boldsymbol{x}). \tag{6}$$

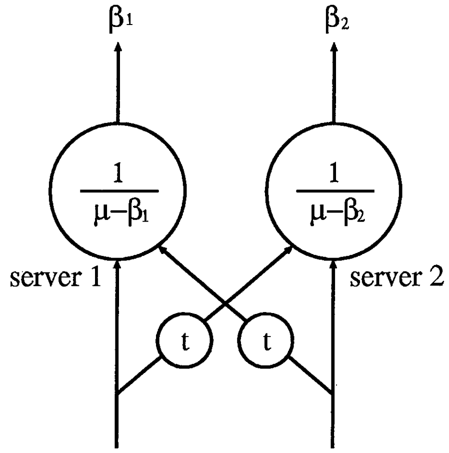

Figure 1: The system model.

3 The Results

We have three optima, the overall, the individual, and the class, as in the following.

(1) [Completely centralized optimization] The overall optimum is given by such $\bar{\boldsymbol{x}}$ as satisfies the following,

$$T(\bar{\boldsymbol{x}}) = \min T(\boldsymbol{x}) \quad \text{with respect to} \quad \boldsymbol{x} \in \boldsymbol{C}.$$

The solution $\bar{\boldsymbol{x}}$ is unique and simply given as follows:

$$\bar{\boldsymbol{x}} = \mathbf{0}, \text{ i.e. } x_{1k} = x_{2k} = 0 \text{ for all } k$$

and

$$T(\bar{\boldsymbol{x}}) = T_{ik}(\bar{\boldsymbol{x}}) = \frac{1}{\mu - 1}, \quad i = 1, 2, \quad k = 1, 2, \cdots, n.$$

This is intuitively clear. Or, this can be easily seen if we note that, since the oveall mean response time $T(\boldsymbol{x})$ is expressed as follows from (1), (2), (3), (4), (5), and (6):

$$2T(\boldsymbol{x}) = \frac{2\mu(\mu - 1)}{\{(\mu - 1)^2 - d^2\}} + st - 2,$$

where $d = \sum_l (x_{1l} - x_{2l})$ and $s = \sum_l (x_{1l} + x_{2l})$, $T(\boldsymbol{x})$ is minimum if and only if $x_{1k} = x_{2k} = 0$ for $k = 1, 2, \cdots, n$.

(2) [Completely distributed optimization] The individual optimum (or Wardrop equilibrium) is given by such $\hat{\boldsymbol{x}}$ as satisfies the following for all i, k

$$T_{ik}(\hat{\boldsymbol{x}}) = \min\{T_{iik}(\hat{\boldsymbol{x}}), T_{ijk}(\hat{\boldsymbol{x}})\} \ (i \neq j) \quad \text{such that} \quad \hat{\boldsymbol{x}} \in \boldsymbol{C}. \tag{7}$$

The solution $\hat{\boldsymbol{x}}$ is unique and given as follows:

$$\hat{\boldsymbol{x}} = \mathbf{0}, \text{ i.e. } \hat{x}_{1k} = \hat{x}_{2k} = 0, \quad \text{for all } k,$$

And, again,

$$T(\hat{\boldsymbol{x}}) = T_{ik}(\hat{\boldsymbol{x}}) = \frac{1}{\mu - 1}, \quad \text{for all } i, k,$$

This can be easily seen in the following way. The solution $\hat{\boldsymbol{x}}$ for (7) is characterized as follows:

$$\begin{array}{rclr} D_1(\hat{\beta}_1) & > & D_2(\hat{\beta}_2) + t, & \hat{x}_{1k} = 1 \quad (8) \\ D_1(\hat{\beta}_1) & = & D_2(\hat{\beta}_2) + t, & 0 \le \hat{x}_{1k} \le 1 \quad (9) \\ D_1(\hat{\beta}_1) & < & D_2(\hat{\beta}_2) + t, & \hat{x}_{1k} = 0 \quad (10) \\ D_2(\hat{\beta}_2) & > & D_1(\hat{\beta}_1) + t, & \hat{x}_{2k} = 1 \quad (11) \\ D_2(\hat{\beta}_2) & = & D_1(\hat{\beta}_1) + t, & 0 \le \hat{x}_{2k} \le 1 \quad (12) \\ D_2(\hat{\beta}_2) & < & D_1(\hat{\beta}_1) + t, & \hat{x}_{2k} = 0 \quad (13) \end{array}$$

for all k. We can easily see that these are satisfied if and only if $\hat{x}_{1k} = \hat{x}_{2k} = 0$ for all k, by noting that, e.g., (8) and (9) contradict with any of (11), (12), and (13), and thus that only (10) and (13) can hold together.

(3) [Intermediately distributed optimization] The class optimum (or Nash equilibrium) is given by such $\tilde{\boldsymbol{x}}$ as satisfies the following for all i, k,

$$T_{ik}(\tilde{\boldsymbol{x}}) = \min_{x_{ik}} T_{ik}(\tilde{\boldsymbol{x}}_{-(ik)}; x_{ik}), \quad \text{such that } (\tilde{\boldsymbol{x}}_{-(ik)}; x_{ik}) \in \boldsymbol{C}.$$

where $(\tilde{\boldsymbol{x}}_{-(ik)}; x_{ik})$ denotes the $2n$ vector in which the element corresponding to $\tilde{x}_{ik}$ has been replaced by x_{ik}.

(A) The case where $t > 1/\{n(\mu - 1)^2\}$: The solution $\tilde{\boldsymbol{x}}$ is unique and given as follows:

$$\tilde{\boldsymbol{x}} = \boldsymbol{0}, \text{ i.e. } \tilde{x}_{1k} = \tilde{x}_{2k} = 0, \quad \text{for all } k.$$

And, again,

$$T(\tilde{\boldsymbol{x}}) = T_{ik}(\tilde{\boldsymbol{x}}) = \frac{1}{\mu - 1}, \quad i = 1, 2, \quad k = 1, 2, \cdots, n.$$

(B) The case where $t \le 1/\{n(\mu - 1)^2\}$: The solution $\tilde{\boldsymbol{x}}$ is unique and given as follows:

$$\tilde{x}_{1k} = \tilde{x}_{2k} = \frac{1}{2}\{\frac{1}{n} - t(\mu - 1)^2\}, \quad \text{for all } k. \tag{14}$$

And in that case, we have

$$\begin{aligned} T(\tilde{\boldsymbol{x}}) &= T_{1k}(\tilde{\boldsymbol{x}}) = T_{2k}(\tilde{\boldsymbol{x}}) \\ &= \frac{1}{\mu - 1} + \frac{t}{2}\{1 - nt(\mu - 1)^2\}, \quad \text{for all } k. \end{aligned} \tag{15}$$

[Proof] From definitions (1), (2), and (3) we have

$$\begin{aligned} (\frac{1}{n})\frac{\partial T_{ik}}{\partial x_{ik}} &= -\frac{\mu - \frac{n-1}{n} + \sum_{l \ne k} x_{il} - \sum_{l} x_{jl}}{(\mu - 1 + \sum_{l} x_{il} - \sum_{l} x_{jl})^2} \\ &\quad + \frac{\mu - 1 - \sum_{l \ne k} x_{il} + \sum_{l} x_{jl}}{(\mu - 1 - \sum_{l} x_{il} + \sum_{l} x_{jl})^2} + t \quad (i \ne j). \end{aligned} \tag{16}$$

By simple inspection of (16), we see that $\frac{\partial T_{ik}}{\partial x_{ik}}$ is monotonically increasing with the increase in x_{ik} with feasible $\boldsymbol{x} \in \boldsymbol{C}$. Thus if we can find a set of such values of $\tilde{\boldsymbol{x}}$ that satisfies

$$\frac{\partial T_{ik}}{\partial x_{ik}}(\tilde{\boldsymbol{x}}) = 0, \text{ for all } i, k, \tag{17}$$

then the set of values is a solution of the class optimum. We have from (16) and defining $d = \sum_l (x_{1l} - x_{2l})$

$$\begin{aligned}
\sum_l \{(\frac{1}{n})\frac{\partial T_{1l}}{\partial x_{1l}} &- (\frac{1}{n})\frac{\partial T_{2l}}{\partial x_{2l}}\} \\
&= \frac{2n\mu - (2n-1)(1+d)}{(\mu - 1 - d)^2} - \frac{2n\mu - (2n-1)(1-d)}{(\mu - 1 + d)^2} \\
&= (\frac{2d}{(\mu-1)^2 - d^2})(\frac{2\mu(\mu-1)}{(\mu-1)^2 - d^2} + 2n - 1),
\end{aligned} \tag{18}$$

If condition (17) holds, then from (18), we have $d = 0$. Then from (16) we have

$$(\frac{1}{n})\frac{\partial T_{ik}}{\partial x_{ik}} = \frac{2x_{ik} - \frac{1}{n}}{(\mu - 1)^2} + t = 0, \quad (i \neq j) \text{ for all } i, k. \tag{19}$$

Therefore

$$x_{ik} = \frac{1}{2}(\frac{1}{n} - t(\mu - 1)^2) \text{ for all } i, k \quad \text{ if } t \leq \frac{1}{n(\mu - 1)^2}. \tag{20}$$

From the above derivation, it is clear that this is a unique solution (in case (B)).

If $t > \frac{1}{n(\mu-1)^2}$ (in case (A))), we have from (19) when $x_{ik} = 0$, for all i, k,

$$(\frac{1}{n})\frac{\partial T_{ik}}{\partial x_{ik}} = t - \frac{1}{n(\mu - 1)^2} > 0, \quad \text{for all } i, k. \tag{21}$$

Considering that $\frac{\partial T_{ik}}{\partial x_{ik}}$ is monotonically increasing with x_{ik}, we have that $\tilde{x}_{ik} = 0$, for every i, k, is a class optimal solution.

We can easily see the uniqueness as in the following. Suppose $\tilde{x}_{1k} > 0$ for some k. From definitions on d and by (16) we have then

$$(\frac{1}{n})\frac{\partial T_{1k}}{\partial x_{1k}} = -\frac{\mu - \frac{n-1}{n} + d - \tilde{x}_{1k}}{(\mu - 1 + d)^2} + \frac{\mu - 1 - d + \tilde{x}_{1k}}{(\mu - 1 - d)^2} + t = 0. \tag{22}$$

Then from the above and condition on t we have

$$\begin{aligned}
\{\frac{1}{(\mu - 1 + d)^2} &+ \frac{1}{(\mu - 1 - d)^2})\}\tilde{x}_{1k} \\
&= -t - \frac{2d}{(\mu-1)^2 - d^2} + \frac{1}{n(\mu - 1 + d)^2} \\
&< \frac{1}{n(\mu - 1 + d)^2} - \frac{1}{n(\mu-1)^2} - \frac{2d}{(\mu-1)^2 - d^2}.
\end{aligned} \tag{23}$$

This implies $d < 0$ for which there must exist some nonzeo $x_{2k'}$. Then by using the argument similar to the above on $x_{2k'}$ we have $d > 0$, which is a contradiction. Thus $\tilde{\boldsymbol{x}} = \boldsymbol{0}$ is the unique class optimal solution.

For the proofs of the existence and uniqueness of those optima for more general setting, see [1, 7, 16]. □

Consider the case (B). We can easily see that $T_{ik}(\tilde{\boldsymbol{x}})(= T(\tilde{\boldsymbol{x}}))$, for every i, k, has its maximum $\tilde{T}(\mu, n)$ (i.e. the worst performance) for given μ, n.

$$\tilde{T}(\mu, n) = \frac{1}{\mu - 1}\{1 + \frac{1}{8n(\mu - 1)}\}, \tag{24}$$

when

$$t = \frac{1}{2n(\mu - 1)^2}. \tag{25}$$

Thus if we add the communication lines with delay t $(= 1/\{2n(\mu-1)^2\})$ to the system that has had no communication means, the response time of each class $T_{ik}(\tilde{\boldsymbol{x}})$ increases in the amount of $\dfrac{1}{8n(\mu - 1)^2}$ (i.e. the performance degrades). This is a Braess-like paradox. We define the *worst ratio of the performance degradation* $\Delta(\mu, n)$ in the paradox for given μ, n to be

$$\Delta(\mu, n) = \frac{\tilde{T}(\mu, n) - T_0(\mu)}{T_0(\mu)}, \tag{26}$$

where $T_0(\mu) = 1/(\mu - 1)$ denotes the mean response time of each class jobs for given μ when the system has no communication means. Then we have

$$\Delta(\mu, n) = \frac{1}{8n(\mu - 1)}. \tag{27}$$

Remark 3. 1 From the above we see that, there occurs no forwarding of jobs in the overall and individual optima, and in the class optimum of the case (A). That is, On the other hand, in the class optimum with the case (B) parameters, each class forwards a part of its jobs through the communication means to the other server for remote processing, and thereby has degradation in its mean response time. *The ratio of such degradation can become unlimitedly large* as the total arrival rate approaches the processing capacity of each server. As the number of classes ($2n$) increases up to infinity, the ratio of degradation and the chances of the paradox decrease and finally disappear.

4 Numerical Examples

For example, we examine the following case: $\mu = 1.01$. Then the mean response time is $T_0(\mu) = 1/(\mu - 1) = 100$ in the overall optimum, in the individual optimum (Wardrop equilibrium), and in the case of no communication line and no forwarding of jobs.

Firstly, we consider the case where $n = 1$, i.e., the number of classes is 2. The mean response time of the class optimum (Nash equilibrium) for various values of t is shown in Fig. 2.

As we can see from the figure, $T = T_{ik}$ takes its maximum value

$$\tilde{T}(\mu, n) = 1350 \text{ (see(24))}$$

and the worst ratio of the performance degradation $\Delta(\mu, n)$ in the paradox is

$$\Delta(\mu, n) = 12.50 \text{ (i.e. 1250\% degradation)} \quad \text{(see (26))}$$

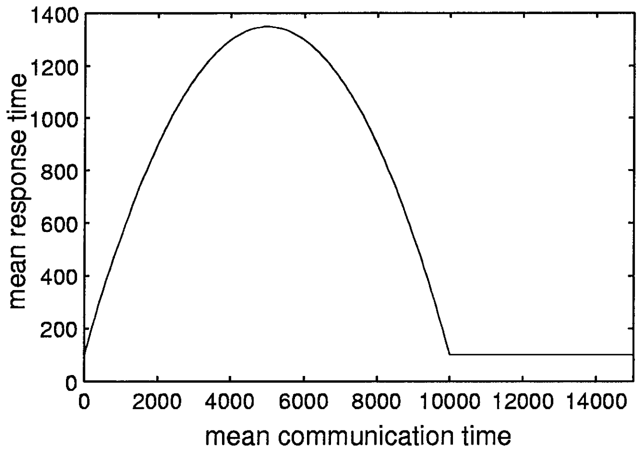

Figure 2: The mean response times for each class jobs in the class optima (or Nash equilibria) with $\mu = 1.01$ and $n = 1$ for the various values of mean communication time t. We see that, in the worst paradoxical case, adding the communication means with $t = 5000$ to the system increases the mean response time up to 1350 from 100, that of no communication means.

when $t = 1/\{2(\mu - 1)^2\} = 5000$ (see (25)). Then

$$\tilde{x}_{1k} = \tilde{x}_{2k} = (1/2)\{1 - t(\mu - 1)^2\} = 1/4 \ (k = 1) \quad \text{(see (14))}.$$

In this case, $\tilde{x}_{1k} = \tilde{x}_{2k}$ decrease from 1/2 down to 0 as t increases from 0 to 10000 ($= 1/(\mu - 1)^2$), and for $t > 10000$, no fowarding of jobs occurs.

It is amazing that each class keeps to forward a part of its jobs to the other server even though the communication delay for forwarding is much greater than the processing delay at the server at which its jobs arrive.

Then we consider the case where $n = 100$, i.e., the number of classes is 200.

The mean response time of the class optimum (Nash equilibrium) for various values of t is shown in Fig. 3.

As we can see from the figure, $T = T_{ik}$ takes its maximum value

$$\tilde{T}(\mu, n) = 112.5 \ \text{(see(24))}$$

and the worst ratio of the performance degradation $\Delta(\mu, n)$ in the paradox is

$$\Delta(\mu, n) = 0.125 \text{ (i.e. 12.50\% degradation)} \quad \text{(see (26))}$$

when $t = 1/\{2n(\mu - 1)^2\} = 50$ (see (25)). Then

$$\tilde{x}_{1k} = \tilde{x}_{2k} = (1/2)\{1/n - t(\mu - 1)^2\} = 1/400 \qquad \text{for all } k \text{ (see (14))}.$$

In this case, $\tilde{x}_{1k} = \tilde{x}_{2k}$ decrease from 1/200 down to 0 as t increases from 0 to 100 ($= 1/(\mu - 1)^2$), and for $t > 100$, no fowarding of jobs occurs.

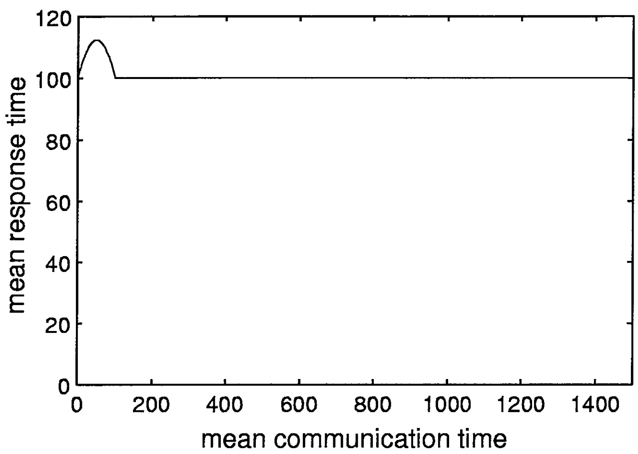

Figure 3: The mean response times for each class jobs in the class optima (or Nash equilibria) with $\mu = 1.01$ and $n = 100$ for the various values of mean communication time t. We see that, in the worst paradoxical case, adding the communication means with $t = 50$ to the system increases the mean response time up to only 112.5 from 100, that of no communication means.

Thus we see that the chances of the paradox and the magnitude in the degradation of the performance in the paradox are greatly reduced from the case of $n = 1$.

Furthermore we consider other values of μ with $n = 1$.
For $\mu = 1.001$, $\Delta(\mu, n) = 125$ (i.e. 12500% degradation), and
for $\mu = 1.00001$, $\Delta(\mu, n) = 12500$ (i.e. 1250000% degradation), etc.
In this way, we see that the worst ratio of the performance degradation $\Delta(\mu, n)$ in the paradox becomes unlimitedly large as μ approaches 1 with $n = 1$.

5 Concluding Remarks

In this paper, we examined the model consisting of two symmetrical servers with identical arrivals to both servers where forwarding of jobs to the other servers through communication means with nonzero delays may clearly lead to performance degradation. We confirmed that in the overall optimization and in the individual optimization (Wardrop equilibrium) such forwarding never occurs. We showed that in some parameter setting of the class optimization (Nash non-cooperative equilibrium) mutual forwarding of jobs for remote processing through communication means incurring positive time delays definitely occurs and the ratio of the performance degradation becomes unlimitedly large.

Such a paradoxical behavior does never occur for the overall and Wardrop optimum in the same setting of this symmetrical two-server model. That may imply that the Nash equilibrium may have more complicated characteristics than the overall optimum and even the Wardrop equilibrium where the Braess paradox may occur.

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