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OPTIMUM PRICING OF INDISPENSABLE RAW MATERIAL UNDER MONOPOLY A TWO STAGE GAME APPROACH

by

Hiroshi Konno

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INSTITUTE
OF
ELECTRONICS AND INFORMATION SCIENCE

UNIVERSITY OF TSUKUBA

Optimum Pricing of Indispensable Raw Material Under Monopoly
A Two Stage Game Approach

H. Konno

1. Introduction

This is a preliminary study on the mathematical modeling of the conflict between two groups of countries G_A which are abundant in several kinds of raw materials like oil, uranium, copper, etc. and its counterpart G_I, the group of industrialized countries producing goods for people of G_A as well as for those of G_I using these raw materials.

G_I wants to minimize the total expenses to meet the demands for goods given the price of raw materials, while G_A wants to maximize their total net income consisting of the following three components:

(i) direct income from the sale of raw materials

(ii) dividend income from the money invested into the industries of G_I

(iii) the expenses for importing the goods produced in G_I.

It is assumed that the dominating amount of raw materials are produced in G_A, so that G_A is entitled to put an arbitrary price on raw materials within some specified range. However, the net income is not necessarily an increasing function of these prices. For example, if the price is beyond some level then item (i) and (ii) could decrease due to the decrease in the activity level of the industry as a result of the decline in the demand for final products.

Our aim here is to find out the level of price optimum from the viewpoint of G_A and the optimal level of industrial activities for G_I given the price of raw materials.

In section 2, we will describe the general model for multi-raw material problem in the framework of two-stage game, first introduced by G.B. Dantzig in [1] and later investigated by the author [3]. In section 3 we will analyze this model in some detail and in section 4 some direction of future research will be briefly touched upon.

2. The Model

We will describe here the game structure of the model

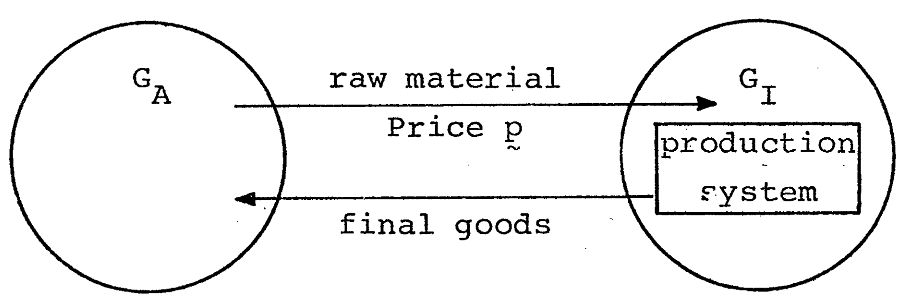


Figure 2.1

i) Players:

There are two groups of players (countries) G_A and G_I.

ii) Production of Raw Materials:

G_A is producing K different kinds of raw materials R_k, $k = 1,2,...,K$, while G_I is producing relatively small amount of these materials. Hence G_A has the control over the price p_k of R_k, $k = 1,\ldots,K$ as long as it satisfies $p_k^{min} \leq p_k \leq p_k^{max}$, where p_k^{min} and p_k^{max} are some positive constants determined by physical or political consideration.

iii) Demand Structure for the Industrial Products:

G_I is producing m kinds of commodities C_i, $i = 1,\ldots,m$ while G_A can produce them very little. Let $b_i^I(\underset{\sim}{p})$ and $b_i^A(\underset{\sim}{p})$ be the demands for C_i in G_I and G_A, respectively, given the price vector $\underset{\sim}{p} = (p_1,\ldots,p_K)^t$ of raw materials. We will assume throughout that $b_i^A(\cdot)$ and $b_i^I(\cdot)$ are continuous, nonnegative functions of p_k for all i and k. Given $b_i^I(\underset{\sim}{p})$ and $b_i^A(\underset{\sim}{p})$, G_I wants to minimize their total net expenses to meet these demands.

iv) Production Mechanism in G_I:

Let us assume here that there are n_i different activities A_{ij}, $j = 1,\ldots,n_i$ to produce C_i and let a_{ijr} be the amount of C_r produced when A_{ij} is operated at a unit level $a_{ijr} > 0$ implies that C_r is produced, while $a_{ijr} < 0$ implies that it is consumed). Let

$$\underset{\sim}{a}_{ij} = (a_{ij1}, a_{ij2}, \ldots, a_{ijm})^t$$

and let

$$A = (\underset{\sim}{a}_{11}, \underset{\sim}{a}_{12}, \ldots, \underset{\sim}{a}_{1n_1}, \underset{\sim}{a}_{21}, \ldots, \underset{\sim}{a}_{mn_m}) \quad .$$

We will assume in the sequel that A is a rectangular Leontief matrix (with substitution), i.e.

(a) $a_{iji} > 0 \quad , \quad a_{ijr} \leq 0 \quad , \quad r \neq i$ for all i and j

(b) $\{\underset{\sim}{y} = (y_{11},\ldots,y_{1n_1}y_{21},\ldots,y_{mn_m})^t \,|\, A\underset{\sim}{y} \geqq \underset{\sim}{b}, \underset{\sim}{y} \geq \underset{\sim}{0}\} \neq \phi$

for some $\underset{\sim}{b} > \underset{\sim}{0}$, where $n = \sum_{i=1}^{m} n_i$ and $\underset{\sim}{0} = (0,0,\ldots,0)^t$.

Hence A has the following structure:

$$A = \left[\begin{array}{cccc|cccc|c|cccc}
\underset{\sim}{a}_{11} & \underset{\sim}{a}_{12} & & \underset{\sim}{a}_{1n_1} & \underset{\sim}{a}_{21} & \underset{\sim}{a}_{22} & & \underset{\sim}{a}_{2n_2} & & \underset{\sim}{a}_{m1} & \underset{\sim}{a}_{m2} & & \underset{\sim}{a}_{mn_m} \\
+ & + & \cdots & + & \ominus & \ominus & \cdots & \ominus & \cdot\cdot & \ominus & \ominus & \cdots & \ominus \\
\ominus & \ominus & \cdots & \ominus & + & + & \cdots & + & \cdot\cdot & \ominus & \ominus & \cdots & \ominus \\
\ominus & \ominus & \cdots & \ominus & \ominus & \ominus & \cdots & \ominus & \cdot\cdot & \vdots & \vdots & & \vdots \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & \ominus & \ominus & \cdots & \ominus \\
\ominus & \ominus & \cdots & \ominus & \ominus & \ominus & \cdots & \ominus & \cdot\cdot & + & + & \cdots & +
\end{array}\right]$$

sector 1 sector 2 sector m

Figure 2.2

where + and $\ominus$ represent positive and non-positive entries, respectively. We will also assume that the unit cost $d_{ij}(\underset{\sim}{p})$ of activity A_{ij} when it is operated at a unit level is a continuous non-negative, non-decreasing function of p_k, $k = 1,\ldots,K$ for all i,j.

v) Consumption of Raw Materials in G_I:

We will assume that the consumption $z_k(\underset{\sim}{y})$ of raw material R_k is a linear function of $\underset{\sim}{y}$, i.e.,

$$z_k = (\lambda^k)^t y \equiv \sum_{i=1}^{m} \sum_{j=1}^{n_i} \lambda_{ij}^k y_{ij} \quad , \qquad k = 1,\ldots,K$$

out of which z_{ko}, $k = 1,\ldots,K$ are accounted for by the domestic production within G_I.

vi) Decision Structure of G_A:

When G_A fixes the price p_k of raw material R_k, $k = 1,\ldots,K$, they will get three different kinds of incomes. Firstly they get the direct income I_1 from the sale of raw materials, which is given by

$$I_1 = \sum_{k=1}^{K} p_k |z_k - z_{ko}|_+$$

where $|\cdot|_+$ is a function defined below:

$$|u|_+ = \begin{cases} u & \text{if } u \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

The second one is the dividend income I_2 from the money G_A has invested into the industries of G_I. Let the amount of money invested into the i^{th} sector (the sector producing C_i) be q_i. We will assume here that the dividend offered by C_i is proportional to the product of q_i and $\sum_{j=1}^{n_i} y_{ij}$, i.e.

$$I_2 = \sum_{i=1}^{m} \alpha_i q_i \sum_{j=1}^{n_i} y_{ij} \quad .$$

Thirdly, G_A has to pay the price E for the import of industrial products C_i, $i = 1,\ldots,m$. Let us assume here that the unit price of C_i is proportional to the production cost of C_i i.e.,

$$E = \sum_{i=1}^{m} b_i^A(\underset{\sim}{p})(1+\mu_i) \sum_{j=1}^{n_i} d_{ij}(\underset{\sim}{p}) \times (y_{ij}/\sum_{j=1}^{n_i} y_{ij}) \quad .$$

The total net income I of G_A is given by

$$I_A = I_1 + I_2 - E \quad ,$$

which they want to maximize by the appropriate choice of $\underset{\sim}{p}$.

vii) Decision Structure of G_I:

Given the price $\underset{\sim}{p}$ of raw materials, G_I naturally wants to minimize their total expenditure E_I, which is given by

$$E_I = \sum_{i=1}^{m} \sum_{j=1}^{n_i} d_{ij}(\underset{\sim}{p}) y_{ij}$$

while satisfying the final demand $b_i^A(\underset{\sim}{p}) + b_i^I(\underset{\sim}{p})$, $i = 1,\ldots,m$.

viii) Structure of the Game between G_A and G_I:

As we have already stated implicitly, G_A plays first in the game fixing the price $\underset{\sim}{p}$ of raw materials and G_I plays second by choosing activity vector $\underset{\sim}{y}$. G_A wants to maximize his total income while G_I wants to minimize total net expenses. We summarize the notation below:

G_A: group of countries producing raw materials

G_I: group of industrialized countries

R_k: k^{th} raw material, $k = 1,\ldots,K$

C_i: i^{th} industrial goods, $i = 1,\ldots,m$

p_k: unit price of R_k

p_k^{max}: upper bound of p_k

p_k^{min}: lower bound of p_k

$\underset{\sim}{p}$, $\underset{\sim}{p}^{max}$, $\underset{\sim}{p}^{min}$: K dimensional price vectors whose k^{th} components are p_k, p_k^{max}, p_k^{min}, respectively

$b_i^I(\underset{\sim}{p})$: final demand for C_i in G_I for given $\underset{\sim}{p}$

$b_i^A(\underset{\sim}{p})$: final demand for C_i in G_A for given $\underset{\sim}{p}$

$\underset{\sim}{b}^I(\underset{\sim}{p})$, $\underset{\sim}{b}^A(\underset{\sim}{p})$: M dimensional demand vectors whose i^{th} components are $b_i^I(\underset{\sim}{p})$, $b_i^A(\underset{\sim}{p})$, respectively

A_{ij}: j^{th} activity to produce C_i

a_{ijr}: amount of C_r produced by A_{ij} when it is operated at a unit level

$\underset{\sim}{a}_{ij}$: activity vector corresponding to A_{ij}, i.e., $\underset{\sim}{a}_{ij} = (a_{ij1},\ldots,a_{ijm})^t$

A: input output matrix of G_I, i.e., $A = (\underset{\sim}{a}_{11}, \underset{\sim}{a}_{1n_1}, \underset{\sim}{a}_{21},\ldots,a_{mn_m})$

$d_{ij}(\underset{\sim}{p})$: cost of A_{ij} when operated at a unit level

y_{ij}: activity level of A_{ij}

$\underset{\sim}{y}$: activity vector, i.e. $\underset{\sim}{y} = (y_{11},\ldots,y_{1n_1},\ldots,y_{nm_n})^t$

λ_{ij}^k: consumption of R_k when A_{ij} is operated at a unit level

$z_k(\underset{\sim}{y})$: consumption of R_k given the activity level $\underset{\sim}{y}$

z_{ko}: amount of R_k produced in G_I

q_i: amount of money G_A has invested into the i^{th} sector in G_I

α_i: dividend rate of i^{th} sector in G_I

μ_i: profit rate of i^{th} sector

p: $p = \{\underset{\sim}{p} = (p_1,\ldots,p_K)^t \mid p_k^{min} \leq p_k \leq p_k^{max},\ k = 1,\ldots,K\}$

3. Mathematical Formulation and Its Analysis

a. Mathematical Formulation

It is quite straightforward to formulate the problems to be solved by G_A and G_A in mathematical terms under the assumptions of the previous section.

$P_I(\underset{\sim}{p})$: G_I's problem for given $\underset{\sim}{p}$

$$P_I(\underset{\sim}{p}): \begin{cases} \text{minimize} & \underset{\sim}{d}(\underset{\sim}{p})^t \underset{\sim}{y} \\ \text{s.t.} & A\underset{\sim}{y} \geq \underset{\sim}{b}(\underset{\sim}{p}) \\ & \underset{\sim}{y} \geq \underset{\sim}{0} \end{cases}$$

where $\underset{\sim}{b}(\underset{\sim}{p}) = \underset{\sim}{b}^A(\underset{\sim}{p}) + \underset{\sim}{b}^I(\underset{\sim}{p})$. This is a standard linear programming problem which will be shown to have an optimal solution $\underset{\sim}{y}^*(\underset{\sim}{p})$ for all $p_k \in [p_k^{min}, p_k^{max}]$ under our assumption on $\underset{\sim}{b}^A(\underset{\sim}{p})$, $\underset{\sim}{b}^I(\underset{\sim}{p})$, $\underset{\sim}{d}(\underset{\sim}{p})$ and A (see Theorem 1 of the next section.)

P_A: G_A's problem

$$P_A: \begin{cases} \text{maximize } f(\underset{\sim}{p}) = \sum_{k=1}^{K} p_k \left| \sum_{i=1}^{m} \sum_{j=1}^{n_i} \lambda_{ij}^{k} y_{ij}^{*}(\underset{\sim}{p}) - z_{ko} \right| + \\ \qquad + \sum_{i=1}^{m} \alpha_i q_i \sum_{j=1}^{n_i} y_{ij}^{*}(\underset{\sim}{p}) \\ \qquad - \sum_{i=1}^{m} b_i^A(\underset{\sim}{p})(1+\mu_1) \sum_{j=1}^{n_i} d_{ij}(\underset{\sim}{p}) \times \dfrac{y_{ij}^{*}(\underset{\sim}{p})}{\sum_{j=1}^{n_i} y_{ij}^{*}(\underset{\sim}{p})} \\ \text{s.t.} \quad p_k^{min} \le p_k \le p_k^{max}, \quad k = 1,\dots,K . \end{cases}$$

where $y^*(\underset{\sim}{p})$ is an optimal solution of $P_I(\underset{\sim}{p})$. The objective function of P_A need not be concave. Also it need not be an increasing or decreasing function of p_k's.

b. Analysis of the General Case

We will first state the general results about $P_I(\underset{\sim}{p})$.

Theorem 1. Let $A \in R^{mxn}$ be a rectangular Leontief matrix. Associated with a linear program:

$$\min\{\underset{\sim}{d}^t \underset{\sim}{y} \mid A\underset{\sim}{y} \ge \underset{\sim}{b}, y \ge \underset{\sim}{0}\} \tag{3.1}$$

where $\underset{\sim}{b} \ge \underset{\sim}{0}$, there exists an optimal basis B with the following properties:

i) B contains exactly one column of A corresponding to each sector i. Hence by the appropriate permutation of columns, B can be transformed to satisfy $b_{ii} > 0$, $b_{ij} \leq 0$, $j \neq i$, for all i.

ii) $B^{-1} \geq 0$.

In particular, B is optimal for all the right hand side vector $\underset{\sim}{b} \geq \underset{\sim}{0}$. Also every feasible basis of (3.1) has the structure described in (i).

Proof. See [2].

Schematically, $B_{\hat{p}}$ looks as follows:

$$\begin{array}{cc} & \begin{array}{ccc} \underset{\sim}{a}_{1j_1}, & \underset{\sim}{a}_{2j_2}, \ldots, & \underset{\sim}{a}_{mj_m} \end{array} \\ B_{\hat{p}} = & \begin{bmatrix} + & \ominus & & \ominus \\ \ominus & + & & \vdots \\ \ominus & \ominus & & \\ \vdots & \vdots & & \ominus \\ \ominus & \ominus & & + \end{bmatrix} \end{array}$$

Figure 3.1

where $\underset{\sim}{a}_{ij_i}$ represents the unique basic column corresponding to the ith sector. It is a very nice property of a Leontief system that the feasible basis always has the structure described in Figure 3.1 with appropriate permutation of columns so that once we know the incoming column in the simplex algorithm, the dropping column is automatically determined as the one having the positive component in the same row as the incoming column.

Theorem 2. Let $B_{\hat{p}}$ be an optimal basis associated with $P_I(\hat{p})$. Then there is a closed set $S_{\hat{p}}$ containing $\hat{p}$ for which $B_{\hat{p}}$ is optimal for all $p \in S_{\hat{p}}$.

Proof. By Theorem 1 (i), $B_{\hat{p}}^{-1} \geq 0$. Also $b(p) \geq 0$ for all $p \in P$, so that $B_{\hat{p}}$ is feasible for all $p \in P$. Therefore, $B_{\hat{p}}$ is optimal for p satisfying

$$\bar{d}^t(p) \equiv d^t(p)_N - d^t(p)_B B_{\hat{p}}^{-1} N_{\hat{p}} \geq 0 \quad ,$$

i.e., for all p for which the reduced cost $\bar{d}^t(p)$ is non-negative. Here, $d(p)_B$ and $d(p)_N$ are the subvectors of $d(p)$ corresponding to basic and non-basic variables, respectively and $N_{\hat{p}}$ is the submatrix of A consisting of non-basic columns. Theorem follows since $\bar{d}(p)$ is a continuous function of p and $\bar{d}(\hat{p}) \geq 0$ by the optimality of $B_{\hat{p}}$. ||

c. Linear Case

Let us now specialize to the case in which $b^A(p)$, $b^I(p)$ and $d(p)$ are linear, i.e.

$$(3.2) \quad \begin{cases} b^A(p) = b^A + E_A p \\ b^I(p) = b^I + E_I p \end{cases}$$

$$(3.3) \quad d(p) = d_o + D_p \quad .$$

Corollary 3. If $b^A(\cdot)$ and $b^I(\cdot)$ are linear functions of p as specified by (3.2), then the objective function $f(p)$ of P_A is a piecewise quadratic function of p.

Proof. Follows from the expression

$$(y^*_{1j_1}(\underset{\sim}{p})\ y^*_{2j_2}(\underset{\sim}{p}),\ldots,y^*_{mj_m}(\underset{\sim}{p}))^t = B_{\hat{p}}^{-1}(\underset{\sim}{b}^A + \underset{\sim}{b}^I + E\underset{\sim}{p}),\quad \underset{\sim}{p} \in P(\hat{p})$$

$$y^*_{ij}(\underset{\sim}{p}) = 0 \quad , \qquad j \neq j_i \quad , \qquad i = 1,\ldots,m \quad ,$$

where j_i is the unique basic variable belonging to the i^{th} sector and $E = E_A + E_I$. ||

Let $\underset{\sim}{\beta}^i_{\hat{p}}$ be the i^{th} row of the matrix $B_{\hat{p}}^{-1}$, then the problem we have to solve is a family of quadratic programming problems:

$$\max f(\underset{\sim}{p}) = \sum_{k=1}^{K} p_k \left| \sum_{i=1}^{m} \lambda^k_{ij_i}\ \underset{\sim}{\beta}^i_{\hat{p}}(\underset{\sim}{b} + E\underset{\sim}{p}) - z_{ko} \right| +$$

$$+ \sum_{i=1}^{m} \alpha_i q_i \underset{\sim}{\beta}^i_{\hat{p}}(\underset{\sim}{b} + E\underset{\sim}{p}) - \sum_{i=1}^{m} (1+\mu_i)(\underset{\sim}{d}+D\underset{\sim}{p})_{ij_i}(\underset{\sim}{b}^A+E_A\underset{\sim}{p})_i$$

$$\text{s.t.} \qquad \underset{\sim}{d}^t_N(\underset{\sim}{p}) - \underset{\sim}{d}^t_B(\underset{\sim}{p})B_{\hat{p}}^{-1}N_{\hat{p}} \geq 0$$

$$p_k^{min} \leq p_k \leq p_k^{max} \quad , \qquad k = 1,\ldots,K \quad , \tag{3.4}$$

where j_i is the index of unique basic variable corresponding to i^{th} sector. We used here the fact that $y^*_{ij}(\underset{\sim}{p}) = 0$, $j \neq j_i$ and hence that $y^*_{ij}(\underset{\sim}{p}) / \sum_{j=1}^{n_i} y^*_{ij}(\underset{\sim}{p}) = 1$ for all i. The objective function of (3.4) need not be concave and it is not an easy task to solve it for large K. Hence we will now concentrate to some of the simplest cases.

d. Single Raw Material Case

Let us now consider the single raw material case, i.e., K = 1. In this case p is a scalar and we can construct a very efficient algorithm. Let us reproduce $P_I(p)$:

$$P_I(p) \quad \begin{cases} \text{minimize} & \underset{\sim}{d}^t(p)\underset{\sim}{y} \\ \text{s.t.} & A\underset{\sim}{y} \geq \underset{\sim}{b}(p) \\ & \underset{\sim}{y} \geq \underset{\sim}{0} \end{cases}$$

where p is now a scalar. This problem can be solved by using the technique of parametric linear programming as follows.

Let $p_o \equiv p_{min}$ and let B_{p_o} be an optimal basis for $P_I(p_o)$. Then B_{p_o} is optimal for all p satisfying

$$\bar{d}_N(p) \equiv d_N^t(p) - d_B^t(p)B_{p_o}^{-1}N_{p_o} \geq 0 \quad , \tag{3.5}$$

where $d_B(p)$ is a subvector of $d(p)$ corresponding to B_{p_o} etc. Note that

$$d_N^t(p_o) - d_B^t(p_o)B_{p_o}^{-1}N_{p_o} \geq 0$$

by the optimality of B_{p_o} for $p = p_o$. Hence by solving this inequality we will get a set of closed intervals $I_{ot} = [\underline{p}_{ot}, \bar{p}_{ot}]$, $t = 0,1,\ldots,T_o$.

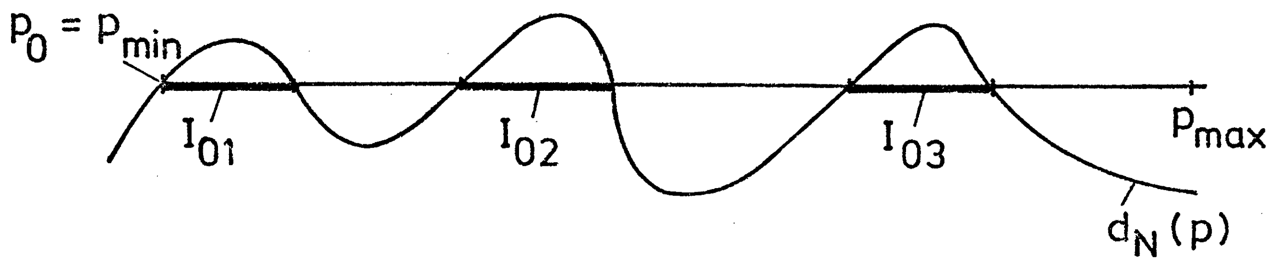


Figure 3.2

Let $\hat{p}_{ot}$ be an optimal solution of the problem

$$\max F_{p_o}(p) = p\left|\sum_{i=1}^{m} \lambda_{ij_i}\beta^{i}_{pt}b(p) - z_o\right|_+ + \sum_{i=1}^{m} \alpha_i q_i \beta^{i}_{p_t} b(p)$$

$$- \sum_{i=1}^{m} (1+\mu_i) d_{ij_i}(p) b^{A}_{i}(p) \qquad (3.6)$$

$$\text{s.t.} \quad \underline{p}_{ot} \le p \le \overline{p}_{ot}$$

and let

$$\max \{F_{p_o}(\hat{p}_{o1}),\ldots,F_{p_o}(\hat{p}_{oT_o})\} = F(\hat{p}_o) \quad .$$

When p_o moves beyond the endpoint of these intervals I_{ot}, the inequality (3.5) is violated and we get another optimal basis (note that the basis change rule for Leontief substitution system is quite simple once the incoming vector is determined). We will continue this process until the entire interval $[p_{min}, p_{max}]$ is covered. This is of course a finite process. The best among all p_j's is certainly the best solution. Also (3.6) is a one dimensional optimization problem and can be solved by any one of the search methods. In particular, if d(p) is linear i.e., if

$$d(p) = \underset{\sim}{d} + p\underset{\sim}{f}$$

then (3.5) reduces to

$$p(\underset{\sim}{f}_N^t - \underset{\sim}{\sigma}_{p_o}^t N_{p_o}) \geq \underset{\sim}{\pi}_{p_o}^t N_{p_o} - \underset{\sim}{d}_N$$

where $\underset{\sim}{\pi}^t = \underset{\sim}{d}_B^t B_{p_o}^{-1}$ and $\underset{\sim}{\sigma}^t = \underset{\sim}{f}_B^t B_{p_o}^{-1}$ and B_{p_o} is optimal for all $p \in [p_o, p_1]$ where

$$p_1 = \max_j \left[\frac{d_{Nj} - \underset{\sim}{\pi}_{p_o}^t N_{p_o j}}{\underset{\sim}{\sigma}_{p_o}^t N_{p_o j} - f_{Nj}} \quad \underset{\sim}{\sigma}_{p_o}^t N_{p_o j} - f_{Nj} > 0 \right] .$$

Moreover, if b(p) is linear and non-increasing, then the objective function of $P_A(p_t)$ is a piecewise concave quadratic function and can be solved by inspection. Let $\hat{p}_t$ be the optimal solution for $P_A(p_t)$. Then the optimal solution for P_A exists among $\hat{p}_t$, $t = 0,1,\ldots,T$ where T is the first index for t for which $p_t \geq p_{max}$.

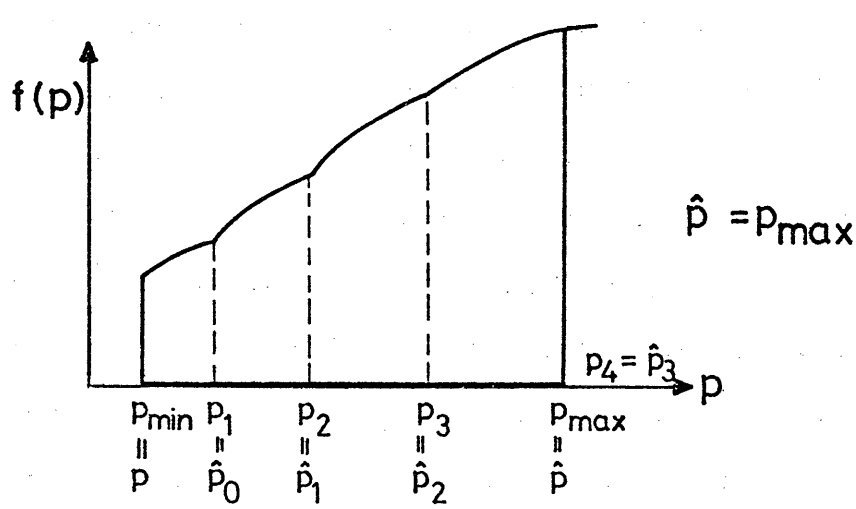


Figure 3.3(a)

Figure 3.3(b)

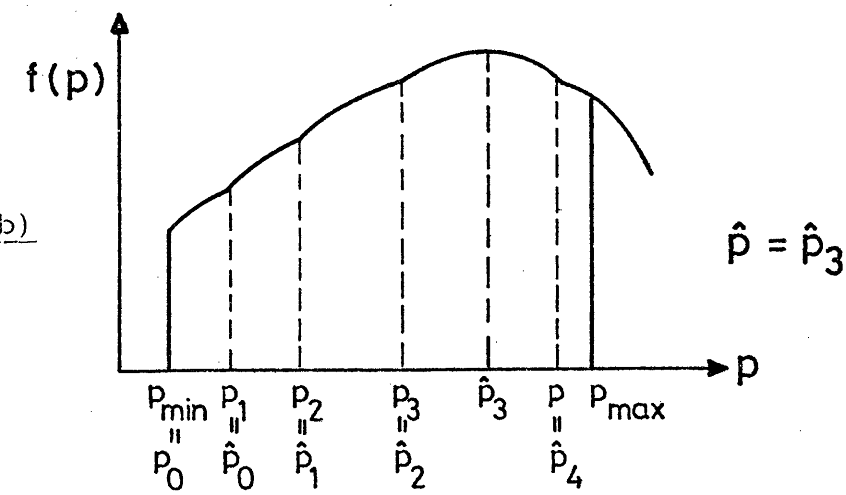
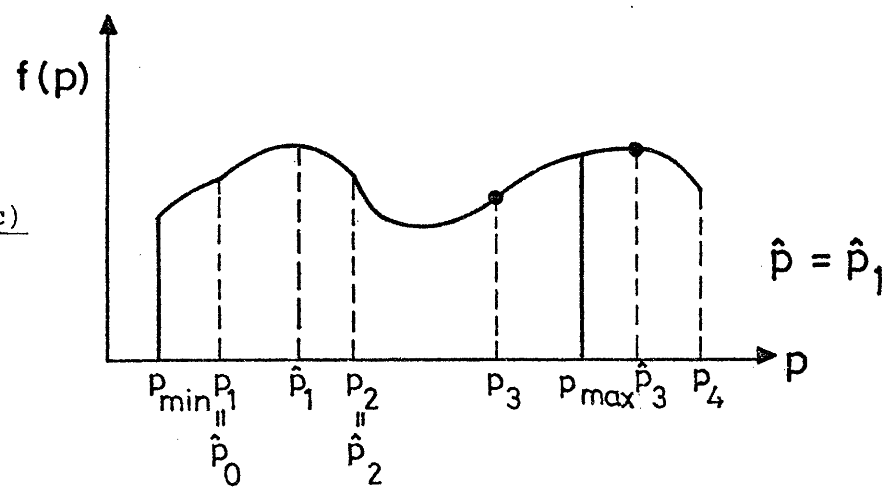


Figure 3.3(c)



Figures 3.3(a)~(c) illustrate the typical shape of the objective function. Typically if the investments q_i, $i = 1,\ldots,m$ are small, the shape will be more like Figure 3.3(a). On the other hand, if q_i are fairly large, then there will be a peak to the left of p_{max} as in Figure 3.3(b). Also f(p) need neither be unimodular nor concave and could have the shape of Figure 3.3(c). The shape of f(p) depends not only on q_i, $i = 1,\ldots,m$, but also on μ_i and $b_i(\underset{\sim}{p})$, $i = 1,\ldots,m$. The important thing to note is that the shape of $f(\underset{\sim}{p})$ can be like Figure 3.3(b), so that the optimum price $\hat{p}$ is less than the specified upper bound under certain circumstances.

4. Concluding Remarks

The model developed in this paper is admittedly quite primitive and needs more elaboration.

First of all, we assumed that the industrialized countries behave like a gentleman even under the strong aggression of G_A. G_I can, of course, take a counter attack by choosing the parameters, e.g., μ_i, $i = 1,\ldots,m$. This possibility opens a brand new dimension on the game structure (i.e., the multi-stage game).

Secondly, we assumed that the price p_k can be chosen independently of each other as long as $p_k^{min} \leq p_k \leq p_k^{max}$, $k = 1,\ldots,K$. It may be, however, in some cases that there is an interdependence between these prices. If this interdependence relation is linear then the problem (3.4) will essentially remain the same, so that we can find an optimum level of $\underset{\sim}{p}$ if we can solve a (non-convex) quadratic program (3.4).

Thirdly, under what condition $f(\underset{\sim}{p})$ has the shape described in Figure 3.3(b) is an interesting question.

The research in these directions will be reported subsequently.

REFERENCES

[1] Dantzig, G.B., *Solving Two-Move Games with Perfect Information*, RAND Report P-1459, The RAND Corporation, Santa Monica, California, 1958.

[2] Koeler, G.J., et al., *Optimization Over Leontief Substitution Systems*, North Holland, New York, 1975.

[3] Konno, H., *Applications of Bilinear Programming*, internal paper, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1975.

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AUTHOR(s)

Hiroshi Konno (Institute of Electronics and Information Sciences)

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two stage game, non-symmetric game, Leontief substitution system, parametric nonlinear programming

ABSTRACT

The conflict between two groups of countries G_A (resourse producing countries) and G_I (industrialized countries) is formulated in the framework of so-called two stage games. The objective of G_I is to minimize the total expenses to meet the demands for goods given the price of raw materials fixed by G_A, while G_A wants to maximize their total net income which consists of (i) direct income from the sale of raw material to G_I, (ii) dividend income from the investment into G_I and (iii) expenses for the import of goods from G_I. An efficient algorithm to compute optimal strategies (optimum price level and production level) is developed under the assumption of the Leontief type production system of G_I.

SUPPLEMENTARY NOTES